1. $\int sechx\, dx$ $\left(dengan\ menggunakan\ ini\ \frac{2}{e^x+e^{-x}} \right)$

2. $\int \frac{1}{\sqrt{x}-\sqrt[3]{x}} dx$

3. $\int \frac{1}{\sqrt[9]{x}+\sqrt[9]{x}} dx$

4. $\int e^t \sqrt{9-e^{2t}}\, dt$

5. $\int \sqrt{e^{2t}-9}\, dt$

6. $\int_{\sqrt{2}}^{2} \frac{dt}{t^3\sqrt{t^2-1}}$

7. $\int_0^2 x^3\sqrt{x^2+4}\, dx$

PENYELESAIAN SOAL-SOAL INTEGRAL

① $\int \operatorname{sech} x \, dx$ (DENGAN MENGGUNAKAN $\operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$)

SOLUSI

$$\int \operatorname{sech} x \, dx = \int \frac{2}{e^x + e^{-x}} dx = \int \frac{2\,dx}{e^x + \frac{1}{e^x}} \cdot \frac{e^x}{e^x}$$

$$= 2\int \frac{e^x\,dx}{(e^x)^2 + 1} = 2\int \frac{d(e^x)}{1 + (e^x)^2}$$

$$\boxed{\text{INGAT RUMUS : } \int \frac{dU}{1+U^2} = \arctan U + C}$$

$$= 2\left[\arctan(e^x) + C\right]$$

$$= 2\arctan(e^x) + 2C$$

$$= 2\arctan(e^x) + C$$

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② $$\int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx = \int \frac{dx}{x^{\frac{1}{2}} - x^{\frac{1}{3}}} = \int \frac{dx}{x^{\frac{3}{6}} - x^{\frac{2}{6}}} = \int \frac{dx}{(x^{\frac{1}{6}})^3 - (x^{\frac{1}{6}})^2} = \boxed{\int \frac{dx}{(\sqrt[6]{x})^3 - (\sqrt[6]{x})^2}}$$

MISALKAN : $U = \sqrt[6]{x} = x^{\frac{1}{6}} \rightarrow \dfrac{dU}{dx} = \dfrac{1}{6} x^{\frac{1}{6}-1} \rightarrow dU = \dfrac{1}{6} x^{-\frac{5}{6}} dx$

$dU = \dfrac{1}{6} \cdot \dfrac{1}{x^{5/6}} dx \rightarrow dU = \dfrac{1}{6(x^{\frac{1}{6}})^5} dx \rightarrow dU = \dfrac{dx}{6(\sqrt[6]{x})^5}$

$\rightarrow dU = \dfrac{dx}{6U^5} \rightarrow \boxed{6U^5\,dU = dx}$

$$= \int \frac{6U^5\,dU}{U^3 - U^2} = 6\int \frac{U^{\cancel{5}\,3}\,dU}{\cancel{U^2}(U-1)} = 6\int \left(\frac{U^3}{U-1}\right) dU = 6\int \left(U^2 + U + 1 + \frac{1}{U-1}\right) dU$$

$$\begin{array}{r|l} & U^2 + U + 1 + \frac{1}{U-1} \\ \hline U-1 & U^3 \\ & U^3 - U^2 \;- \\ \hline & U^2 \\ & U^2 - U \;- \\ \hline & U \\ & U - 1 \;- \\ \hline & 1 \end{array}$$

$$= 6\left[\int (U^2 + U + 1)\,dU + \int \frac{dU}{U-1}\right]$$

$$= 6\left[\tfrac{1}{3}U^3 + \tfrac{1}{2}U^2 + U + \int \frac{d(U-1)}{(U-1)}\right]$$

$$= 6\left[\tfrac{1}{3}U^3 + \tfrac{1}{2}U^2 + U + \ln|U-1| + C\right]$$

$$= 2U^3 + 3U^2 + 6U + 6\ln|U-1| + 6C$$

$$= 2(\sqrt[6]{x})^3 + 3(\sqrt[6]{x})^2 + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C$$

$$= 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + C$$

(3) $$\int \frac{1}{\sqrt[9]{x}+\sqrt[9]{x}}\,dx = \int \frac{dx}{2\sqrt[9]{x}} = \frac{1}{2}\int \frac{dx}{x^{\frac{1}{9}}} = \frac{1}{2}\int x^{-\frac{1}{9}}\,dx$$

