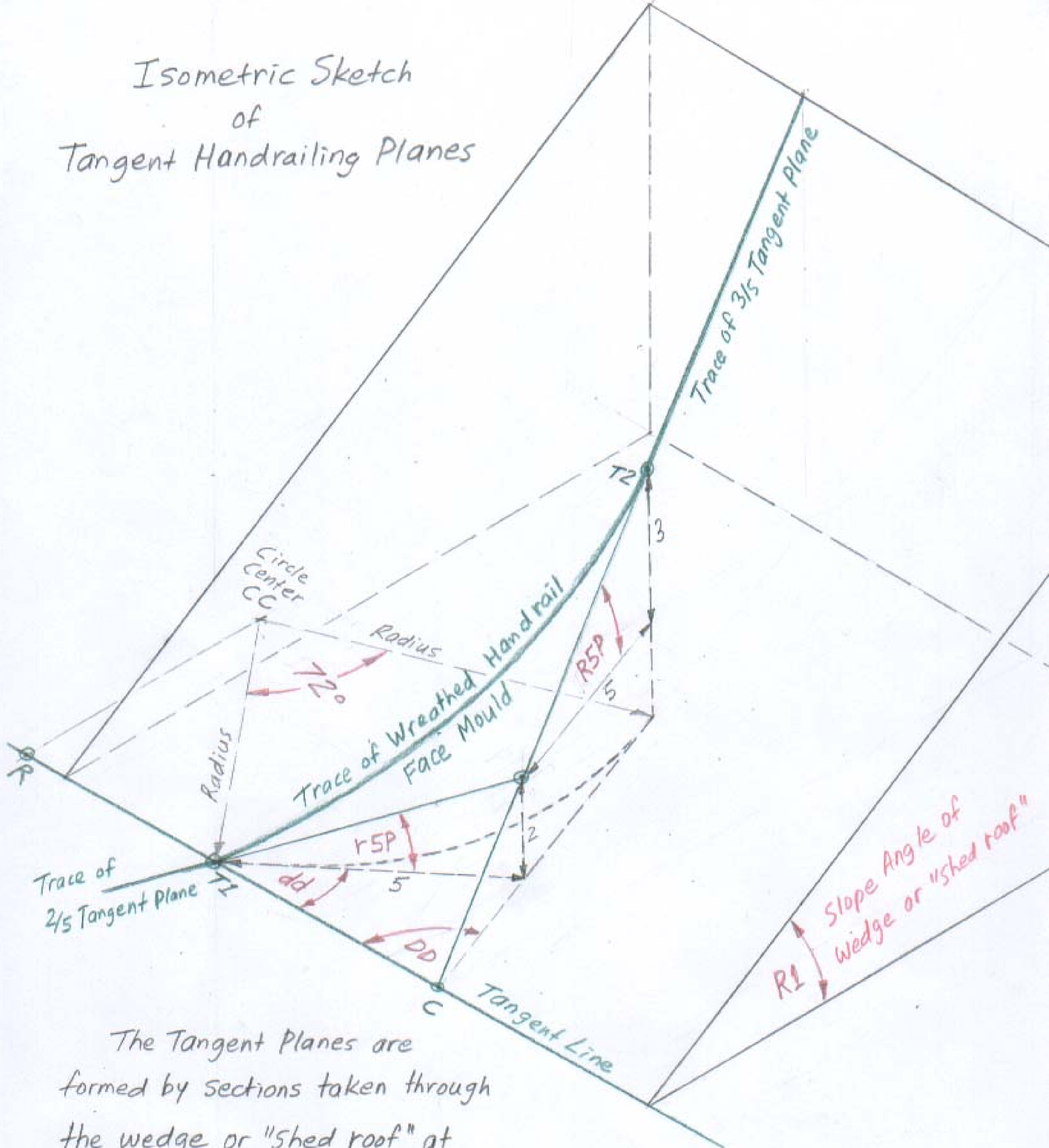
Study of Tangent Handrail Geometry

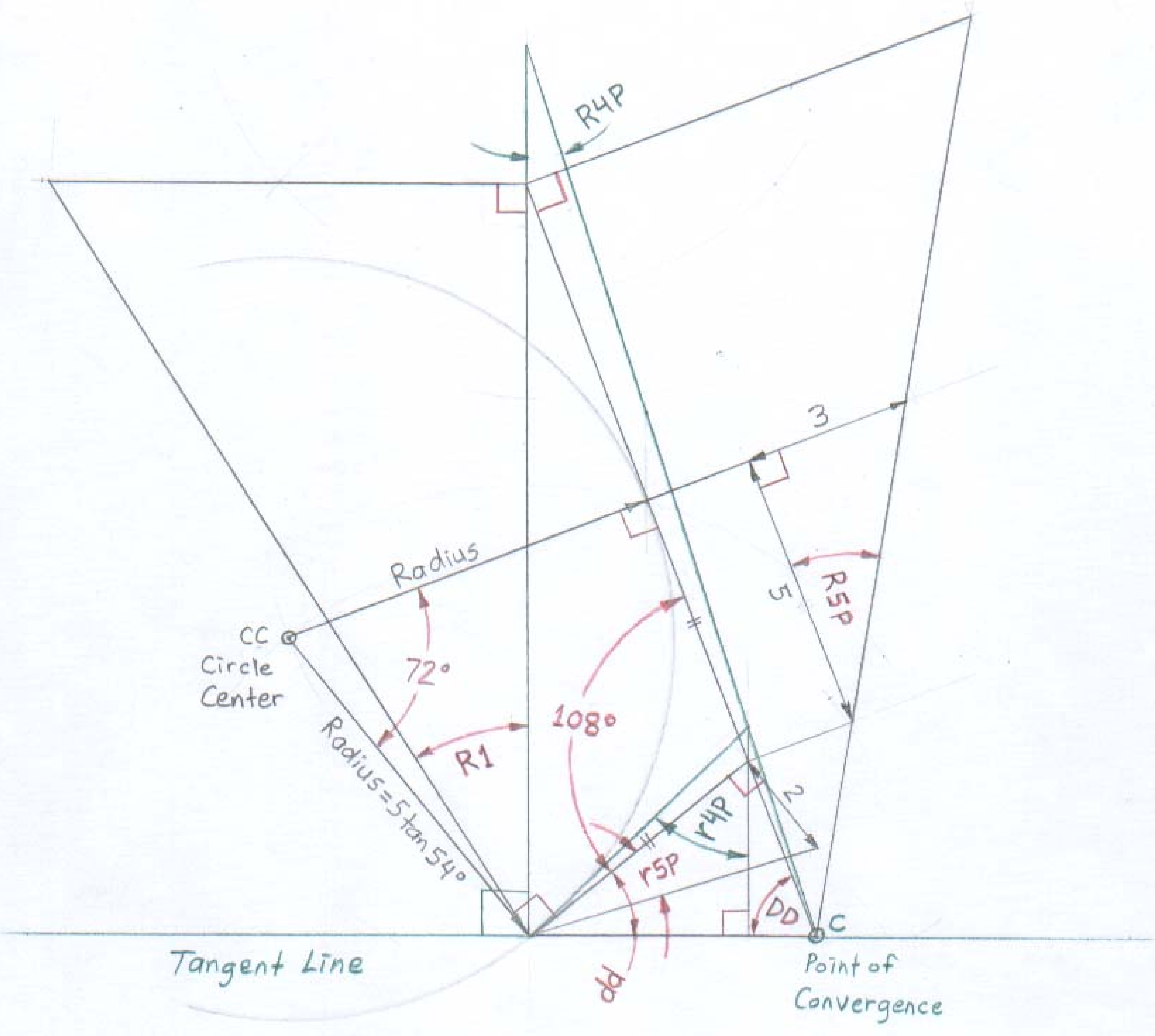
2/5 Slope, 3/5 Slope and 108° Corner Angle between Tangents in Plan View (Handrail negotiates a 72° Turn)

Isometric Sketch of Tangent Handrailing Planes



The Tangent Planes are formed by sections taken