# The Benefits of Bezier, B-Spline Algorithm in the Industry World

Djunaidy Santoso and Wikaria Gazali, *Member, IAENG*

***Abstract*—Computer Graphic has many applications in various fields such as industries, corporates, banking, and healthcare (hospital). Advanced computer graphic with latest technologies are widely used in strategic industries like aviation, marine, and automotive for solving issues related to images. Using similar methodologies, image analysis can be done to produce more accurate results with better quality (for example CAD or CAM) which will be useful for doctors and patients in hospitals, architects and civil engineers, or other decision makers. The objective of this research is to combine algorithms to produce accurate images with better quality which can be used for decision making based on selected curve. This is performed by associating the Hermite, Bezier, and B-Spline algorithms to its harmonic functions so that the resulting curves / images becomes better and well formed. This research will produce a picture/image/shape that will serve the needs of the designers.**

***Index Terms*— Computer Graphics, image analysis, algorithms, harmonic functions, curves , images, CAD / CAM**

## I. Introduction

With more algorithms in the field of computer graphics, more time will be required to decide and determine the appropriate algorithms to use. Therefore a research is necessary to determine the best algorithms to be used when dealing with curves and need to be according to the users needs. Problem Identification : From the industry's or other institution's statistical data, many curves are generated however the curves are not smooth, such that it can not be used by the industry/organization. [1][5][6][7]

### *Specific Objectives*

This research is developed with the objectives and benefits as follows:

To develop a curve/image for automotive industries (machinery), ships and aeroplanes and building designs.

To analyze and develop well-shaped curves or to develop a smooth curve from images obtained from sattelites (GIS).

To be used in the fields of statistics, hospitals, and other industries.

### *Advanced Research Urgency*

Designers working in industries nowadays need high quality images/pictures/shapes to accomplish their tasks. Therefore it is necessary to have a special method to handle the images by utilizing technology. This can be done in the form of a software package to produce images/pictures/shapes in their best quality to meet users' requirement (for example using AutoCad, Microsoft word, Paintbrush, 3D Max, etc.)[1][7]. In this case, for this research, it is necessary to have the accuracy, precision, and the smoothness of the curves/images/pictures/shapes which is according to the user's requirements[1][4][5][6][7]. Therefore this research is very important.

## II. Research Methodology

As described earlier, this research will be conducted using the following methodology:

Literature studies, is performed by searching data and information on the internet. This is conducted in order to get the latest and most updated information regarding the methods and model in this research, and also to study the existing and currently used methods. Data analysis is performed to analyze all the collected data. System development is performed to obtain the user's requirements and the system requirements. System implementation is performed to create and develop a model/simulation using a software/application which is developed by writing a program. Testing dan evaluating the system. Testing is conducted using the system application and the developed model, checking wether it is according to the user's requirements.

### *Research Plan Schedule*

*First Step: Creating the design mode*

The first step is to perform a literature study, data and information searching, from books or the internet, in order to get the latest information on the development of this method and the research model, and also to study the existing model. Then searching for actual information and facts from other institution. The data collected [3] empirically is used as a means to develop the model and the relationships among data sources, in terms of the advantages of using Bezier algorithm and B-Spline[5][6].

Djunaidy Santoso is with the School of Computer Science of Bina Nusantara University, Indonesia ( e-mail: djunaidys0533@binus.ac.id ).
Wikaria Gazali is with the School of Computer Science of Bina Nusantara University, Indonesia (e-mail: wikaria@binus.edu ).

*Second Step: Model Implemetation*

The model obtained from the survey is then implemented in a software that can be used by the institution or department in the areas surveyed. In addition to the software, a guide is also developed for users to facilitate users whom working with the software. Feedback from users are used as an evaluation to assess the implementation of the model for further refinement. Evaluation of the implementation is done by using a questionnaire to obtain feedback.

*Third Step: Model verification*

Verification of the early model equations are used and the results of a survey on the use of the software is developed. In order to study the design, the influence of the system software is analyzed. It includes the possibility of research in other domain of Soft-Computing. Main focus on graphics system will be placed around numbers, letters, pictures and objects which are commonly used as key factors in industry.

This research need to be developed further because it serves as a starting point. The end product using the algorithm will be beneficial and is widely needed by industries.

