Study of Tangent Handrail Geometry

40° Slope, 50° Slope and 120° Corner Angle between Tangents in Plan View (Handrail negotiates a 60° Turn)

Developments of Tetrahedra

Developments of Posterboard Polyhedra

Drawings of Face Mould

Supplementary Data

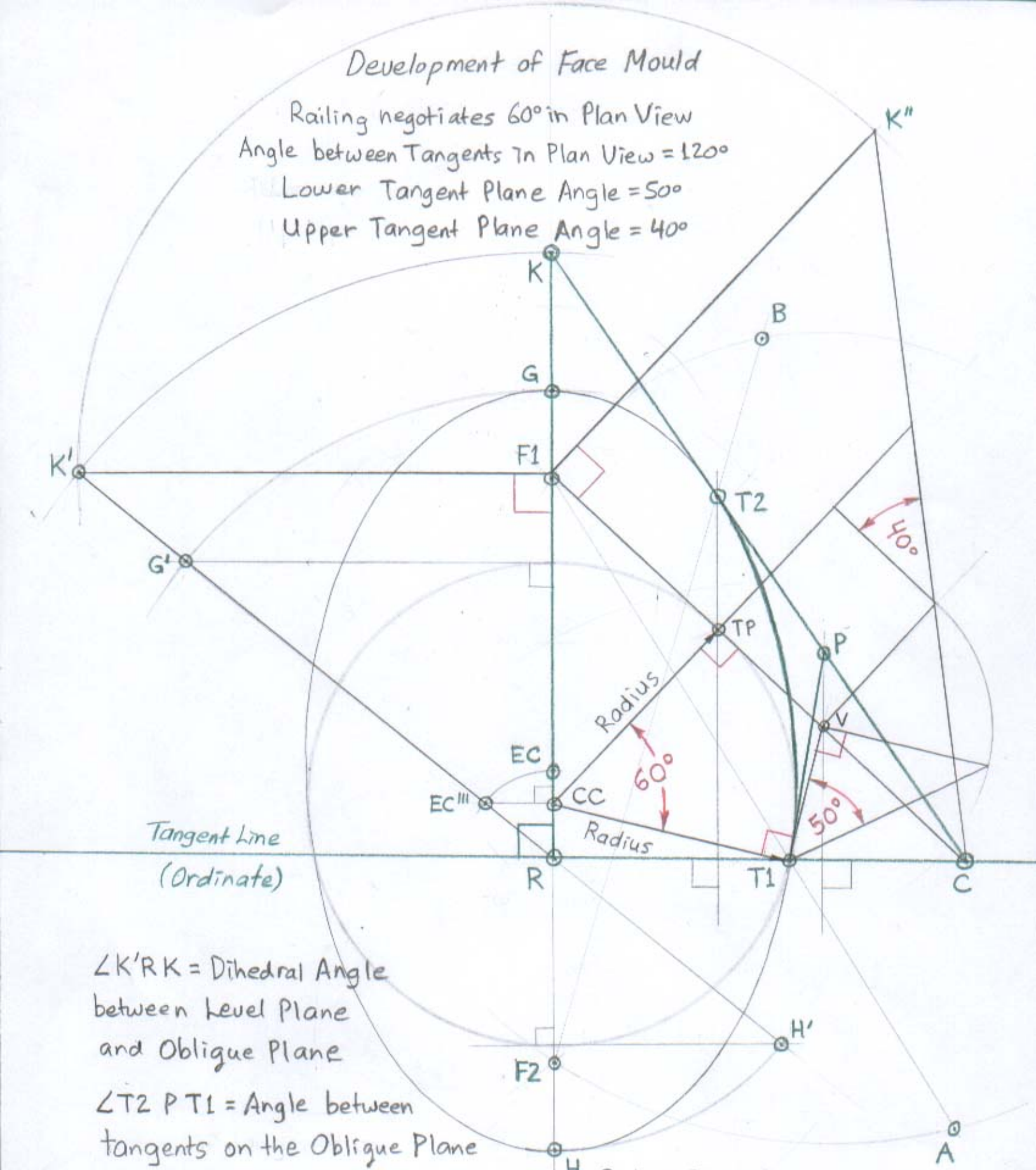
Development of Face Mould

Railing negotiates 60° in Plan View

Angle between Tangents In Plan View = 120°

Lower Tangent Plane Angle = 50°

Upper Tangent Plane Angle = 40°