through the wedge or "shed roof" at plan angles dd and DD

r5P = Slope Angle of 2/5 Tangent Plane
R5P = Slope Angle of 3/5 Tangent Plane



Development of Tangent Handrailing Angles
Unequal Slopes, 108° Corner Angle

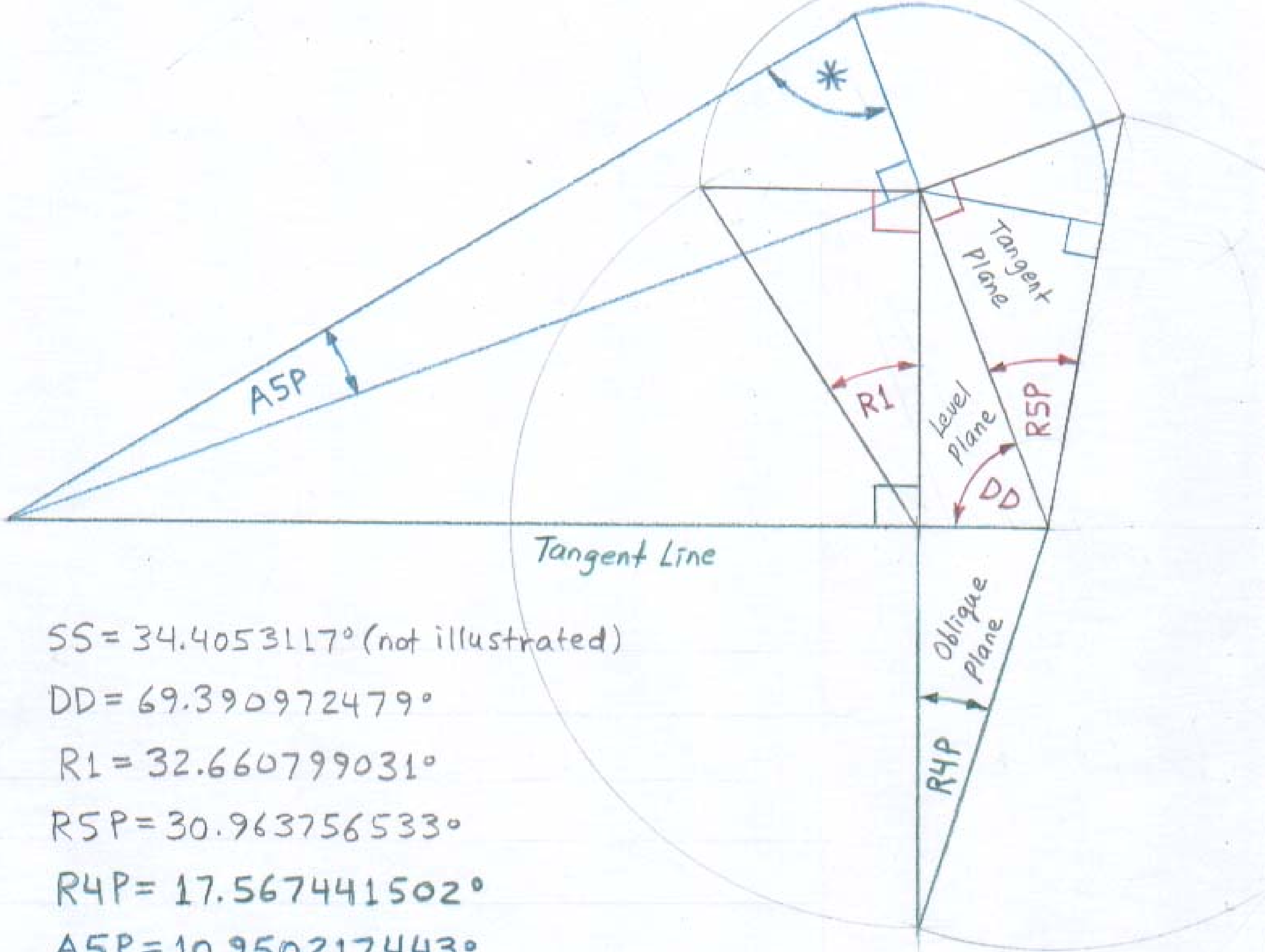
R5P = Angle of 3/5 Tangent Plane , r5P = Angle of 2/5 Tangent Plane