In order to design for the prototype, there is a need of System design, then a requirement of a reliable database system, and a convenient method of designing user interface, so that the output obtained can be fast, accurate and effective, and the software is developed with focus on user experience (easy to understand menus), thus is easy to digest. The ultimate goal is helping the decision making process by users. The development method is based on the system development life cycle, such that it will make it is easier for the developer to develop and maintain the system.

That all the points will study the statistics of the curve traversed either from the starting point to the end point, the only difference approximation with the Bezier curve and B-Spline is only the starting point and end point just passed. The point of the midpoint between the starting point and end point only averaged averaged only (on Lagrange Interpolation curve, Newton and Hermite and others in the area at the point of starting point and ending point point area strain curves are very steep). Another characteristic feature on Bezier curves and B-spline curve [5][6] that is when

desired curve becomes smooth tangent direction must be made sustainable and turning points of maximum and minimum turning points need to be considered very significant. The Formulation of BEZIER Function :

*That need to be investigated first Bezier functions:*

By taking the point $P_o$ = (2/6); $P_1$ = (4/12)

$P_2$ = (7/3) ; $P_3$ = (9/10)

Mean n = 3; after the binomial coefficients are calculated, the calculated blending function obtained

$b_{0,3}(t) = (1 - t)^3$

$b_{1,3}(t) = 3 * t * (1 - t)^2$

$b_{2,3}(t) = 3 * t^2 (1 - t)$

$b_{3,3}(t) = t^3$

Obtained after blending functions, the new calculated n Bezier polynomial function $P(t)=\Sigma B_i * J_{n,i}(t)$ $0 \le t \le 1$ i=0

$B_i$= control Points ; $J_{n,i}(t)= C(n,i)*t_i*(1-t)^{n-i}$

$J_{n,i}$=Blending function of Bezier $C(n,i)=n! /( i! * (n-i) !)$

Then obtained:

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.15) = (2.9586 / 7.7902)
$P_2$ (0.35) = (4386/8125)
$P_3$ (t = 0.5 = (5575/7625)
$P_4$ (t = 0.65) = (6625/7208))
$P_5$ (t = 0.85) = (8.0374 / 7.8242)
$P_6$ (t = 1) = (9/10)

*The second study B-Spline functions:*

With the same point for research Bezier functions: first calculate the value knotvektor and incorporated into the B-spline blending function obtained polynom B-spline functions are:

With order k = 2 obtained:

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.5) = (3/9)
$P_2$ (1) = (4/12)
$P_3$ (t = 1.5) = (5.5 / 7.5)
$P_4$ (t = 2.0) = (7/3))
$P_5$(t = 2.5) = (8.0 / 6.5)
$P_6$(t = 3.0) = (9/10)

With order k = 3 obtained:

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.5) = (3,875 / 9,375)
$P_2$ (t = 1) = (5.5 / 7.5)
$P_3$ (t = 1.5) = (7125/5875))
$P_4$ (t = 2.0) = (9/10)

With order k = 4 obtained:

$P_0$ (t = 0) = (2/6)
$P_1$(t=0.5) = ( 5.5 / 7.625)
$P_2$(t=1) = ( 9/ 10)

Results and Discussion about previous Research (Filing and Quoting)

From the calculation of Bezier function

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.15) = (2.9586 / 7.7902)
$P_2$ (0.35) = (4386/8125)
$P_3$ (t = 0.5 = (5575/7625)
$P_4$ (t = 0.65) = (6625/7208))
$P_5$ (t = 0.85) = (8.0374 / 7.8242)
$P_6$ (t = 1) = (9/10)

*Discussion for Bezier:*

The first curve yield curve polygon and the second produces the same curve as the last curve of the B-Spline, the point $P_0$, $P_3$ and $P_6$ together with B-Spline curve of order k = 4 (always order the last k equals the number of points).

And the results of B-Spline function calculation by Order k = 2:

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.5) = (3/9)
$P_2$ (1) = (4/12)
$P_3$ (t = 1.5) = (5.5 / 7.5)
$P_4$ (t = 2.0) = (7/3)

$P_5$ (t = 2.5) = (8.0 / 6.5)
$P_6$ (t = 3.0) = (9/10)

Order k = 3:

$P_0$ (t = 0) = (2/6)
$P_1$ (t = 0.5) = (3,875 / 9,375)
$P_2$ (t = 1) = (5.5 / 7.5)
$P_3$ (t = 1.5) = (7125/5875))
$P_4$ (t = 2.0) = (9/10)

Order k=4 obtained :

