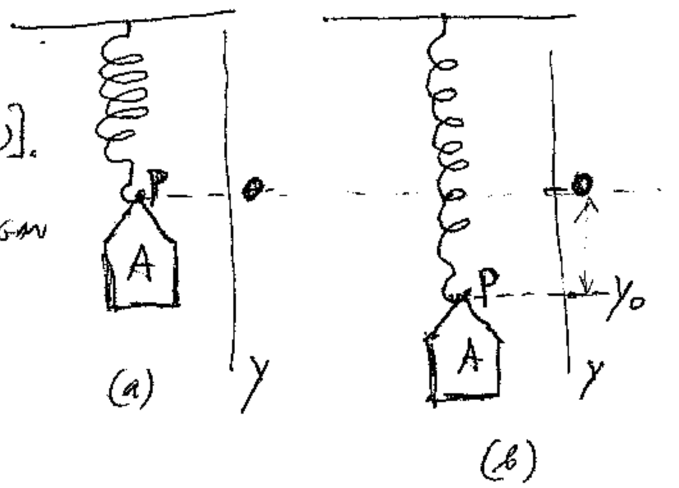
PENGGUNAAN PERSAMAAN DIFERENSIAL ORDE 2

- MENENTUKAN PERSAMAAN GERAK PEGAS (DENGAN ATAU TANPA REDAMAN)
- MENENTUKAN PERSAMAAN MUATAN DAN ARUS PADA RANGKAIAN LISTRIK R-L-C.

PEGAS BERGETAR / BEROSILASI

DIKETAHUI SEBUAH PEGAS DIGANTUNG SECARA VERTIKAL, DAN DIBEBANI SUATU OBJEK A [Gbr (a)]. PEGAS TERSEBUT DITARIK SEJAUH y_0 SATUAN DI BAWAH TITIK KESETIMBANGAN-NYA [Gbr (b)], LALU DILEPAS DENGAN KECEPATAN AWAL V_0. MAKA PEGAS AKAN BEROSILASI.



PEGAS BERGETAR / BEROSILASI

JIKA GESEKAN DENGAN UDARA DIABAIKAN, MAKA MENURUT HUKUM HOOKE, GAYA F YANG CENDERUNG MENGEMBALIKAN TITIK UJUNG PEGAS (P) KE TITIK KESETIMBANGANNYA (O) AKAN SEBANDING DENGAN SIMPANGANNYA, YAKNI

$$\boxed{F = -ky},$$

DENGAN $k > 0$ KONSTANTA PEGAS DAN y MENYATAKAN SIMPANGAN PEGAS (JARAK P DARI O)

DALAM HAL INI, y MERUPAKAN FUNGSI DARI WAKTU (t).

MENURUT HUKUM II NEWTON, $F = m.a$

$w = m.g \rightarrow \boxed{m = \frac{w}{g}} \Rightarrow F = \left(\frac{w}{g}\right).a$,

DENGAN m = massa objek A, w = berat objek A, a = percepatan titik P, DAN g = KONSTANTA PERCEPATAN AKIBAT GRAVITASI.

JADI $\boxed{F = m.a = -k.y}$

$$\boxed{\frac{w}{g} \cdot \frac{d^2y}{dt^2} = -k.y}$$

SOLUSINYA : $\boxed{y = y(t)}$, HARUS MEMENUHI SYARAT AWAL $y(0) = y_0$ DAN $y'(0) = 0$

$$\frac{w}{g} \cdot \frac{d^2y}{dt^2} = -k.y \quad \left(\frac{g}{w}\right)$$

$$\frac{g}{w} \cdot \frac{w}{g} \cdot \frac{d^2y}{dt^2} = \frac{g}{w} \cdot (-ky)$$

$$\frac{d^2y}{dt^2} = \frac{1}{m}(-k.y)$$

$$\frac{d^2y}{dt^2} = -\frac{k}{m} \cdot y$$

$$\boxed{\frac{d^2y}{dt^2} + \left(\frac{k}{m}\right) \cdot y = 0}$$

$w = m.g$

$\boxed{\frac{1}{m} = \frac{g}{w}}$

JIKA MISALKAN $\boxed{B^2 = \frac{k}{m}}$, MAKA PDB AKAN

MENJADI $\boxed{\frac{d^2y}{dt^2} + B^2 y = 0}$

$$D^2y + B^2y = 0$$

$$(D^2 + B^2).y = 0$$

PERS. EIGEN

$$\alpha^2 + B^2 = 0$$

$$\alpha^2 = -B^2$$

$$\alpha_{1,2} = \pm\sqrt{-B^2} = \pm B\sqrt{-1}$$

$$\alpha_{1,2} = 0 \pm B\,i$$

$a = 0$ $\quad$ $b = B$

SOLUSI UMUM PDB :