R1 = Dihedral Angle between Level Plane and Oblique Plane

180° - (R4P + r4P) = Angle between Tangents on Oblique Plane

DD and dd are plan angles measured on Level Plane

Development of Angles associated with the 3/5 Tangent Plane



SS = 34.4053117° (not illustrated)

DD = 69.390972479°

R1 = 32.660799031°

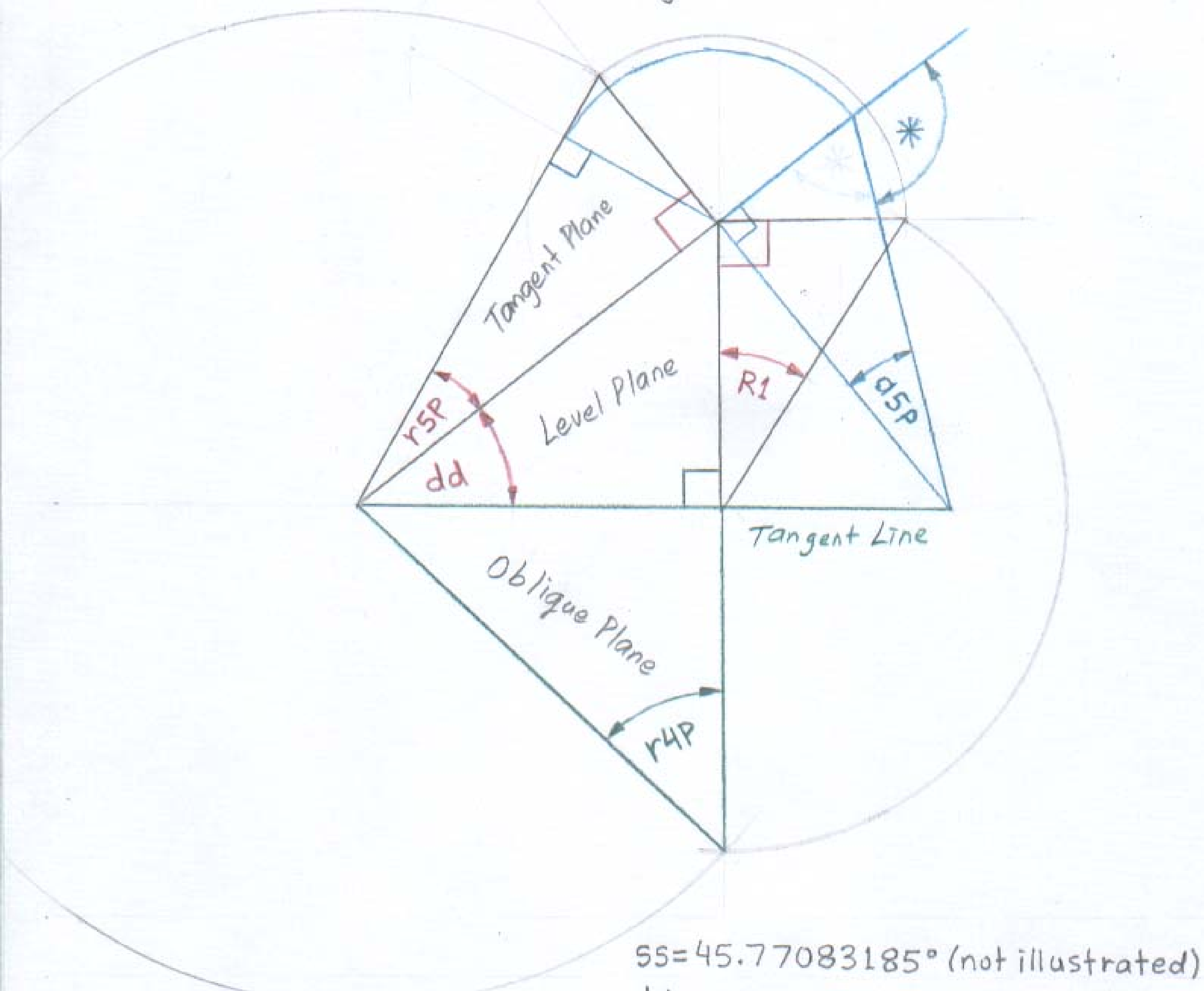
R5P = 30.963756533°

R4P = 17.567441502°

A5P = 10.950217443°

* 90° − A5P is the dihedral angle between the 3/5 Tangent Plane and the Oblique Plane