$P_0$(t=0) = ( 2 / 6 )
$P_1$(t=0.5) = ( 5.5 / 7.625)
$P_2$(t=1) = ( 9/ 10)

*Discussion for B-Spline:*

Curve B is a B-spline Bernstein with special properties. This base is non-global, so each point of Bi relation with a different basis functions. So that every point of the curve affects only in areas with base parameter values where the function is not zero

Basically the general requirements of B-Spline curve is almost equal to the requirements of the Bezier curve. For the B-spline curve of order k (degree k-1) each point of the curve should be in the convex hull of k adjacent point. Then the whole point of the B-Spline curve must be in join from around the convex hull formed by k control points. In the figure below are shown the effects of differences in the value of k. Note that k = 2 convex hull is a polygon defining itself. Here B-Spline curve is generated is also a defining polygon itself.

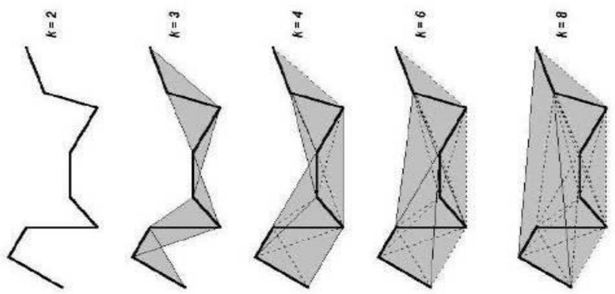


Figure 1. Curves of Polygon depent on k

B-Spline method is the development of Bezier methods, but differ with Bezier curves, on the B-Spline curve control points influence the shape of a curve as a whole. Change one of the existing control point coordinates will only change the shape of the curve on the segment near the control point. This is because the segment of the curve is only affected by a number of control points which was near Mathematical equation B-Spline curve with a parameter defined as follows:

Number of B-Spline function for some value of parameter 1 to be in the show: Ref. [5], [6]

$$\sum_{i=1}^{n+1} N_{i,k}(t) = blending\_\ function \qquad (1)$$

$$P(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t)\ , \quad t_{\min} \le t < t_{\max} \qquad (2)$$

$$2 \le k \le n+1$$

With

$$N_{i,k}(t) = \begin{cases} 1\ , & x_i \le t < x_{i+1} \\ 0\ , & \text{otherwise} \end{cases} \qquad (3)$$

$$N_{i,k}(t) = \frac{(t - x_i) N_{i,k-1}(t)}{x_{i+k-1}} + \frac{(x_{i,k-t}) N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}} \qquad (4)$$

Description :

- Each basis function is positive or zero for all values of the parameters, namely Ni, k ≥ 0.
- Except for k = 1 each basis function has a maximum value of 1.
- The maximum level of the curve equals the number of control points polygon.
- Curve follows the shape of a polygon control point.
- Curve lies in the convex hull of the polygon points.

Open uniform knot vector B-spline, which has a number of points at the ends of the value equal to k levels of B-Spline functions. Some of the open uniform sample point with the vector k:
K = 2 [0 0 1 2 3 4 4]
K = 3 [0 0 0 1 2 3 3 3]
K = 4 [0 0 0 0 1 2 2 2 2]
For dependence of a triangular diagram is given as follows

$$\begin{array}{llllll} N_{i,k} \\ N_{i,k-1} & N_{i+1,k-1} \\ N_{i,k-2} & N_{i+1,k-2} & N_{i+2,k-2} \\ : & : & : \\ : & : & : & : \\ N_{i,1} & N_{i+1,1} & N_{i+2,1} & N_{i+3,1} & : \quad : & N_{i+k-1,1} \end{array} \qquad (5)$$

For N(1,3) is given as the following diagram:

$$\begin{array}{llllll} N_{1,3} & N_{2,3} & N_{3,3} & N_{4,3} \\ N_{1,2} & N_{2,2} & N_{3,2} & N_{4,2} & N_{5,2} \\ N_{1,1} & N_{2,1} & N_{3,1} & N_{4,1} & N_{5,1} & N_{6,1} \end{array} \qquad (6)$$

Value of the required knots from 0 to n + k, while the number of the knots is n + k +1

