$$\underset{2\times 2}{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow |A| = ad - bc$$

$$\boxed{\det(A) = ad - bc}$$

PERSAMAAN EIGEN

$$|\lambda I - A| = 0$$

$$\left| \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - a & -b \\ -c & \lambda - d \end{vmatrix} = 0$$

$$(\lambda - a)(\lambda - d) - (-b)(-c) = 0$$

$$\lambda^2 + (-a-d)\lambda + ad - bc = 0$$

$$\boxed{\lambda^2 - \underbrace{(a+d)}_{tr(A)}\lambda + \underbrace{(ad-bc)}_{\det(A)} = 0}$$

$$\rightarrow \boxed{\lambda^2 - tr(A) + \det(A) = 0}$$

$$A_{3\times3} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \longrightarrow |A| = \left|\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right|$$

$$- \quad - \quad - \quad + \quad + \quad +$$

$$\boxed{\det(A) = (aei + bfg + cdh) - (ceg + afh + bdi)}$$

PERSAMAAN EIGEN : $|\lambda I - A| = 0$

$$\left| \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \right| = 0$$

$$\left|\begin{array}{ccc|cc} \lambda - a & -b & -c & \lambda - a & -b \\ -d & \lambda - e & -f & -d & \lambda - e \\ -g & -h & \lambda - i & -g & -h \end{array}\right| = 0$$

$$- \quad - \quad - \quad + \quad + \quad +$$

$$[(\lambda-a)(\lambda-e)(\lambda-i) - bfg - cdh] - [eg(\lambda-e) + fh(\lambda-a) + bd(\lambda-i)] = 0$$

$$[(\lambda-a)\{\lambda^2 - (e+i)\lambda + ei\} - bfg - cdh] - [\lambda eg - eeg + \lambda fh - afh + \lambda bd - bdi] = 0$$

$$\begin{cases} \lambda^3 - (e+i)\lambda^2 + \lambda ei - a\lambda^2 + (ae+ai)\lambda - aei - bfg - cdh \\ -\lambda eg + ceg - \lambda fh + afh - \lambda bd + bdi = 0 \end{cases}$$

$$\lambda^3 - (a+e+i)\lambda^2 + (ae+ai+ei-eg-fh-bd)\lambda + (-aei-bfg-cdh) + (ceg+afh+bdi) = 0$$

$$\lambda^3 - (a+e+i)\lambda^2 + (ae+ai+ei-eg-fh-bd)\lambda - (aei+bfg+cdh) + (ceg+afh+bdi) = 0$$

$$\lambda^3 - (a+e+i)\lambda^2 + (ae+ai+ei-eg-fh-bd)\lambda - \underbrace{[(aei+bfg+cdh) - (ceg+afh+bdi)]}_{\det(A)} = 0$$

KESIMPULAN :

$$A_{2\times2} \rightarrow +\lambda^2 - tr(A) + \underline{\det(A)} = 0$$

$$A_{3\times3} \rightarrow +\lambda^3 - (a+e+i)\lambda^2 + (ae+ai+ei-eg-fh-bd)\lambda - \underline{\det(A)} = 0$$

$$A_{4\times4} \rightarrow +\lambda^4 - \ldots\lambda^3 + \ldots\lambda^2 - \ldots\lambda + \underline{\det(A)} = 0$$