Development of the Angles associated with the 2/5 Tangent Plane



SS=45.77083185° (not illustrated)

dd=38.6090275215°

R1=32.660799042°

r5P=21.80140949°

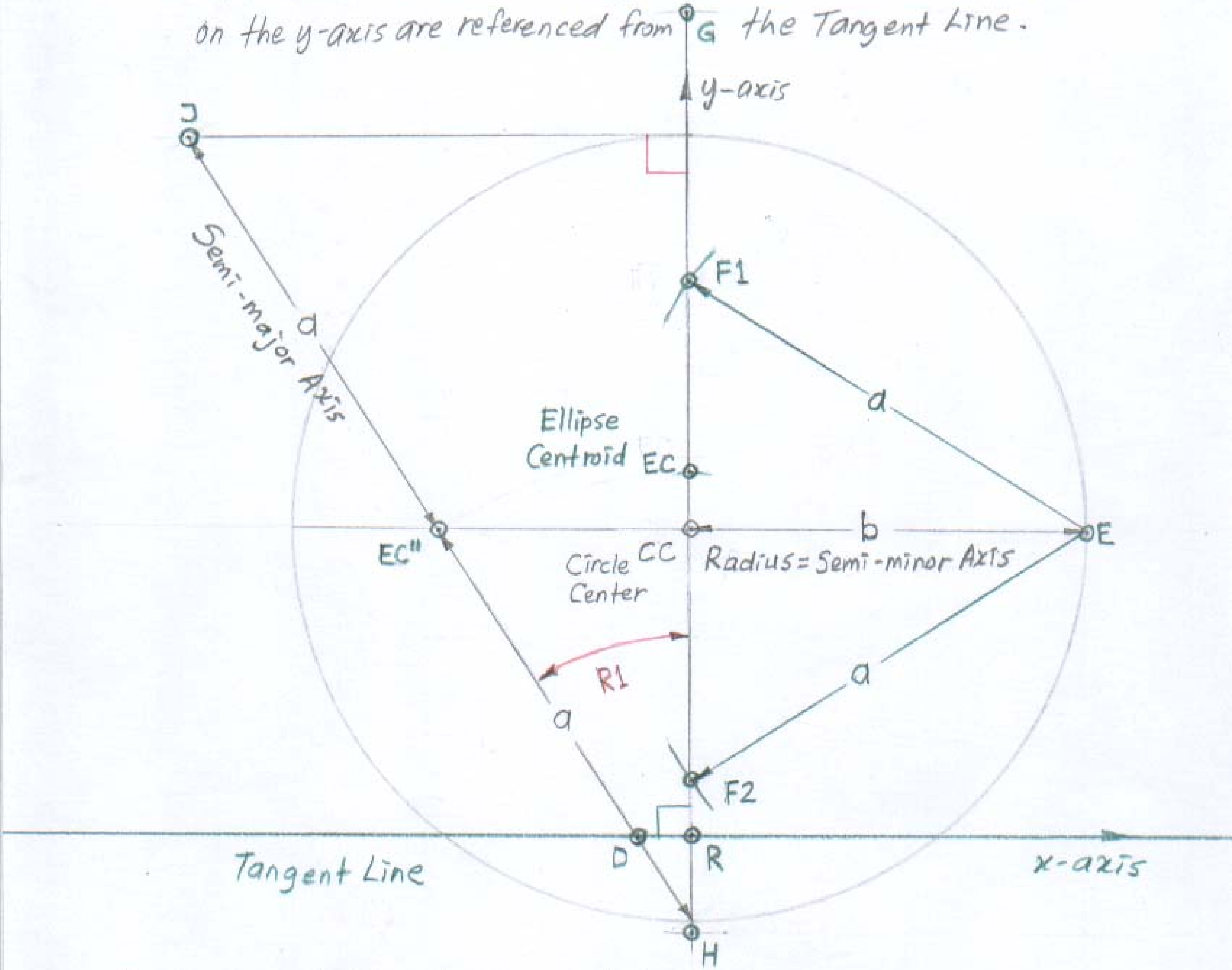
r4P=46.513144047°

a5P=24.942326051°

* 90°+a5P is the dihedral angle between the 2/5 Tangent Plane and the Oblique Plane