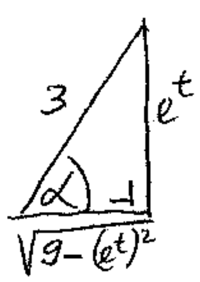
$$= \frac{1}{2}\cdot\left[\frac{1}{\left(-\frac{1}{9}\right)+1}\, x^{\left(-\frac{1}{9}\right)+1} + C\right]$$

$$= \frac{1}{2}\cdot\frac{1}{\left(\frac{8}{9}\right)}\, x^{\frac{8}{9}} + \left(\frac{1}{2}\cdot C\right)$$

$$= \frac{1}{2}\cdot\frac{9}{8}\,\sqrt[9]{x^{8}} + C$$

$$= \underline{\frac{9}{16}\,\sqrt[9]{x^{8}} + C}$$

(4) $$\int e^{t}\sqrt{9-e^{2t}}\,.dt = \int \sqrt{9-(e^{t})^{2}}\,.\,e^{t}.dt = \int \sqrt{(3)^{2}-(e^{t})^{2}}\,.\,d(e^{t})$$



SUBSITUSI TRIGONOMETRI

$$\boxed{\frac{e^{t}}{3} = \sin\alpha} \longrightarrow \boxed{e^{t} = 3\sin\alpha}$$

$$\frac{d(e^{t})}{d\alpha} = 3\cos\alpha \rightarrow \underline{d(e^{t}) = 3\cos\alpha\,.d\alpha}$$

$$\boxed{\frac{\sqrt{9-(e^{t})^{2}}}{3} = \cos\alpha} \longrightarrow \underline{\sqrt{9-e^{2t}} = 3\cos\alpha}$$

$$\int e^{t}\sqrt{9-e^{2t}}\,.dt = \int\sqrt{9-e^{2t}}\,.\,d(e^{t})$$

$$= \int 3\cos\alpha\,.\,(3\cos\alpha\,.\,d\alpha)$$

$$= 9\int\cos^{2}\alpha\,.\,d\alpha$$

$$= 9\int\left(\frac{1+\cos 2\alpha}{2}\right)d\alpha$$

$$= \frac{9}{2}\int(1+\cos 2\alpha)\,d\alpha$$

$$= \frac{9}{2}\int d\alpha + \frac{9}{2}\cdot\left(\frac{1}{2}\right)\int\cos 2\alpha\,.\,\overbrace{d(2\alpha)}^{2d\alpha}$$

$$= \frac{9}{2}\,.\,\alpha + \frac{9}{4}\sin 2\alpha + C$$

$$= \frac{9}{2}\arcsin\left(\frac{e^{t}}{3}\right) + \cancel{\frac{9}{4}}\cdot\cancel{\frac{2}{9}}\,e^{t}\sqrt{9-e^{2t}} + C$$

$$= \underline{\frac{9}{2}\arcsin\left(\frac{e^{t}}{3}\right) + \frac{1}{2}\,e^{t}\sqrt{9-e^{2t}} + C}$$

Rumus :

$$\cos 2\alpha = 2\cos^{2}\alpha - 1$$

$$1+\cos 2\alpha = 2\cos^{2}\alpha$$

$$\cos^{2}\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin\alpha = \frac{e^{t}}{3}$$

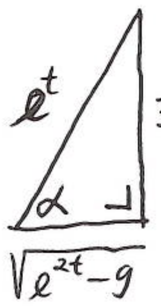
$$\alpha = \arcsin\left(\frac{e^{t}}{3}\right)$$

$$\sin 2\alpha = 2\sin\alpha\,.\cos\alpha$$

$$= 2\,.\,\frac{e^{t}}{3}\,.\,\frac{\sqrt{9-e^{2t}}}{3}$$

$$= \frac{2}{9}\,e^{t}\sqrt{9-e^{2t}}$$

⑤ $\int \sqrt{e^{2t}-9}\,.\,dt = \int \sqrt{(e^t)^2-(3)^2}\,.\,dt$



$\boxed{\dfrac{e^t}{3} = \operatorname{cosec}\alpha} \longrightarrow e^t = 3\operatorname{cosec}\alpha$