$$y = e^{at}(\underset{\downarrow \atop c_1}{A} \cos bt + \underset{\downarrow \atop c_2}{B} \sin bt)$$

$$\boxed{y = e^{ax}(c_1 \cos bt + c_2 \sin bt)}$$

$$y(t) = \underset{1}{e^{0t}}(c_1 \cos Bt + c_2 \sin Bt)$$

$$y(t) = c_1 . \cos Bt + c_2 . \sin Bt$$

JIKA $\boxed{y(0) = y_0}$

$$y(0) = c_1 \cos B(0) + c_2 \sin B(0) = y_0$$

$$c_1 \underset{1}{\cos 0} + c_2 \underset{0}{\sin 0} = y_0$$

$$\boxed{c_1 = y_0}$$

$$\longrightarrow y'(t) = -c_1 . B . \sin Bt + c_2 . B \cos Bt$$

JIKA $\boxed{y'(0) = 0}$

$$y'(0) = -c_1 B . \sin B(0) + c_2 . B \cos B(0) = 0$$

$$-c_1 B \underset{0}{\sin 0} + c_2 B \underset{1}{\cos 0} = 0$$

$$c_2 B = 0$$

$$\boxed{c_2 = 0}$$

SOLUSI PDB :

$$y = c_1 . \cos Bt + c_2 . \sin Bt$$

$$y = y_0 \cos Bt + \underline{0 . \sin Bt}$$

$$\boxed{y = y_0 . \cos Bt}$$

DALAM HAL INI PEGAS BEROSILASI DENGAN AMPLITUDO y_0 DAN PERIODE $\frac{2\pi}{B}$ (TIDAK KEMBALI KE POSISI SETIMBANG).

PEGAS BEROSILASI TEREDAM

JIKA PEGAS MENGALAMI GESEKAN SEBANDING DENGAN KECEPATAN $\frac{dy}{dt}$, MAKA PERSAMAAN GERAK PEGAS TERSEBUT MENJADI :

$$\boxed{\frac{W}{g} \cdot \frac{d^2y}{dt^2} = -K \cdot y - \underbrace{q \cdot \frac{dy}{dt}}_{\text{GAYA GESEKAN}}}$$

$$\frac{W}{g} \cdot \frac{d^2y}{dt^2} + K \cdot y + q \cdot \frac{dy}{dt} = 0$$

$$\frac{W}{g} \cdot \frac{d^2y}{dt^2} + q \cdot \frac{dy}{dt} + K \cdot y = 0 \quad \cdot \left(\frac{g}{W}\right)$$

$$\frac{g}{W} \cdot \frac{W}{g} \cdot \frac{d^2y}{dt^2} + \frac{g}{W} \cdot q \cdot \frac{dy}{dt} + \frac{g}{W} \cdot K \cdot y = 0$$

$W = m \cdot g$

$\boxed{\frac{1}{m} = \frac{g}{W}}$

$$\frac{d^2y}{dt^2} + \frac{1}{m} \cdot q \cdot \frac{dy}{dt} + \frac{1}{m} \cdot K \cdot y = 0$$

$$\boxed{\frac{d^2y}{dt^2} + \underbrace{\frac{q}{m}}_{E} \cdot \frac{dy}{dt} + \underbrace{\frac{K}{m}}_{B^2} \cdot y = 0}$$

$$\Longrightarrow \frac{d^2y}{dt^2} + E \cdot \frac{dy}{dt} + B^2 y = 0$$

di mana $\boxed{E = \frac{q}{m}}$ DAN $\boxed{B^2 = \frac{K}{m}}$

$$D^2y + E\,Dy + B^2 y = 0$$

$$(D^2 + E D + B^2) \cdot y = 0$$

PERS. EIGEN

$$\alpha^2 + E\alpha + B^2 = 0$$

$a = 1$ $\quad b = E \quad$ $c = B^2$

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha_{1,2} = \frac{-E \pm \sqrt{E^2 - 4(1)B^2}}{2(1)}$$