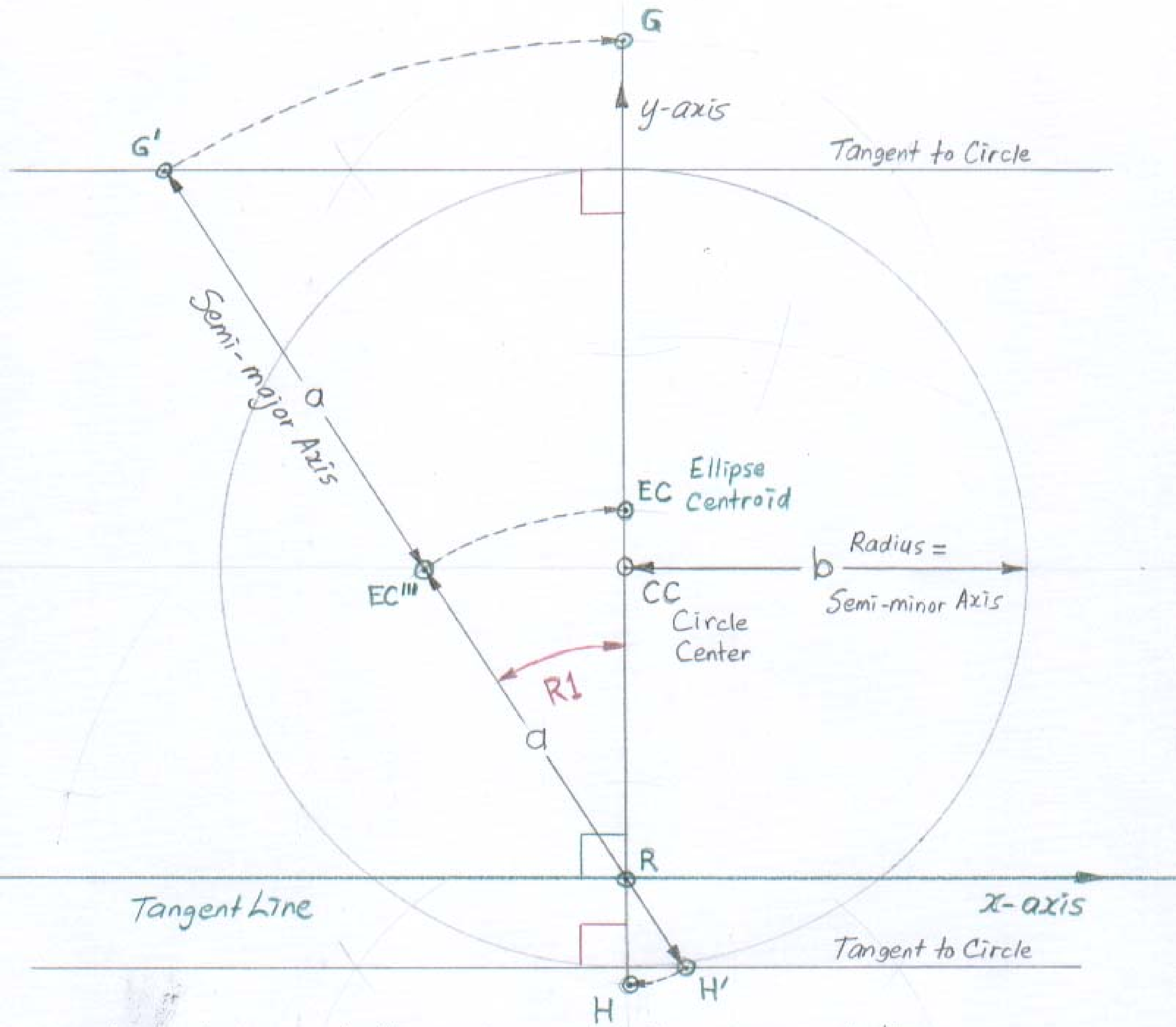
Locating the Centroid and Foci of the Ellipse on the Oblique Plane

The Tangent Line, Ordinate or x-axis is the intersection of the Level Plane and the Oblique Plane. All measurements on the y-axis are referenced from the Tangent Line.



- Construct angle R1, with respect to the diameter of the circle
- Bisect hypotenuse JH; lengths a are the semi-major axes
- With point R as the center, scribe arc of length D EC", the intersect at point EC on the y-axis is the centroid of the ellipse
- With point EC as the center, mark lengths a on the y-axis, the intersects at G and H define the major axis
- Using radius a, with point E on the semi-minor axis as the center, mark foci F1 and F2 on the major axis.

