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FINDING A PARTICULAR SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS WITH BALANCED METHOD

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Abstract. This research is conducted to obtain a new method in finding the particular solution for an Ordinary Differential Equation. Until now, the method in finding the particular solution of an Ordinary Differential Equations have not changed significantly in literature. In this talk, we propose a new method called the Balanced Method, in which the sum of its parts on the left should be the same as part on the right.

Key words and Phrases: Particular Solution, Ordinary Differential Equation, Balanced Method

1. Introduction

This research is conducted to obtain a new method in finding the particular solution for an Ordinary Differential Equation. Until now, the method in finding the particular solution of an Ordinary Differential Equations have not changed significantly in literature. In this talk, we propose a new method called the Balanced Method, in which the sum of its parts on the left should be the same as part on the right.

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2. Results and Discussion

To illustrate the method, let us consider the following example in finding the particular solution of :

(1)

$$\frac{dy}{dx} + 2y = e^{3x}$$

In order to find the particular solution, divide the equation into two parts as follow :

$2y$	$\frac{dy}{dx}$
e^{3x}	$\frac{3}{2}e^{3x}$
$-\frac{3}{2}e^{3x}$	$-\frac{9}{4}e^{3x}$
$\frac{9}{4}e^{3x}$	$\frac{27}{8}e^{3x}$
$-\frac{27}{8}e^{3x}$	$\frac{81}{16}e^{3x}$

The left column starts with the non homogenous function, which means

$$y = e^{3x}$$

, and then move to the right column by differentiating and dividing by two in order to obtain the differential. And then move to the left column in the new row with changing the last result. Do this procedure up to the pattern of the series is clear.

Finally the particular solution is :

$$2y = \left(1 - \frac{3}{2} + \frac{9}{4} - \frac{27}{8} + \frac{81}{16} - \ldots\ldots\right)e^{3x}$$

$$2y = \left(\frac{1}{1+\frac{3}{2}}\right)e^{3x}$$

So Particular Solution is : $y = \frac{1}{5}e^{3x}$

Note :

$(1 + x - x^2 + x^3 - x^4 + x^5 - \ldots\ldots = \frac{1}{1+x})$

(2)

$$\frac{dy}{dx} + 2y = sin2x$$

In order to find the particular solution, divide the equation into two parts as follow :

Settlement of the case to 2y = sin (2x)

2y	$\frac{dy}{dx}$
sin(2x)	cos(2x)
-cos(2x)	sin(2x)
-sin(2x)	-cos(2x)
cos(2x)	-sin(2x)

Settlement of the case for dy / dx = sin (2x)

$\frac{dy}{dx}$	2y
sin(2x)	-cos(2x)
cos(2x)	sin(2x)
-sin(2x)	cos(2x)
-cos(2x)	-sin(2x)

For Settlement of the case to 2y = sin (2x),

2y = sin(2x)-cos(2x)-sin(2x)+cos(2x)+sin(2x)- ...

$y = \frac{1}{2}[sin(2x) - cos(2x) - sin(2x) + cos(2x) + sin(2x) - ...]$

$y = \frac{1}{2}[1 - 1 + 1 - 1 + ...]sin(2x) - \frac{1}{2}[1 - 1 + 1 - 1 + ...]cos(2x)$

$y = \frac{1}{2}[\frac{1}{1+1}]sin(2x) - \frac{1}{2}[\frac{1}{1+1}]cos(2x)$

$y = \frac{1}{4}sin(2x) - \frac{1}{4}cos(2x)$

For Settlement of the case for dy / dx = sin (2x),

2y = -cos(2x)+sin(2x)+cos(2x)-sin(2x)-cos(2x)+sin(2x)+cos(2x)+...
$y = \frac{1}{2}[-cos(2x) + sin(2x) + cos(2x) - sin(2x) - cos(2x) + sin(2x) + cos(2x) + ...]$
$y = \frac{1}{2}[-cos(2x) + cos(2x) - cos(2x) + cos(2x) - ...] + \frac{1}{2}[sin(2x) - sin(2x) + sin(2x) - sin(2x) + ...]$
$y = -\frac{1}{2}[\frac{1}{1+1}]cos(2x) + \frac{1}{2}[\frac{1}{1+1}]sin(2x)$
$y = -\frac{1}{4}cos(2x) + \frac{1}{4}sin(2x)$

Both are equal.

3. Simultaneous Differential Equations Settlement

Simultaneous Differential Equations settlement can be done by the same method as follows:

$$\frac{dx}{dt} - y = e^{2t} \text{ , } \frac{dy}{dt} + 4x = 0$$

Simultaneous Differential Equations of the particular settlement

$\frac{dx}{dt}$	$4x$	$\frac{dy}{dt}$	$-y$	$\frac{dx}{dt}$
e^{2t}				e^{2t}
	$+2e^{2t}$ →	$-2e^{2t}$ →	e^{2t} →	$-e^{2t}$ → to column (2)
	$-2e^{2t}$ →	$+2e^{2t}$ →	$-e^{2t}$ →	$+e^{2t}$ → to column (2)
	$+2e^{2t}$ →	$-2e^{2t}$ →	$+e^{2t}$ →	$-e^{2t}$ → and so on

From column (2) obtained

4x = $2e^{2t} - 2e^{2t} + 2e^{2t} - ...$
$x = \frac{1}{2}[\frac{1}{91+1}]e^{2t}$
$x = \frac{1}{4}e^{2t}$

$-y = e^{2t} - e^{2t} + e^{2t} - ...$
$-y = (1 - 1 + 1 - ...)e^{2t}$
$-y = [\frac{1}{1+1}]e^{2t} = \frac{1}{2}e^{2t}$
$y = -\frac{1}{2}e^{2t}$

3. Concluding Remarks

With Balance Method can determine the particular solution of an Ordinary Differential Equation.

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