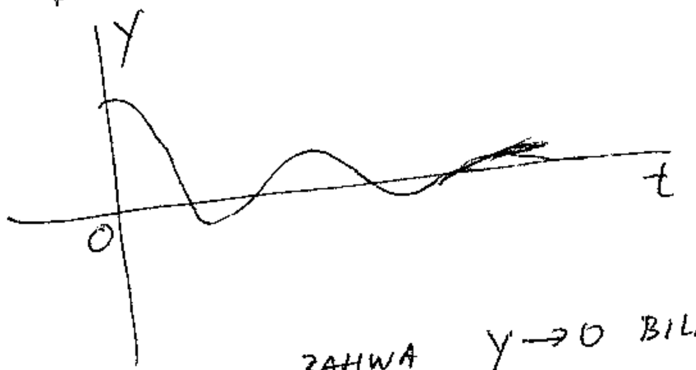
$$\boxed{\alpha_{1,2} = \frac{-E \pm \sqrt{E^2 - 4B^2}}{2}}$$

<u>KASUS 1</u> : $\boxed{E^2 - 4B^2 < 0}$

DALAM KASUS INI, PERSAMAAN ~~KARAK~~ EIGEN MEMPUNYAI 2 AKAR KOMPLEKS $\rightarrow$ $\boxed{\alpha_{1,2} = -a \pm bi}$,

DAN SOLUSI UMUM PDB-NYA :

$$\boxed{y = e^{-at}(c_1 \cos bt + c_2 \sin bt)}$$



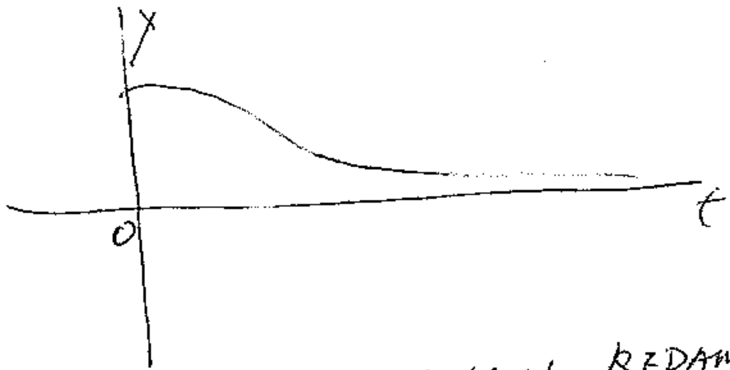
PERHATIKAN BAHWA $y \rightarrow 0$ BILA $t \rightarrow \infty$.

KASUS 2 : $\boxed{E^2 - 4B^2 = 0}$

DALAM KASUS INI, PERSAMAAN EIGEN MEMPUNYAI 1 AKAR REAL KEMBAR $\longrightarrow$ $\boxed{\alpha_{1,2} = -\frac{E}{2}} = -a$,

DAN SOLUSI UMUM PDB-NYA :

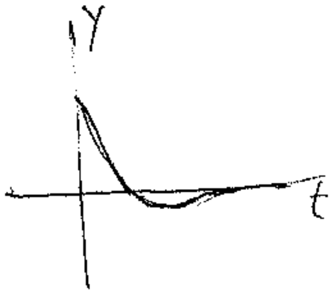
$$\boxed{\begin{aligned} y &= (c_1 + c_2 t)\, e^{-at} \\ y &= c_1 e^{-at} + c_2 . t . e^{-at} \end{aligned}}$$



DI SINI PEGAS MENGALAMI REDAMAN KRITIS.

KASUS 3 : $\boxed{E^2 - 4B^2 > 0}$

DALAM KASUS INI, PERSAMAAN EIGEN MEMPUNYAI 2 AKAR REAL BERBEDA $\longrightarrow$ $\alpha_1 = -a_1$ DAN $\alpha_2 = -a_2$ (dua-duanya bernilai negatif) DAN SOLUSI UMUM PDB-NYA :



$$\boxed{y(t) = c_1 e^{-a_1 t} + c_2 e^{-a_2 t}}$$

$\boxed{y(0) = y_0} \longrightarrow y(0) = c_1 e^0 + c_2 e^0 = y_0$

$$\boxed{c_1 + c_2 = y_0}$$

$y'(t) = -a_1 c_1 e^{-a_1 t} - a_2 c_2 e^{-a_2 t}$

$\boxed{y'(0) = 0} \longrightarrow y'(0) = -a_1 c_1 e^0 - a_2 c_2 e^0 = 0$

$$\boxed{-a_1 c_1 - a_2 c_2 = 0}$$

$$-a_1 e_1 - a_2 e_2 = 0$$

$$-a_2 e_2 = a_1 e_1$$

$$e_2 = \frac{a_1 e_1}{-a_2} \longrightarrow \boxed{e_2 = -\frac{a_1}{a_2} e_1}$$

$$e_1 + e_2 = y_0$$

$$e_1 - \frac{a_1}{a_2} e_1 = y_0 \quad (a_2)$$

$$a_2 e_1 - a_1 e_1 = a_2 y_0$$

$$(a_2 - a_1) e_1 = a_2 y_0 \longrightarrow \boxed{e_1 = \frac{a_2 y_0}{a_2 - a_1}}$$