∠K'RK = Dihedral Angle between Level Plane and Oblique Plane

∠T2 P T1 = Angle between tangents on the Oblique Plane

$\overline{K'H'} = \overline{KH}$ = Major Axis of Ellipse

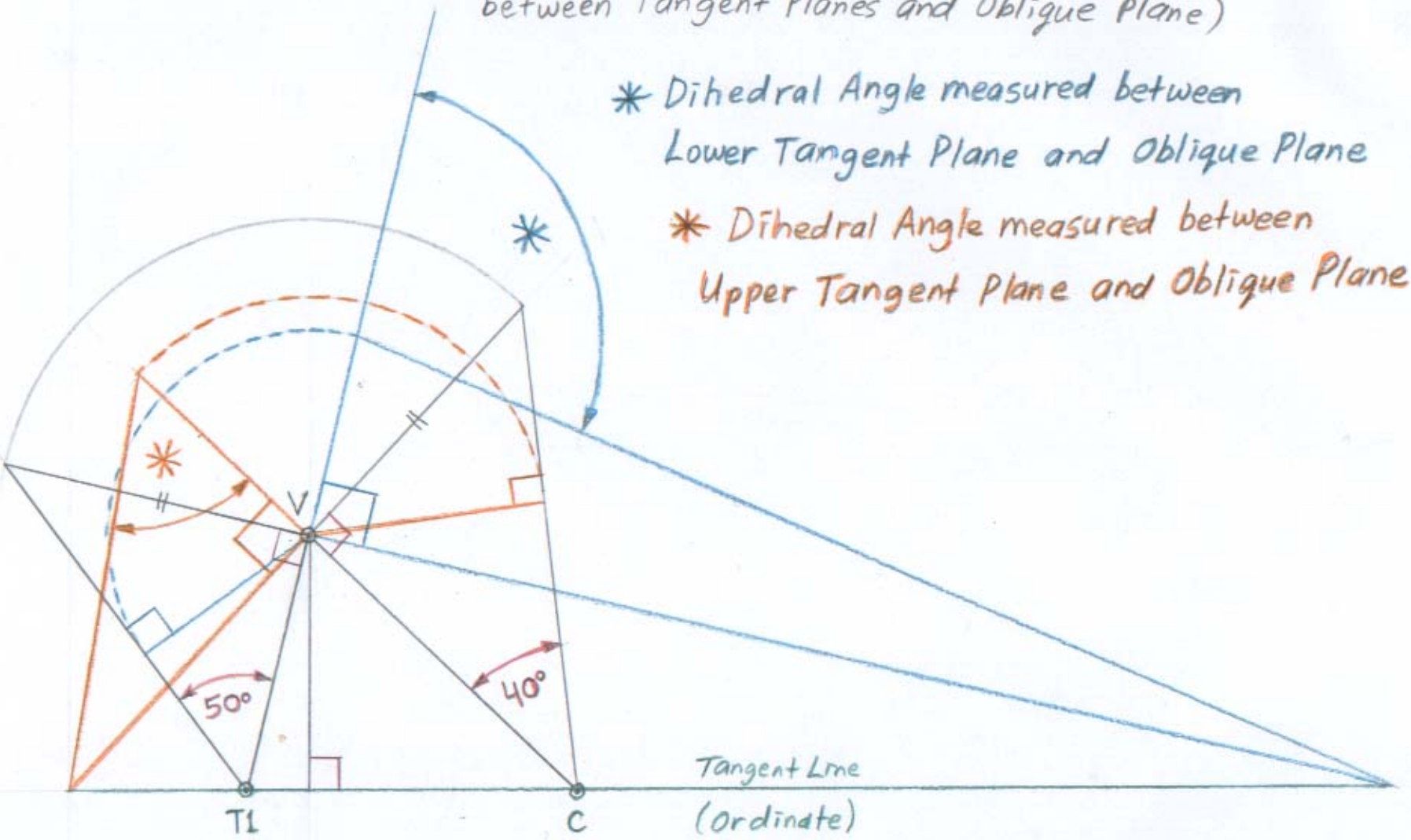
2x Radius = Minor Axis of Ellipse

Points F1, T1 and A are collinear

Points F2, T2 and B are collinear

Points K, T2, P and C are collinear

Construction of Twist Angles (Dihedral Angles measured between Tangent Planes and Oblique Plane)