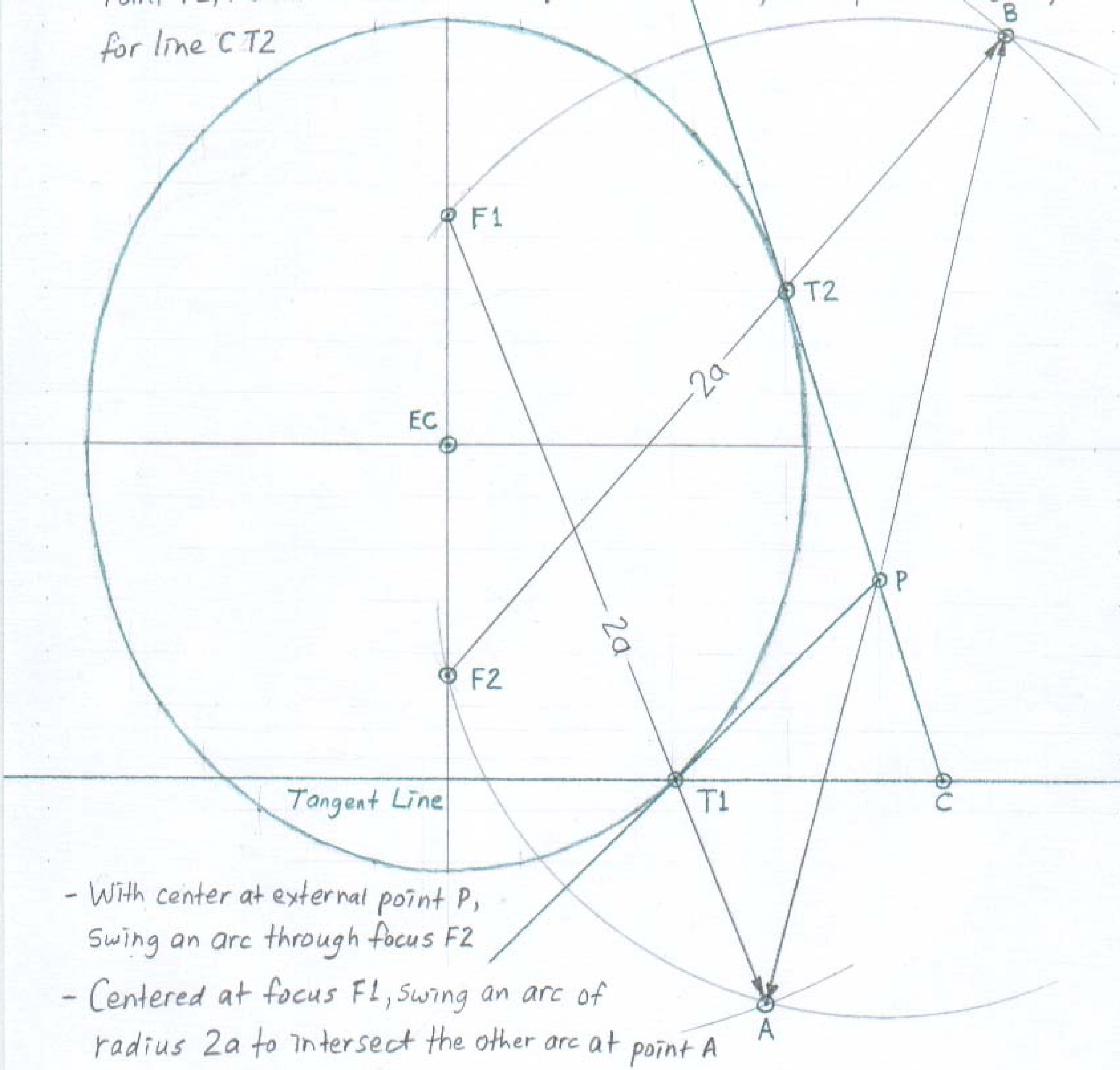
Alternative Method of finding the Centroid and Semi-Major Axes



- Construct tangents to the circle at its intersections with the y-axis
- Construct angle R1 through point R, intersecting the tangents at points G' and H'
- Bisect line G'H' at EC''' into equal lengths a, the semi-major axes
- With point R as the center, draw arcs to intersect the y-axis at point EC, the centroid of the ellipse, and points G and H, defining the extent of the major axis