TITIK POTONG DENGAN Sb.t $\longrightarrow y(t) = 0$.

$$y(t) = e_1 e^{-a_1 t} + e_2 e^{-a_2 t}$$

$$0 = e_1 e^{-a_1 t} + e_2 e^{-a_2 t}$$

$$e_1 e^{-a_1 t} = -e_2 e^{-a_2 t}$$

$$\frac{e^{-a_1 t}}{e^{-a_2 t}} = -\frac{e_2}{e_1}$$

$$e^{(a_2 - a_1)t} = -\frac{e_2}{e_1}$$

$$t = \ldots \quad (\text{ada})$$

CONTOH / LATIHAN

SEBUAH PEGAS DENGAN KONSTANTA PEGAS $K = 10$ DIGANTUNG DENGAN BEBAN BERMASSA $m = 2{,}5$ (SATUAN K DAN m TELAH DISESUAIKAN).
JIKA BEBAN TSB DITARIK KE BAWAH SEJAUH 5 cm DARI POSISI SETIMBANG DAN KEMUDIAN DILEPASKAN, TENTUKAN SIMPANGAN PEGAS TSB SETIAP SAAT, APABILA

a) PEGAS TIDAK MENGALAMI GESEKAN

b) PEGAS MENGALAMI GESEKAN DENGAN FAKTOR REDAMAN $\lambda = 0{,}2$.

---

a) PEGAS TIDAK MENGALAMI GESEKAN

$$\boxed{\frac{d^2y}{dt^2} + \frac{k}{m} \cdot y = 0}$$

$$\frac{d^2y}{dt^2} + \frac{10}{2{,}5} \cdot y = 0$$

$$\frac{d^2y}{dt^2} + 4y = 0$$

$$D^2y + 4y = 0$$

$$(D^2 + 4) \cdot y = 0$$

Pers eigen

$$\alpha^2 + 4 = 0$$

$$\alpha^2 = -4$$

$$\alpha_{1,2} = \pm\sqrt{-4} = \pm 2\sqrt{-1}$$

$$\alpha_{1,2} = 0 \pm 2i$$

SOLUSI UMUM PDB nya:

$$y(t) = e^{at}(C_1 \cos bt + C_2 \sin bt)$$

$$y(t) = e^{0t}(C_1 \cos 2t + C_2 \sin 2t).$$

$$\boxed{y(t) = c_1 \cos 2t + c_2 \sin 2t}$$

$y(0) = 5$ cm

$$\rightarrow y(0) = c_1 \underbrace{\cos 0}_{1} + c_2 \underbrace{\sin 0}_{0} = 5$$

$$\boxed{c_1 = 5}$$

$$y'(t) = -2c_1 \sin 2t + 2c_2 \cos 2t$$

$$y'(0) = -2c_1 \underbrace{\sin 0}_{0} + 2c_2 \cos(0) = 0$$

$$2c_2 \underbrace{\cos 0}_{1} = 0$$

$$\boxed{c_2 = 0}$$

SOLUSI PDB-NYA

$$y(t) = c_1 \cos 2t + c_2 \sin 2t$$

$$y(t) = 5 \cos 2t + \underbrace{0 \cdot \sin 2t}$$

$$\boxed{y(t) = 5 \cos 2t}$$

b) PEGAS MENGALAMI GESEKAN DENGAN FAKTOR REDAMAN (10)