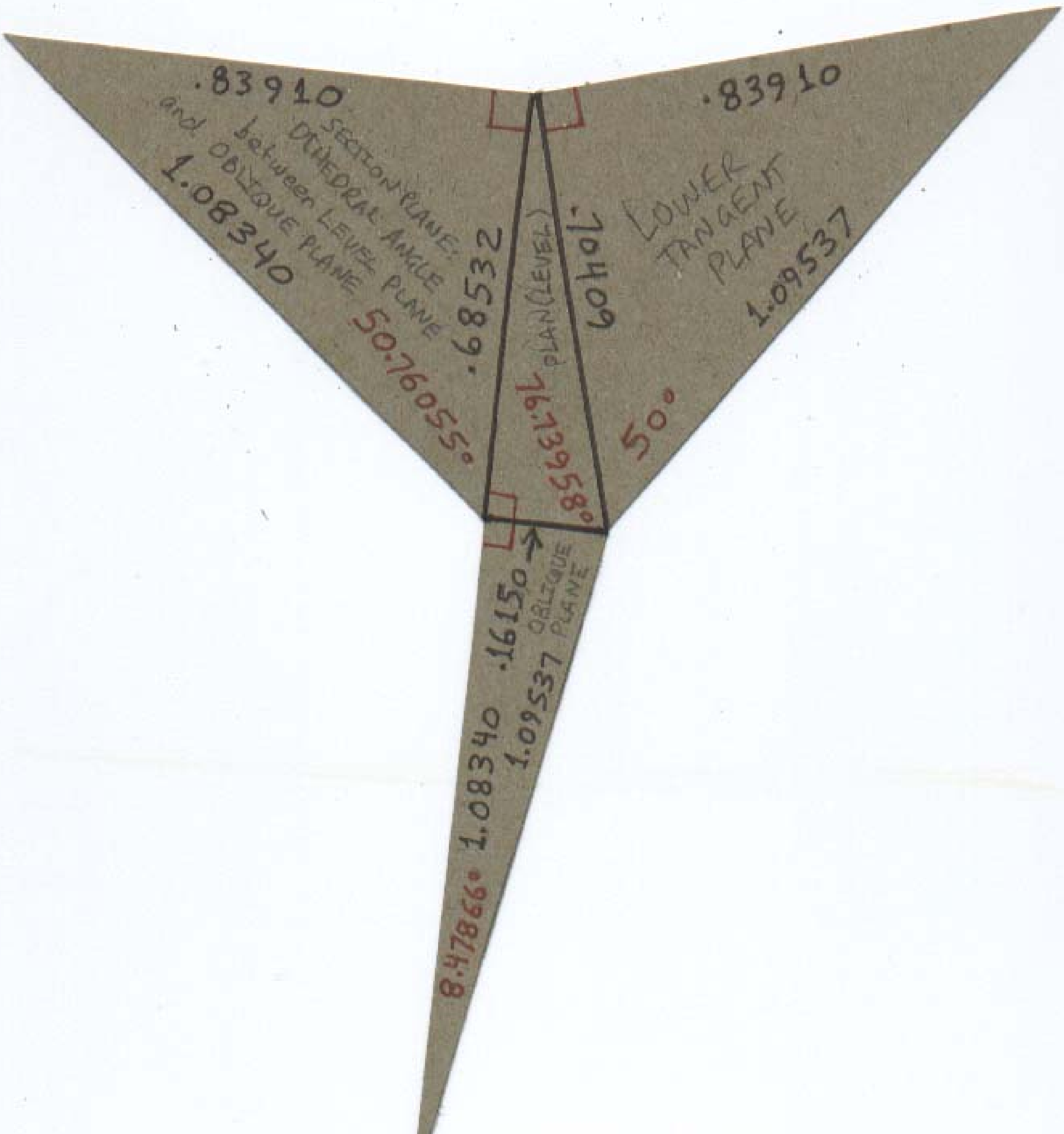
Lower Tangent Plane Angle = 50°

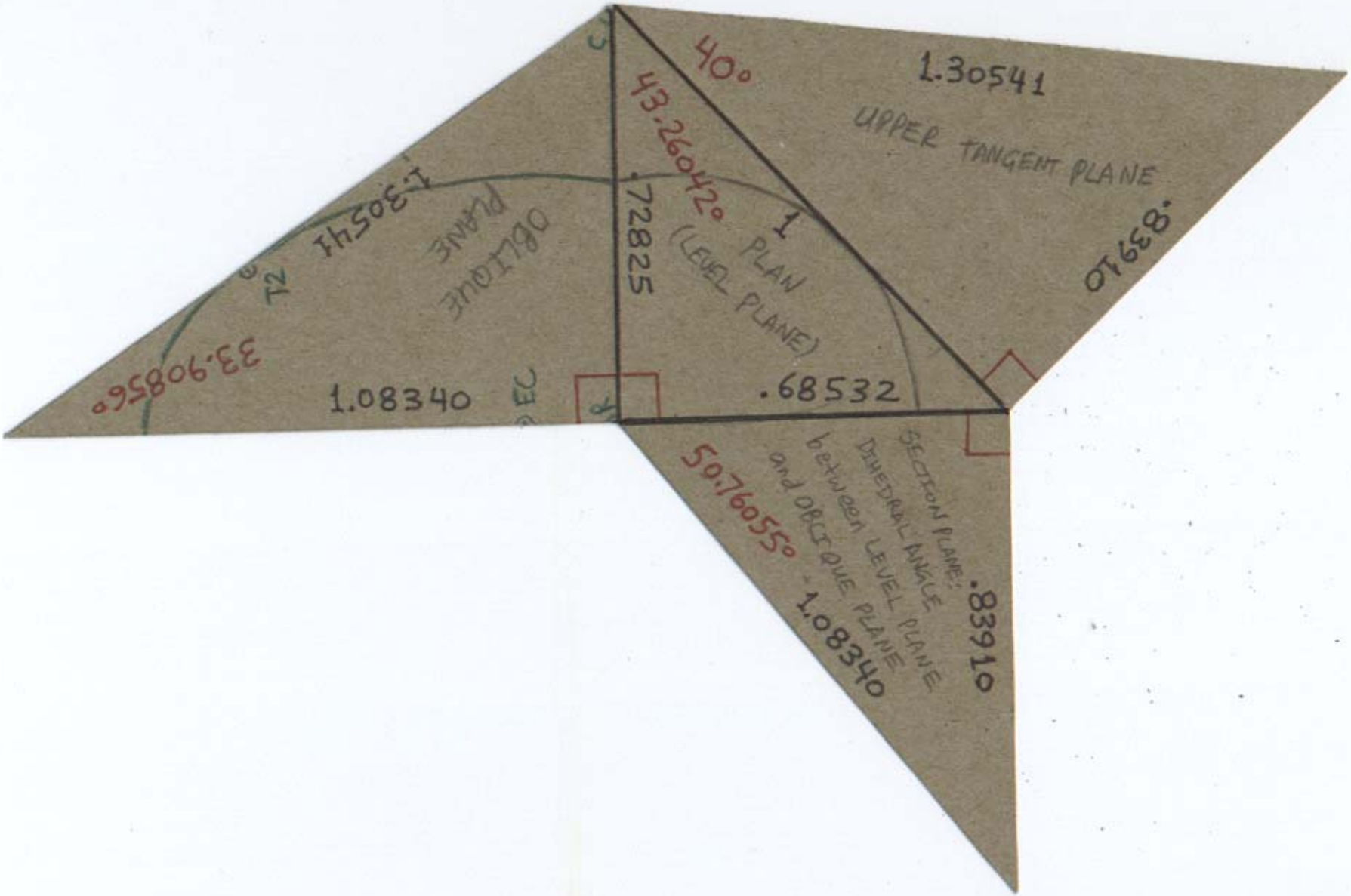
Upper Tangent Plane Angle = 40°

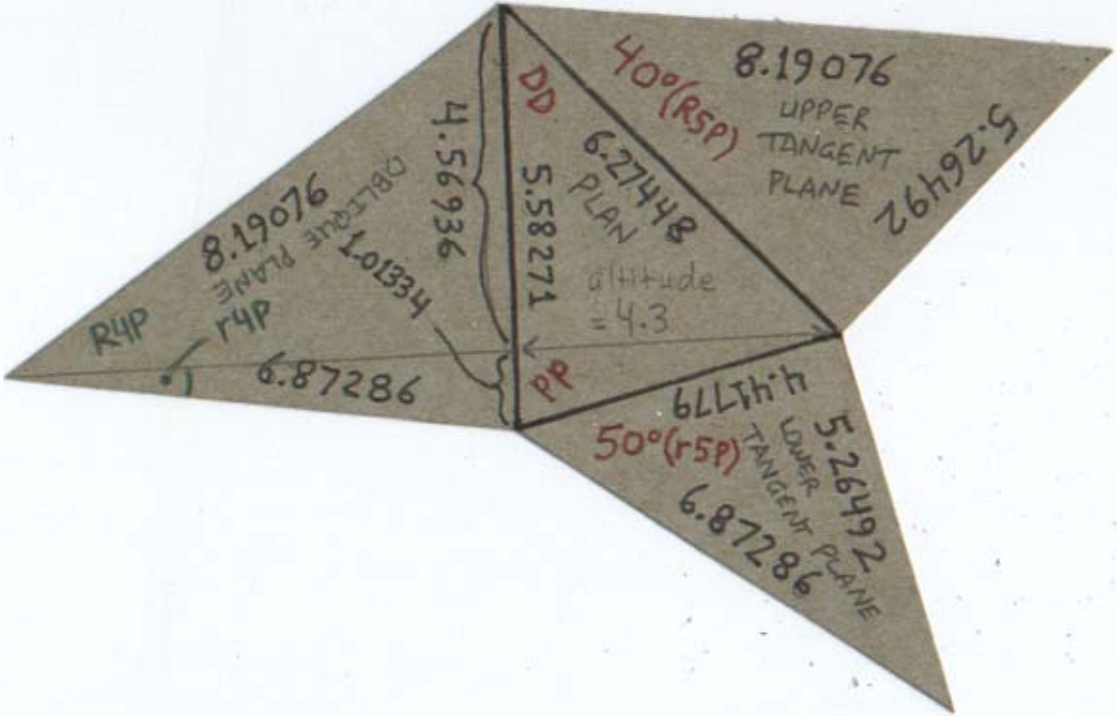
Point V is the center of all radii