*Examples of Questions*

Value of the required knots from 0 to n+k, while number of the knots is n+k+1

$0 \le t < 1$

$$\begin{array}{lll} N_{3,1}(t) = 1; & N_{i,1}(t) = 0\,, & \mathrm{i} \ne 3 \\ N_{2,2}(t) = 1 - t\,; & N_{3,2}(t) = t\,; & N_{i,2}(t) = 0\,, \quad \mathrm{i} \ne 2, 3 \\ N_{1,3}(t) = (1-t)^2\,; & N_{2,3}(t) = t(1-t)\dfrac{(2-t)}{2}t\,; & \\ N_{3,3}(t) = \dfrac{t^2}{2}\,; & N_{i,3}(t) = 0\,, & \mathrm{i} \ne 1, 2, 3 \end{array} \qquad (7)$$

$1 \le t < 2$

$$\begin{array}{lll} N_{2,1}(t) = 1; & N_{i,3}(t) = 0\,, & \mathrm{i} \ne 4 \\ N_{2,2}(t) = (2-t); & N_{4,2}(t) = (t-1); & N_{i,2}(t) = 0\,, \quad \mathrm{i} \ne 3, 4 \\ N_{2,3}(t) = \dfrac{(2-t)^2}{2}; & N_{2,3}(t) = t(1-t)\dfrac{(2-t)}{2}t\,; & \\ N_{4,2}(t) = (t-1)^2\,; & N_{i,3}(t) = 0\,, & \mathrm{i} \ne 2, 3, 4 \end{array} \qquad (8)$$


ISBN: 978-988-14047-3-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Calculation results can be seen in the following figure :

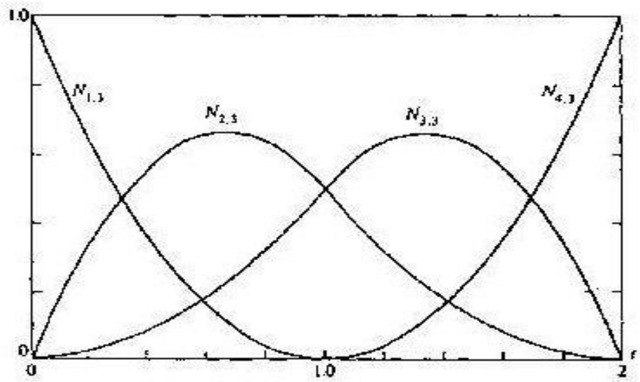


Figure 2. Calculation results

## III. B-SPLINE RESEARCH

At the same point as the research Bezier functions: first calculate knotvektor and put those values to the function of B-Spline blending and obtained polynom B-Spline function from a given point:

$P_0(2,6)$ $P_1(4,12)$ $P_2(7,3)$ $P_3(9,10)$

$P_0(t=0) = (2/6)$
$P_1(t=0.5) = (3/9)$
$P_2(t=1) = (4/12)$
$P_3(t=1.5) = (5.5/7.5)$
$P_4(t=2.0) = (7/3))$
$P_5(t=2.5) = (8.0/6.5)$
$P_6(t=3.0) = (9/10)$

With order k=3, we have :

$P_0(t=0) = (2/6)$
$P_1(t=0.5) = (3.875/9.375)$
$P_2(t=1) = (5.5/7.5)$
$P_3(t=1.5) = (7.125/5.875)$
$P_4(t=2.0) = (9/10)$

With order K=4, we have :

$P_0(t=0) = (2/6)$
$P_1(t=0.5) = (5.5/7.625)$
$P_2(t=1) = (9/10)$

*Curve Obtained :*

$P_0(2,6)$ $P_1(4,12)$ $P_2(7,3)$ $P_3(9,10)$

K = 2 form Polygons
K = 3 form curves like sine
K = 4 B-Spline curve shape is pure = Bezier curves.