$\eta = 0,2$

$$E = \frac{\eta}{m} = \frac{0,2}{2,5} = \frac{2}{25} \times \frac{4}{4} = \frac{8}{100} = 0,08$$

$$B^2 = \frac{k}{m} = \frac{10}{2,5} = 4$$

$$\boxed{\frac{d^2y}{dt^2} + E \cdot \frac{dy}{dt} + B^2 \cdot y = 0}$$

$$\frac{d^2y}{dt^2} + 0,08 \frac{dy}{dt} + 4y = 0$$

$$D^2 y + 0,08\, Dy + 4y = 0$$

$$(D^2 + 0,08\, D + 4) \cdot y = 0$$

PERS. EIGEN

$$\alpha^2 + 0,08\,\alpha + 4 = 0$$

$a = 1$, $b = 0,08$, $c = 4$

$$\alpha_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha_{1,2} = \frac{-0,08 \pm \sqrt{(0,08)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-0,08 \pm \sqrt{0,0064 - 16}}{2}$$

$$= \frac{-0,08}{2} \pm \frac{\sqrt{(16 - 0,0064)(-1)}}{2}$$

$$\approx -0,04 \pm \frac{i}{2}(4)$$

$$\alpha_{1,2} \approx -0,04 \pm 2i$$

$a = -0,04$ $\quad b = 2$

SOLUSI UMUM PDB

$$y = e^{at}(c_1 \cos bt + c_2 \sin bt)$$

$$\boxed{y = e^{-0,04t}(c_1 \cos 2t + c_2 \sin 2t)}$$

$\boxed{y(0)=5} \longrightarrow y(0) = \underset{1}{e^0}\left(c_1 \cdot \underset{1}{\cos 0} + c_2 \underset{0}{\sin 0}\right) = 5$ (11)

$$\boxed{c_1 = 5}$$

$$y(t) = e^{-0,04t}\left(c_1 \cdot \cos 2t + c_2 \sin 2t\right)$$

$$\longrightarrow y'(t) = -0,04 \cdot e^{-0,04t}\left(c_1 \cdot \cos 2t + c_2 \cdot \sin 2t\right) + e^{-0,04t}\left(-2c_1 \sin 2t + 2c_2 \cdot \cos 2t\right)$$

$\boxed{y'(0)=0}$

$$\longrightarrow y'(0) = -0,04 \cdot e^0\left(c_1 \underset{1}{\cos 0} + \cancel{c_2 \underset{0}{\sin 0}}\right) + e^0\left(\cancel{-2c_1 \underset{0}{\sin 0}} + 2c_2 \cdot \underset{1}{\cos 0}\right) = 0$$

$$-0,04\, c_1 + 2c_2 = 0$$

$$2c_2 = 0,04\, c_1$$

$$\boxed{c_2 = 0,02\, c_1}$$

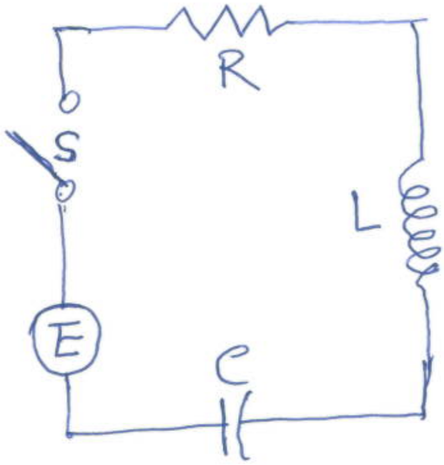
$$c_2 = 0,02\,(5)$$

$$\boxed{c_2 = 0,1}$$

SOLUSI PDB

$$\therefore \boxed{y(t) = e^{-0,04t}\left(\underbrace{5 \cos 2t + 0,1 \sin 2t}_{R \cdot \cos(2t - \alpha)}\right)}$$

RANGKAIAN LISTRIK R-L-C



PDB ORDE 2 JUGA MUNCUL PADA RANGKAIAN LISTRIK R-L-C. BERDASARKAN HUKUM KIRCHHOFF, MUATAN Q PADA KAPASITOR AKAN MEMENUHI PDB

$$\boxed{L \cdot \frac{d^2Q}{dt^2} + R \cdot \frac{dQ}{dt} + \frac{1}{C} \cdot Q = E(t)}$$

SEMENTARA ITU, ARUS $\boxed{I = \frac{dQ}{dt}}$,

MEMENUHI PDB

$$\boxed{L \cdot \frac{d^2I}{dt^2} + R \cdot \frac{dI}{dt} + \frac{1}{C} I = E(t)}$$

CONTOH / LATIHAN

DIKETAHUI RANGKAIAN LISTRIK R-L-C DENGAN $R=16$, $L=0{,}02$, $C=2\times10^{-4}$, DAN $E=20$ (SATUAN TELAH DISESUAIKAN).
TENTUKAN MUATAN DAN ARUS PADA RANGKAIAN TSB, SEBAGAI FUNGSI DARI WAKTU.
ASUMSIKAN BAHWA $Q=0$ DAN $I=0$ PADA SAAT $t=0$.