$\angle V T1 C$ = Plan Angle associated with Lower Tangent Plane

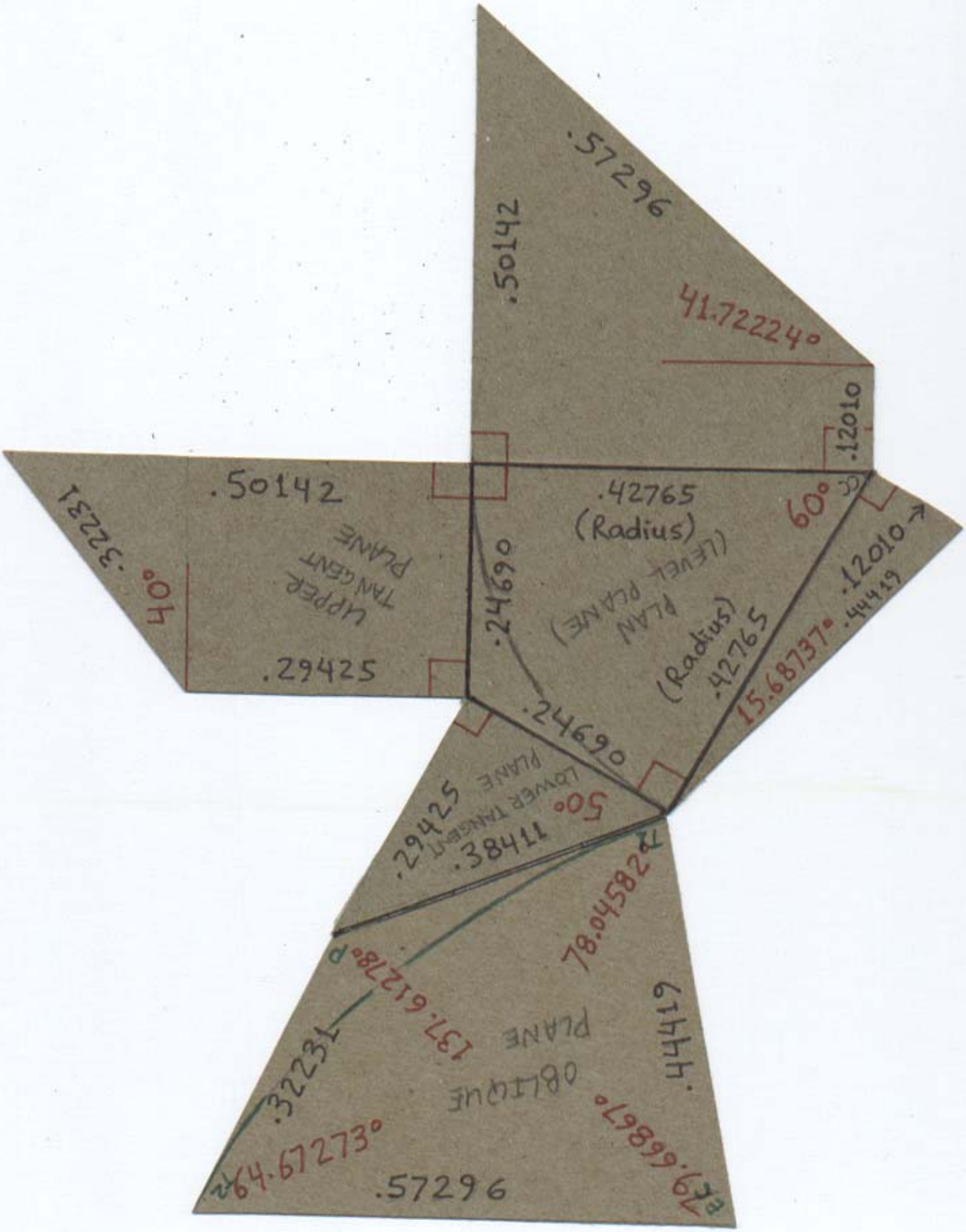
$\angle V C T1$ = Plan Angle associated with Upper Tangent Plane

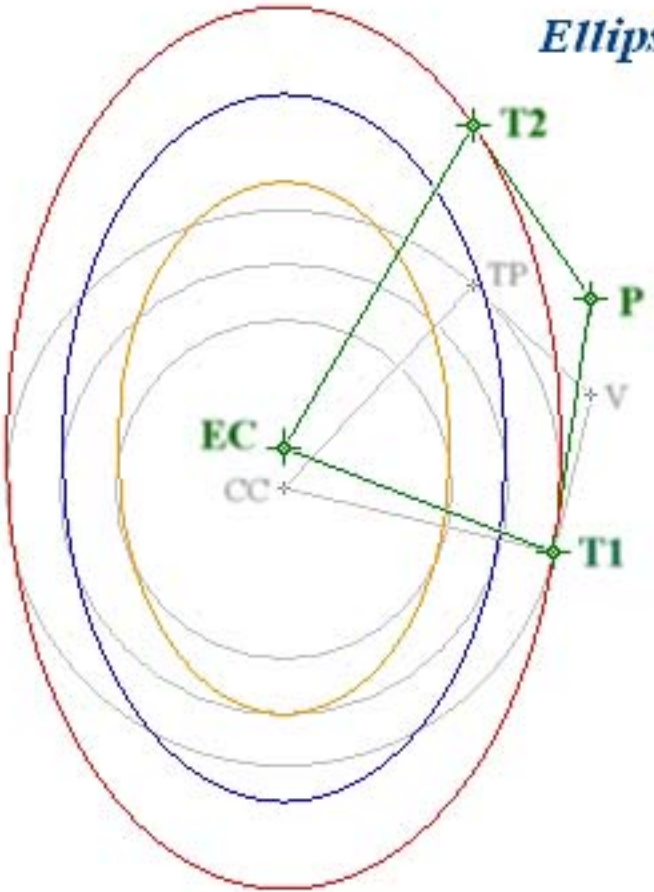

.83910
SECTION PLANE: DIHEDRAL ANGLE between LEVEL PLANE and OBLIQUE PLANE
1.08340
.68532
50.76055°
.83910
LOWER TANGENT PLANE
1.09537
PLAN (LEVEL)
76.23958°
.60404
50°
OBLIQUE PLANE
.16150
1.09537
1.08340
8.47866°


