TRANSFORMASI LAPLACE DARI INTEGRAL

* PEMBAGIAN DENGAN t

JIKA $L\{F(t)\} = f(s)$, MAKA

$$\boxed{L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(u)\,du}$$

BUKTI :

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\,dt = f(s)$$

$$\longrightarrow \quad f(s) = \int_0^\infty e^{-st} \cdot F(t)\,dt$$

$$\boxed{f(u) = \int_0^\infty e^{-ut} \cdot F(t)\,dt}$$

$$\int_s^\infty f(u)\,du = \int_{u=s}^\infty \left[\int_{t=0}^\infty e^{-ut} \cdot F(t)\,dt\right] \cdot du$$

$$= \int_{t=0}^\infty \int_{u=s}^\infty e^{-ut} F(t) \cdot du \cdot dt$$

$$= \int_{t=0}^\infty F(t)\,dt \left[\int_{u=s}^\infty e^{-ut}\,du\right]$$

$$= \int_{t=0}^\infty F(t)\,dt \left[\frac{1}{-t} \lim_{p\to\infty} \int_s^p e^{-ut} \cdot \frac{(-t)\,du}{d(-ut)}\right]$$

$$= \int_{t=0}^\infty F(t)\,dt \left(-\frac{1}{t}\right) \cdot \lim_{p\to\infty} \left[e^{-ut}\Big|_s^p\right]$$

$$= \int_{t=0}^\infty F(t)\,dt \left(-\frac{1}{t}\right) \left[\lim_{p\to\infty} \frac{1}{e^{pt}} - \lim_{p\to\infty} \frac{1}{e^{st}}\right]$$

$$= \int_{t=0}^\infty F(t)\,dt \left(-\frac{1}{t}\right) \left[\underbrace{\lim_{p\to\infty} \frac{1}{e^{\infty t}}}_{0} - e^{-st}\right]$$

$$\int_s^\infty f(u)\,du = \int_0^\infty e^{-st} \cdot \frac{F(t)}{t}\,dt$$

$$\int_s^\infty f(u)\,du = L\left\{\frac{F(t)}{t}\right\} \longrightarrow \boxed{L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(u)\,du}$$