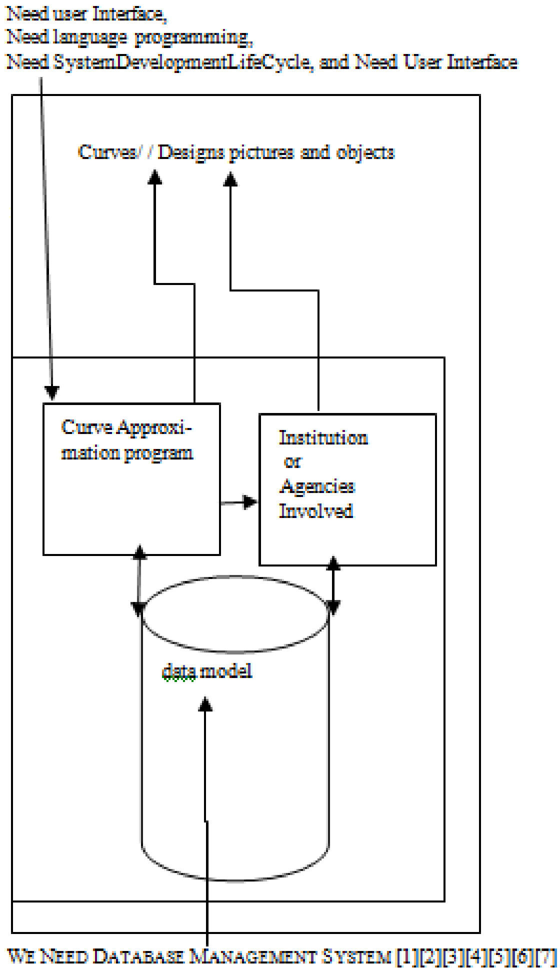


Figure 3. Framework Research

## IV. RESULTS AND DISCUSSIONS

*Testing and Results*

The result obtained by running the above program under C++ with Open GL will be used to test the accuracy of the Bezier and B-Spline algorithm through Display monitor, not a printer, as the smoothness of the curve is better shown on screen rather than on paper.

CD ROM is included. (When necessary, the CD can be given, as the code of this program is not included in this paper/article.

Table 1. Test Results

| Test | Successful | Problems |
|---|---|---|
| The first | partly | in navigation menu |
| | partly | In saving and editing of data |
| | partly | In drawing data |
| The second | ok | Menu dynamically |
| | ok | Delete data model and append data model |
| | ok | For creating and drawing model |


ISBN: 978-988-14047-3-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

## V. Conclusion

From the test result, a conclusion can be drawn as follows: That the program is sufficient, the data model is drawing according to the algorithms and the result is that the curves are better (more smooth) as required by the user.

The Suggestion for further research:

- The requirement to evaluate the hardware, especially the monitor with high resolution and adequate CPU requirements, as the data can be very large in size, and a comparison of the algorithm with other curves.
- Menu displays can be either stactic or dynamic.

## Acknowledgements

Appreciation to the Ministry of Research Technology and Higher Education Republic of Indonesia, which has provided support to the author through 2017 Research Grants.

## References

[1] Connolly, Thomas and Begg, Carolyn, ”Database Systems 3rd Edition”, Addison Wesley Longman, Inc, USA, 2002.
[2] Date, C. J., “An Introduction to Database System”, Addison Wesley Longman, Inc, USA, 1990.
[3] Edward, A., “Computer Graphics”, 1 Addison-Wesley Publi. Reading, Massachuset. ISBN: 0-201-13548-5, 1990.
[4] Elmasri, Ramez, and Navathe, Shamkant B., “Fundamental of Database Systems Third Edition”, Addison – Wesley Publishing Company, Inc, United States of America, 2000.
[5] F.David, Rogers and Adams J.A., "Mathematical Elements for Computer Graphics", New York: McGraw-Hill, 1998.
[6] Foley J. D. et al., ” Computer graphics : principle and practice”, 2 Addison-Wesley Publi.Reading, Massachuset, ISBN: 0-201-12110-7, 1996.
[7] Hearn D., and Baker M. P., “Computer Graphics”, 1 Prentice-Hall Intern. London. ISBN: 0-13-165598-1, 1996.
[8] Newman,W. M., Sproull R.F., ”Principles of Interactive Computer Graphics”, Kogakusha, Mc.Graw-Hill, 1981.
[9] Pressman, Roger S., “Software Engineering : A Practioner’ s Approach, 5th Ed.”, MC Graw - Hill, New York, 2001.
[10] Ramakrishnan, Raghu, and. Gehrke, Johannes, “Database Management Systems 3rd Edition”, The Mc. Graw-Hill Companies, Inc, USA, 2003.
[11] Shneiderman, Ben, “Designing the User Interface: Strategies for Effective Human-Computer Interaction”, Third Edition. Addison-Wesley, 1998.
[12] Theo Pavlidis, “Algorithms for Graphics and Image Processing”, Computer Science Press Inc, Rockville USA, 1997.
[13] Wolfgang K.Giloi, "Interactive Computer Graphics", Prentice Hall,Inc., Englewood Cliffs, New Jersey 07632, 1999.
[14] Woo, M. et al., “OpenGL Programming Guide”, 5 Addison-Wesley Publi.Reading, Massachuset. ISBN: 0-201-60458-2, 2006.