$\ln e^{t} = \ln(3\operatorname{cosec}\alpha)$

$t = \ln 3 + \ln \operatorname{cosec}\alpha$

$\dfrac{dt}{d\alpha} = 0 + \dfrac{1}{\operatorname{cosec}\alpha}\cdot(\operatorname{cosec}\alpha)'$

$\dfrac{dt}{d\alpha} = \dfrac{1}{\cancel{\operatorname{cosec}\alpha}}\cdot(-\cancel{\operatorname{cosec}\alpha}\,.\cot\alpha)$

$dt = -\cot\alpha\,.\,d\alpha$

$\boxed{\dfrac{\sqrt{e^{2t}-9}}{3} = \cot\alpha}$

$\sqrt{e^{2t}-9} = 3\cot\alpha$

$\int \sqrt{e^{2t}-9}\,.\,dt = \int 3\cot\alpha\,.\,(-\cot\alpha\,.\,d\alpha)$

$= -3\int \cot^2\alpha\,.\,d\alpha$

$\boxed{\begin{aligned}1+\cot^2\alpha &= \operatorname{cosec}^2\alpha\\ \cot^2\alpha &= \operatorname{cosec}^2\alpha-1\end{aligned}}$

$= -3\int(\operatorname{cosec}^2\alpha-1)\,.\,d\alpha$

$= -3\int\operatorname{cosec}^2\alpha\,.\,d\alpha + 3\int d\alpha$

$\boxed{\begin{aligned}\operatorname{cosec}\alpha &= \frac{e^t}{3}\\ \alpha &= \operatorname{arc\,cosec}\left(\frac{e^t}{3}\right)\end{aligned}}$

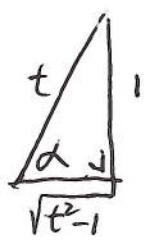
$= -3(-\cot\alpha) + 3\alpha$

$= 3\cot\alpha + 3\alpha$

$= \sqrt{e^{2t}-9} + 3\operatorname{arc\,cosec}\left(\dfrac{e^t}{3}\right)$

INTEGRAL DENGAN SUBSTITUSI TRIGONOMETRI

⑥ $$\int_{\sqrt{2}}^{2} \frac{dt}{t^3\sqrt{t^2-1}} =$$



$\boxed{\sqrt{t^2-1} = \sqrt{t^2-1^2}}$

$\boxed{\frac{t}{1} = \operatorname{cosec}\alpha} \rightarrow t = \operatorname{cosec}\alpha \rightarrow t^3 = \operatorname{cosec}^3\alpha$

$\frac{dt}{d\alpha} = -\operatorname{cosec}\alpha \cdot \cot\alpha$

$dt = -\operatorname{cosec}\alpha \cdot \cot\alpha \cdot d\alpha$

$\boxed{\frac{\sqrt{t^2-1}}{1} = \cot\alpha}$

$\sqrt{t^2-1} = \cot\alpha$

$\boxed{\begin{array}{l} \sin\alpha = \frac{1}{t} \\ \cos\alpha = \frac{\sqrt{t^2-1}}{t} \end{array}}$

$$\int \frac{dt}{t^3\sqrt{t^2-1}} = \int \frac{-\cancel{\operatorname{cosec}\alpha} \cdot \cancel{\cot\alpha} \cdot d\alpha}{\operatorname{cosec}^{\cancel{3}\,2}\alpha \cdot \cancel{\cot\alpha}}$$

$$= -\int \frac{d\alpha}{\operatorname{cosec}^2\alpha} = -\int \sin^2\alpha \cdot d\alpha$$

$\sin\frac{\pi}{4} = \frac{1}{2}\sqrt{2} = \frac{\sqrt{2}}{2}$

$\operatorname{cosec}\frac{\pi}{4} = \frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