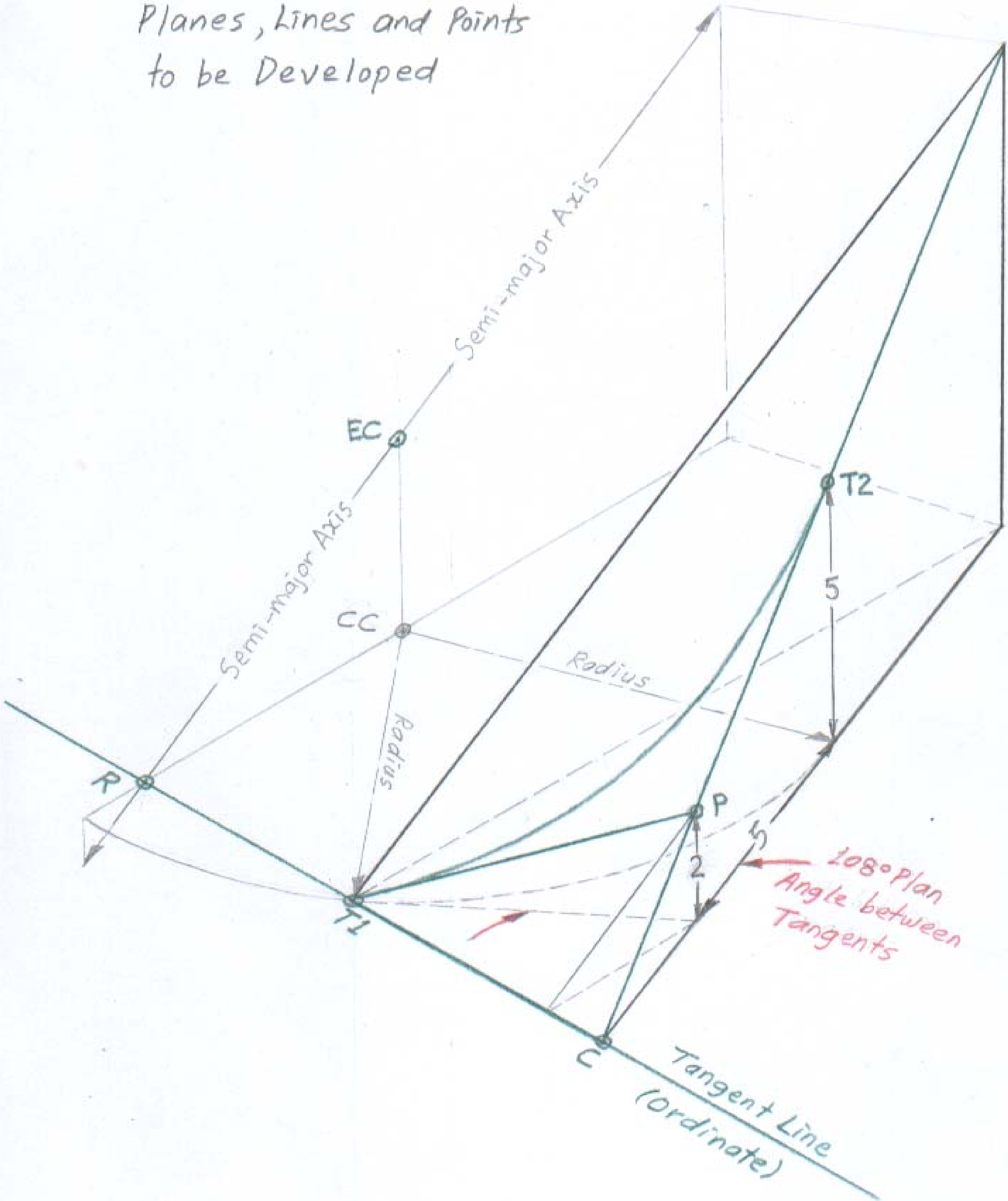
Checking the Points of Tangency

- With center at external point P, swing an arc through focus F1
- Centered at focus F2, swing an arc of radius 2a to intersect the first arc at point B
- Point T2, the intersection of the ellipse and line F2 B, is the point of tangency for line C T2

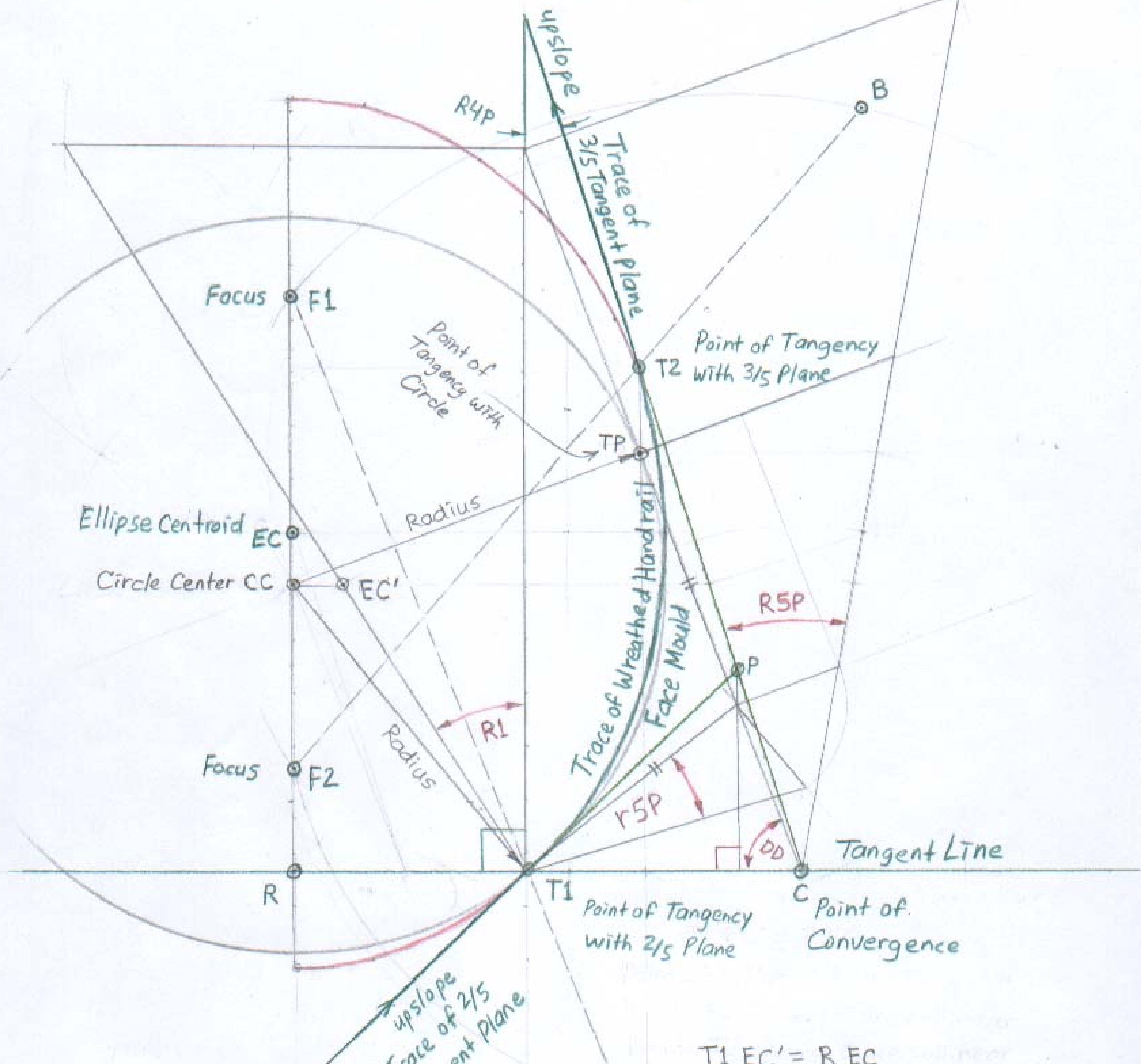


- With center at external point P, swing an arc through focus F2
- Centered at focus F1, swing an arc of radius 2a to intersect the other arc at point A
- Point T1, the intersection of the ellipse and line F1 A, is the point of tangency for line P T1