40°
1.30541
UPPER TANGENT PLANE
43.26042°
.72825
1
PLAN
(LEVEL PLANE)
.83910
1.30541
OBLIQUE PLANE
T2
33.90856°
1.08340
.68532
SECTION PLANE:
DIHEDRAL ANGLE
between LEVEL PLANE
and OBLIQUE PLANE
50.76055°
.83910
1.08340


8.19076
UPPER
TANGENT
PLANE
40°(RSP)
5.26492
DD
6.27448
PLAN
5.58271
altitude
=4.3
4.56936
8.19076
OBLIQUE PLANE
1.01334
R4P
r4P
6.87286
PP
4.41779
5.26492
LOWER TANGENT PLANE
50°(rSP)
6.87286


.84765
.83910
UPPER TANGENT PLANE
40°
.29425
.84765
46.73958°
120°
PLAN (LEVEL PLANE)
.24690
103.26042°
.68532
.83910
SECTION PLANE: ANGLE between LEVEL PLANE and OBLIQUE PLANE (DIHEDRAL ANGLE)
50.76055°
1.08340
.29425
.38411
.41624
88.47866°
.38411
132.61278°
OBLIQUE PLANE
1.08340
.84765
33.90856°


.57296
.50142
41.72224°
.12010
.50142
.42765
(Radius)
(LEVEL PLANE)
PLAN
60°
.12010
.44419
.32231
40°
UPPER TANGENT PLANE
.29425
.24690
.42765
(Radius)
15.68737°
.24690
.29425
LOWER TANGENT PLANE
.38411
50°
78.04582°
.44419
OBLIQUE PLANE
137.61278°
.32231
64.67273°
.57296
79.66867°



Ellipses and Quadrilateral (Oblique Plane) superimposed on Circles and Kite (Plan View)

Semi-Minor Axis = Radius = 5
Semi-Major Axis = 7.90435

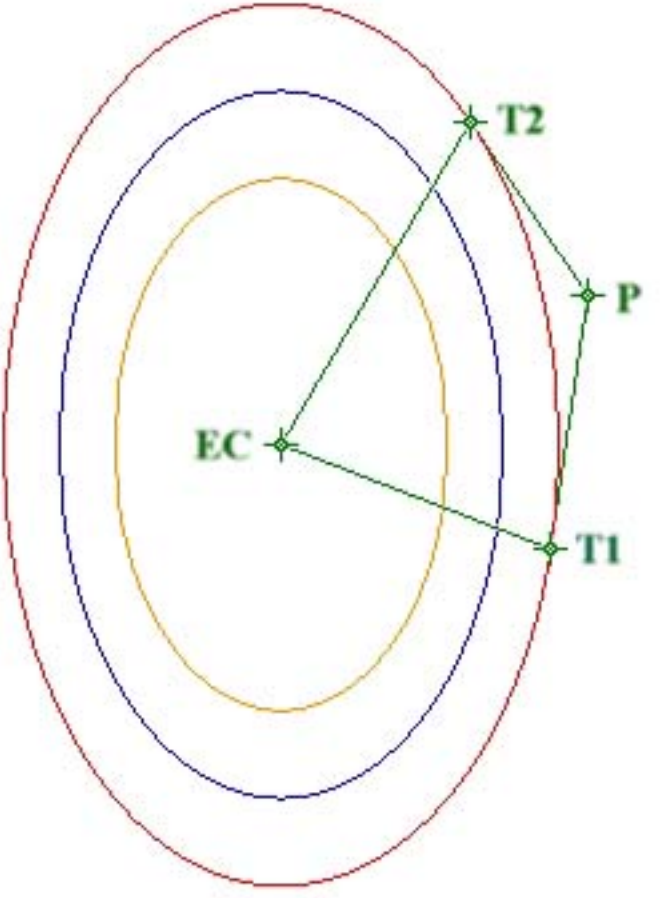
Semi-Minor Axis = Radius = 4
Semi-Major Axis = 6.32348