$\operatorname{cosec}\frac{\pi}{4} = \sqrt{2}$

$\operatorname{arc\,cosec}\sqrt{2} = \frac{\pi}{4}$

$\boxed{\begin{array}{l} \cos 2\alpha = 1 - 2\sin^2\alpha \\ 2\sin^2\alpha = 1 - \cos 2\alpha \\ \sin^2\alpha = \frac{1-\cos 2\alpha}{2} \end{array}}$

$$= -\int \frac{1-\cos 2\alpha}{2}\, d\alpha$$

$$= -\int \left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\alpha\right) d\alpha$$

$\sin\frac{\pi}{6} = \frac{1}{2}$

$\operatorname{cosec}\frac{\pi}{6} = \frac{2}{1}$

$\operatorname{arc\,cosec} 2 = \frac{\pi}{6}$

$$= -\tfrac{1}{2}\int d\alpha + \tfrac{1}{2}\int \cos 2\alpha \cdot d\alpha$$

$$= -\tfrac{1}{2}\cdot\alpha + \tfrac{1}{2}\cdot\left(\tfrac{1}{2}\right)\int \cos 2\alpha \cdot \overbrace{d(2\alpha)}^{2\,d\alpha}$$

$\boxed{\begin{array}{l} \operatorname{cosec}\alpha = t \\ \alpha = \operatorname{arc\,cosec} t \end{array}}$

$$= -\tfrac{1}{2}\alpha + \tfrac{1}{4}\sin 2\alpha + C$$

$\boxed{\begin{array}{l} \sin 2\alpha = 2\sin\alpha \cdot \cos\alpha \\ \quad = 2\left(\frac{1}{t}\right)\left(\frac{\sqrt{t^2-1}}{t}\right) \\ \quad = \frac{2\sqrt{t^2-1}}{t^2} \end{array}}$

$$= -\tfrac{1}{2}\cdot\operatorname{arc\,cosec} t + \tfrac{1}{4}\cdot\frac{2\sqrt{t^2-1}}{t^2} + C$$

$$= -\tfrac{1}{2}\operatorname{arc\,cosec} t + \frac{\sqrt{t^2-1}}{2t^2} + C$$

$$\int_{\sqrt{2}}^{2} \frac{dt}{t^3\sqrt{t^2-1}} = -\tfrac{1}{2}\operatorname{arc\,cosec} t + \frac{\sqrt{t^2-1}}{2t^2}\Bigg|_{\sqrt{2}}^{2}$$

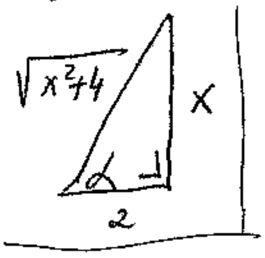
$$= -\tfrac{1}{2}\left(\operatorname{arc\,cosec} 2 - \operatorname{arc\,cosec}\sqrt{2}\right) + \left[\frac{\sqrt{2^2-1}}{2(2)^2} - \frac{\sqrt{(\sqrt{2})^2-1}}{2(\sqrt{2})^2}\right]$$

$$= -\tfrac{1}{2}\left(\underbrace{\operatorname{arc\,cosec} 2}_{\frac{\pi}{6}} - \underbrace{\operatorname{arc\,cosec}\sqrt{2}}_{\frac{\pi}{4}}\right) + \frac{\sqrt{3}}{8} - \frac{1}{4}$$

$$= -\tfrac{1}{2}\left(\tfrac{\pi}{6} - \tfrac{\pi}{4}\right) + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{4} = -\tfrac{1}{2}\left(\frac{2\pi - 3\pi}{12}\right) + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{4}$$

$$= -\tfrac{1}{2}\left(-\tfrac{\pi}{12}\right) + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{4} = \tfrac{1}{24}\pi + \tfrac{1}{8}\sqrt{3} - \tfrac{1}{4}$$

⑦ $\int_0^2 x^3 \sqrt{x^2+4}\,.\,dx$



$\sqrt{x^2+4} = \sqrt{x^2+2^2}$

$\boxed{\frac{x}{2} = \tan\alpha} \longrightarrow \frac{x = 2\tan\alpha}{\frac{dx}{d\alpha} = 2\sec^2\alpha} \begin{array}{l} \longrightarrow x^3 = 2^3\tan^3\alpha = \underline{8\tan^3\alpha} \\ \rightarrow \underline{dx = 2\sec^2\alpha\,.\,d\alpha} \end{array}$