Isometric Sketch of
Planes, Lines and Points
to be Developed



Trace of Handrail on the Oblique Plane



T1 EC′ = R EC

Points F1, T1 and A are collinear

Points F2, T2 and B are collinear

Points T2, P and C are collinear

Semi-Minor Axis = Radius = $5\tan 54°$

$= 6.881909602$

Semi-Major Axis = Radius / cos R1

$= 8.174452282$

Analytic Data

The Origin is the Centroid of the Ellipse

Circle Center ... Point **CC** (0, 1.010019339)
Trace of 2/5 Tangent Plane, Point of Tangency with Ellipse
Point **T1** (4.294330347, – 6.387698198)
Trace of 3/5 Tangent Plane, Point of Tangency with Ellipse
Point **T2** (6.441495519, 2.877318344)
Intersection of Traces of Tangent Planes
Point **P** (8.201441168, 2.681691581)
Intersection of Trace of 3/5 Tangent Plane and Tangent Line
Point **C** (9.374738266, – 6.387698198)
Intersection of Trace of Major Axis of Ellipse and Tangent Line
Point **R** (0, – 6.387698198)
Focus of Ellipse ... Point **F1** (0, 4.411461248)
Focus of Ellipse ... Point **F2** (0, – 4.411461248)
Point **A** (6.041095803, – 10.78037346)
Point **B** (10.82644303, 7.839041257)

Equation of the Ellipse

$y = \pm\, 8.174452282 \sqrt{(6.881909602^2 - x^2)} \div 6.881909602$

Equation of Trace of 2/5 Tangent Plane (tangent to Ellipse at Point **T1**)

$y = .948528666x - 10.46099364$

Equation of Trace of 3/5 Tangent Plane (tangent to Ellipse at Point **T2**)

$y = -\, 3.15862591x + 23.22359299$

Equation of Line through F1 A

$y = -\, 2.51474818x + 4.411461248$

Equation of Line through F2 B

$y = -\, 1.131535304x - 4.411461248$

Definitions of Angles

W = Corner Angle measured between traces of Tangent Planes in Plan View
R5P = Upper Tangent Plane Slope Angle
r5P = Lower Tangent Plane Slope Angle
R1 = Dihedral Angle measured between Level Plane and Oblique Plane
DD = Plan Angle of Upper Tangent Plane
dd = Plan Angle of Lower Tangent Plane
R4P = Angle created by trace of Upper Tangent Plane on the Oblique Plane
r4P = Angle created by trace of Lower Tangent Plane on the Oblique Plane
90° – **A5P** = Dihedral Angle between Upper Tangent Plane and Oblique Plane
90° + **a5P** = Dihedral Angle between Lower Tangent Plane and Oblique Plane

Solution of General Plan Angle Formula given unequal Tangent Plane Slope Angles and any Corner Angle

Given ...
tan **R1** = tan **R5P** / sin **DD**
tan **R1** = tan **r5P** / sin **dd**

Since both expressions are equal ...
tan **R5P** / sin **DD** = tan **r5P** / sin **dd**

Rearranging the terms ...
sin **dd /** sin **DD =** tan **r5P** / tan **R5P**

Since ...
dd = **W** – **DD**

Substituting ...
sin (**W** – **DD**) **/** sin **DD =** tan **r5P** / tan **R5P**

Applying the Sine of the Difference of Angles Identity ...
(sin **W** cos **DD** – cos **W** sin **DD**) **/** sin **DD =** tan **r5P** / tan **R5P**

Dividing by sin **DD** ...
sin **W** / tan **DD** – cos **W** = tan **r5P** / tan **R5P**

Adding cos **W** to both sides of the equation ...
sin **W** / tan **DD** = tan **r5P** / tan **R5P** + cos **W**

tan **DD** = sin **W** / (tan **r5P** / tan **R5P** + cos **W**)

Employing a similar chain of reasoning ...
tan **dd** = sin **W** / (tan **R5P** / tan **r5P** + cos **W**)

Tangent Handrailing Angle Formulas

tan **R4P** = cos **R1** / tan **DD**
tan **r4P** = cos **R1** / tan **dd**

tan **A5P** = tan **R1** sin **R4P**
tan **a5P** = tan **R1** sin **r4P**

Angle measured between traces of Tangent Planes on the Oblique Plane:
180° – (**R4P** + **r4P**)

Slope Angles entered in Javascript Calculator:
tan **SS** = tan **R5P** $\sin^{2}$ **DD**
tan **ss** = tan **r5P** $\sin^{2}$ **dd**

DD + **dd** + **Angle negotiated by Handrail** = 180°