Semi-Minor Axis = Radius = 3
Semi-Major Axis = 4.74261

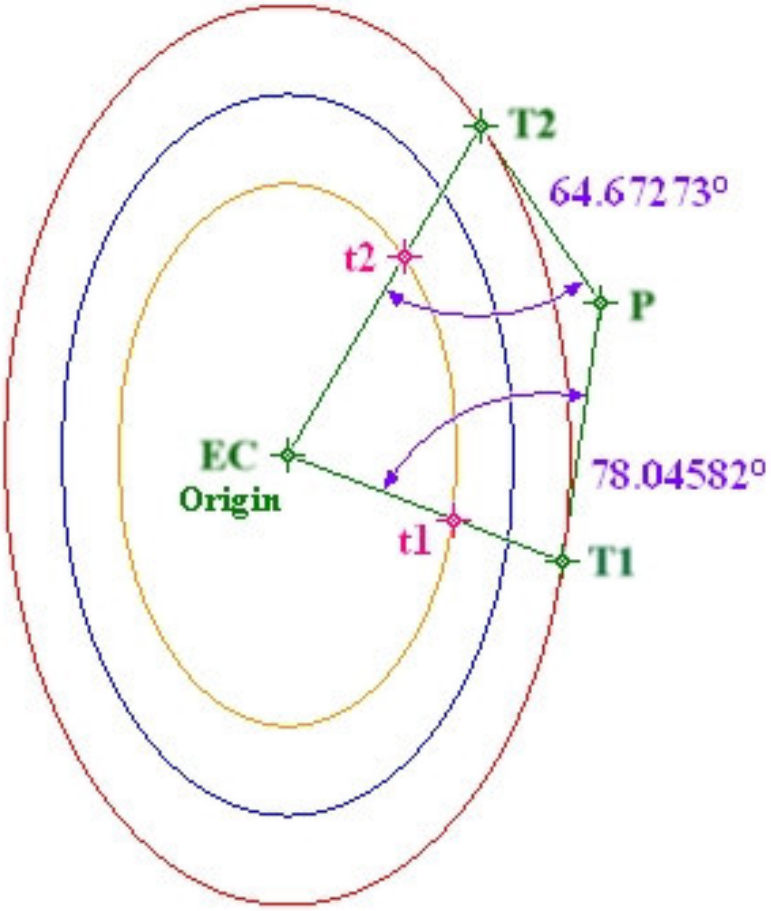
Ellipses superimposed on Quadrilateral

$\angle$ T2 P T1 = 180° – (R4P + r4P)

= *Angle between Tangents on the Oblique Plane*



Oblique Plane ...
Data for Analytic Solution



T1 = (4.866688, – 1.813080)
t1 = (2.920013, – 1.087848)
Distance between Ellipses from t1 to T1 = 2.077379

T2 = (3.426577, 5.756318)
t2 = (2.055946, 3.453791)
Distance between Ellipses from t2 to T2 = 2.679601

Equation of Line through EC T1 = – tan 20.43282 *x*

Equation of Line through EC T2 = tan 59.23583 *x*

Equations of Ellipses and Lines as entered in WZ Grapher

```
7.90435(25-x^2)^.5/5;
-7.90435(25-x^2)^.5/5;
6.32348(16-x^2)^.5/4;
-6.32348(16-x^2)^.5/4;
4.74261(9-x^2)^.5/3;
-4.74261(9-x^2)^.5/3;
-.372549*x;
1.679903*x
```

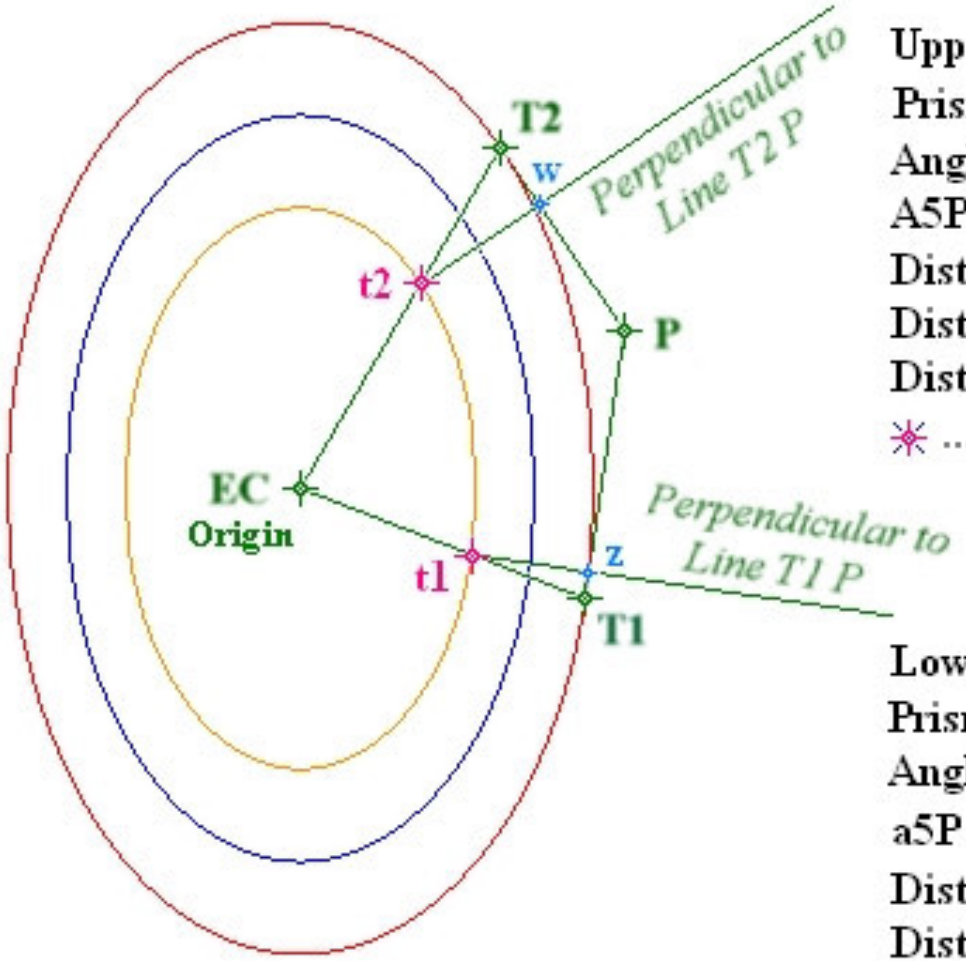
... points validated using the grapher's "Trace" function