$\boxed{\frac{\sqrt{x^2+4}}{2} = \sec\alpha} \longrightarrow \underline{\sqrt{x^2+4} = 2\sec\alpha}$

$$\int x^3\sqrt{x^2+4}\,.\,dx = \int (8\tan^3\alpha).(2\sec\alpha).(2\sec^2\alpha\,.\,d\alpha)$$

$$= 32\int \sec^3\alpha\,.\,\tan^3\alpha\,.\,d\alpha = 32\int \sec^2\alpha\,.\,\tan^2\alpha\,(\sec\alpha\,.\tan\alpha\,.\,d\alpha)$$

$$= 32\int \sec^2\alpha\,(\sec^2\alpha - 1)\,.\,d(\sec\alpha)$$

$$\boxed{\begin{array}{r} 1+\tan^2\alpha = \sec^2\alpha \\ \tan^2\alpha = \sec^2\alpha - 1 \end{array}}$$

$$= 32\int (\sec^4\alpha - \sec^2\alpha)\,.\,d(\sec\alpha)$$

$$= 32\left[\int (\sec\alpha)^4 . d(\sec\alpha) - \int (\sec\alpha)^2 . d(\sec\alpha)\right]$$

$$= 32\left[\tfrac{1}{5}(\sec\alpha)^5 - \tfrac{1}{3}(\sec\alpha)^3\right] + C$$

$$= 32\left[\tfrac{1}{5}\left(\frac{\sqrt{x^2+4}}{2}\right)^5 - \tfrac{1}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3\right] + C$$

$$= \frac{32}{5}\,\frac{\left(\sqrt{x^2+4}\right)^5}{2^5} - \frac{32}{3}\,\frac{\left(\sqrt{x^2+4}\right)^3}{2^3} + C$$

$$= \frac{\cancel{32}}{5}.\frac{(x^2+4)^2\sqrt{x^2+4}}{\cancel{32}} - \frac{\cancel{32}^{\,4}}{3}.\frac{(x^2+4)\sqrt{x^2+4}}{\cancel{8}_{\,1}} + C$$

$$= \underline{\frac{(x^2+4)^2\sqrt{x^2+4}}{5} - \frac{4(x^2+4)\sqrt{x^2+4}}{3} + C}$$

$$\int_0^2 x^3\sqrt{x^2+4}\,.\,dx = \left.\frac{(x^2+4)^2\sqrt{x^2+4}}{5} - \frac{4(x^2+4)\sqrt{x^2+4}}{3}\right|_0^2$$

$$= \left[\frac{(2^2+4)^2\sqrt{2^2+4}}{5} - \frac{(0^2+4)^2\sqrt{0^2+4}}{5}\right] - \left[\frac{4(2^2+4)\sqrt{2^2+4}}{3} - \frac{4(0^2+4)\sqrt{0^2+4}}{3}\right]$$

$$= \left(\frac{64\sqrt{8}}{5} - \frac{16\sqrt{4}}{5}\right) - \left(\frac{32\sqrt{8}}{3} - \frac{16\sqrt{4}}{3}\right)$$

$$= \left(\frac{64(2\sqrt{2})}{5} - \frac{16(2)}{5}\right) - \left(\frac{32(2\sqrt{2})}{3} - \frac{16(2)}{3}\right)$$

$$= \left(\frac{128\sqrt{2} - 32}{5}\right) - \left(\frac{64\sqrt{2} - 32}{3}\right) = \frac{128}{5}\sqrt{2} - \frac{32}{5} - \frac{64}{3}\sqrt{2} + \frac{32}{3}$$

$$= \cancel{\left(\frac{384}{15}\sqrt{2} - \frac{320}{15}\sqrt{2}\right) + \left(\frac{160}{15} - \frac{96}{15}\right)} = \left(\frac{384}{15}\sqrt{2} - \frac{320}{15}\sqrt{2}\right) + \left(\frac{160}{15} - \frac{96}{15}\right)$$

$$= \underline{\underline{\frac{64}{15}\sqrt{2} + \frac{64}{15}}}$$