$$L\cdot\frac{d^2Q}{dt^2}+R\cdot\frac{dQ}{dt}+\frac{1}{C}\cdot Q=E(t)$$

$$0{,}02\cdot\frac{d^2Q}{dt^2}+16\cdot\frac{dQ}{dt}+\frac{1}{2\times10^{-4}}Q=20 \qquad \times(50)$$

$$\frac{d^2Q}{dt^2}+800\frac{dQ}{dt}+250.000\,Q=1000$$

PERS. EIGEN

$$\alpha^2+800\alpha+250.000=0$$

$$\alpha_{1,2}=\frac{-800\pm\sqrt{(800)^2-4(1)(250.000)}}{2(1)}$$

$$=\frac{-800\pm\sqrt{640.000-1.000.000}}{2}$$

$$=-400\pm300\,i$$

$a=-400$ $\qquad$ $b=300$

SOLUSI UMUM PDB

$$Q_h(t)=e^{at}(c_1\cos bt+c_2\sin bt)$$

$$Q_h(t)=e^{-400t}(c_1\cos300t+c_2\sin300t)$$

$$\frac{d^2Q}{dt^2} + 800\frac{dQ}{dt} + 250.000\,Q = 1000$$

$$D^2Q + 800\,DQ + 250.000\,Q = 1000$$

$$(D^2 + 800\,D + 250.000) \cdot Q = 1000$$

$$Q_p = \frac{1.000}{D^2 + 800\,D + 250.000}$$

$D(1000) = 0$

$D^2(1000) = 0$

$$Q_p = \frac{1}{250.000 + 800D + D^2}(1000) = \frac{1}{250.000}(1000) = \frac{1}{250} \times \frac{4}{4} = \frac{4}{1000} = 0{,}004.$$

$$250.000 + 800D + D^2 \,\Big/\, 1 \quad \frac{1}{250.000}$$

$$1 + \frac{800D}{250.000} + \frac{1}{250.000}D^2 -$$

$$\frac{8D}{250} + \frac{D^2}{250.000}$$

$$Q(t) = Q_h + Q_p$$

$$\boxed{Q(t) = e^{-400t}(c_1 \cos 300t + c_2 \sin 300t) + 0{,}004}$$

$$Q(0) = e^0(c_1 \underbrace{\cos 0}_{1} + c_2 \underbrace{\sin 0}_{0}) + 0{,}004 = 0$$

$$\boxed{c_1 = -0{,}004}$$

$$I = \frac{dQ}{dt} = -400\,e^{-400t}(c_1 \cdot \cos 300t + c_2 \sin 300t) + e^{-400t}(-300c_1 \sin 300t + 300c_2 \cos 300t) + 0$$

$$\boxed{I(t) = -400\,e^{-400t}(c_1 \cdot \cos 300t + c_2 \sin 300t) + e^{-400t}(-300c_1 \sin 300t + 300c_2 \cos 300t)}$$

$$I(0) = -400\,e^0(c_1 \underbrace{\cos 0}_{1} + c_2 \underbrace{\sin 0}_{0}) + e^0(-300c_1 \underbrace{\sin 0}_{0} + 300c_2 \underbrace{\cos 0}_{1}) = 0$$

$$-400\,c_1 + 300c_2 = 0 \longrightarrow 300c_2 = 400c_1$$

$$c_2 = \frac{4}{3}c_1$$

$$c_2 = \frac{4}{3} \cdot (-0{,}004)$$

$$\boxed{c_2 = \frac{-0{,}016}{3}}$$

$$Q(t) = e^{-400t}\left(c_1 \cos 300t + c_2 \sin 300t\right) + 0{,}004$$

$$\boxed{Q(t) = e^{-400t}\left(-0{,}004 \cos 300t - \frac{0{,}016}{3} \sin 300t\right) + 0{,}004}$$

$$I(t) = -400\, e^{-400t}\left(c_1 \cos 300t + c_2 \sin 300t\right) + e^{-400t}\left(-300 c_1 \cdot \sin 300t + 300\, c_2 \cos 300t\right)$$

$$\boxed{I(t) = -400\, e^{-400t}\left(-0{,}004 \cdot \cos 300t - \frac{0{,}016}{3} \sin 300t\right) + e^{-400t}\left(-300(-0{,}004) \sin 300t + 300\left(-\frac{0{,}016}{3}\right) \cos 300t\right)}$$