T1 (*x*) = 5 sin 76.739582° = 4.866688
T1 (*y*) = – 5 cos 76.739582° / cos 50.760553° = – 1.813080
T2 (*x*) = 5 sin 43.260418° = 3.426577
T2 (*y*) = 5 tan 30° (tan 40° + tan 50°) / sin 50.760553° – 1.813080 = 5.756318

t1 (*x*) = 3 sin 76.739582° = 2.920013
t1 (*y*) = – 3 cos 76.739582° / cos 50.760553° = – 1.087848
t2 (*x*) = 3 sin 43.260418° = 3.426577
t2 (*y*) = 3 tan 30° (tan 40° + tan 50°) / sin 50.760553° – 1.0878748 = 3.453791

Trigonometric Solution of Dimensions ...
relationships between lengths on the Level Plane and the Oblique Plane

Distance between Radii in Plan View = 2



Upper Tangent Plane Slope Angle = 40°
Prism Angle on Plumb Plane through EC T2 = 41.72224°
Angle t2 T2 w on Oblique Plane = 64.67273°
A5P = arctan (tan 41.72224° cos 40°) = 34.33520°
Distance T2 t2 = 2 / cos 41.72224° = 2.679601 ✳
Distance t2 w = 2 / cos 34.33520° = 2.422035
Distance T2 t2 = 2.422035 / sin 64.67273° = 2.679601 ✳
✳ ... agrees with analytic solution

Lower Tangent Plane Slope Angle = 50°
Prism Angle on Plumb Plane through EC T1 = 15.68737°
Angle t1 T1 z on Oblique Plane = 78.04582°
a5P = arctan (tan 15.68737° cos 50°) = 10.23319°
Distance T1 t1 = 2 / cos 15.68737° = 2.077379 ✳
Distance t1 z = 2 / cos 10.23319° = 2.032329
Distance T1 t1 = 2.032329 / sin 78.04582° = 2.077379 ✳
✳ ... agrees with analytic solution

Table of Angles

50° Slope, 40° Slope and 120° Corner Angle between Tangents in Plan View (Handrail negotiates a 60° Turn)

Angles associated with the 40° (Upper) Tangent Plane

SS = 60.76357° ... arctan (tan **R5P** ÷ (sin **DD**)^2) ... angle entered calculator *
DD = 43.26042° ... Upper Tangent Plane Plan Angle
R5P = 40.00000° ... Upper Tangent Plane Slope Angle
R4P = 33.90856°... produced on Oblique Plane by trace of Upper Tangent Plane
A5P = 34.33520°
90° – **A5P** ... Dihedral Angle between Oblique Plane and Upper Tangent Plane

Angles associated with the 50° (Lower) Tangent Plane

ss = 51.51701° ... arctan (tan **r5P** ÷ (sin **dd**)^2) ... angle entered calculator *
dd = 76.73958° ... Lower Tangent Plane Plan Angle
r5P = 50.00000° ... Lower Tangent Plane Slope Angle
r4P = 8.47866°... produced on Oblique Plane by trace of Lower Tangent Plane
a5P = 10.23320°
90° + **a5P** ... Dihedral Angle between Oblique Plane and Lower Tangent Plane

R1 = 50.76055° ... Slope of Plank: Dihedral Angle measured between the Level Plane and the Oblique Plane

180° – (**R4P** + **r4P**) = 137.61278° ... Angle between Tangents: angle produced on the Oblique Plane by traces of Upper and Lower Tangent Planes

DD + **dd** = Angle between Tangents in Plan View
DD + **dd** + **Angle negotiated by Handrailing** = 180°

* **SS** and **ss** are theoretical angles used to obtain a trigometric solution of the tangent handrailing angles with a Javascript calculator. These two angles are not needed to understand the geometric constructions and developments.

Tangent = **Radius** tan (.5 × **Angle negotiated by Handrailing**) = 2.86675

Distance from Point **R** to Point **C**
= **Radius** sin **dd** + **Tangent** cos **dd** + **Tangent** sin **dd** ÷ tan **DD**
= 8.51464