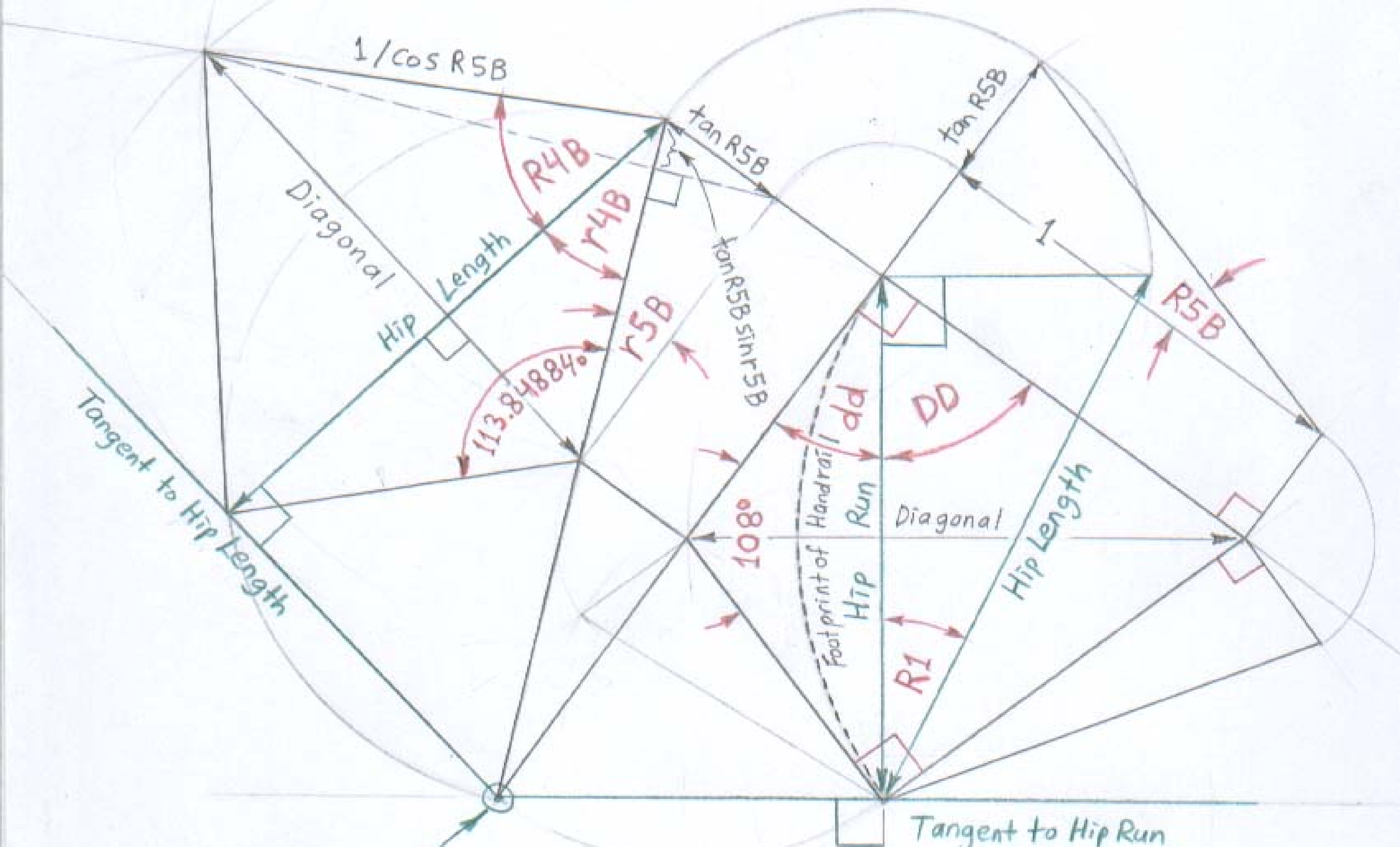
Study of Tangent Handrail Geometry

Equal Slopes, 108° Corner Angle between Tangents in Plan View (Handrail negotiates a 72° Turn)

Development of Tangent Handrail
Equal Slopes, Corner Angle = 108°

$$R4B + r4B = \arccos(\tan R5B \sin r5B \cos R5B)$$
$$= \arccos(\sin R5B \sin r5B) = 84.05093°$$



Tangent to Hip Length,
Tangent to Hip Run, and Hypotenuse
of triangle of r5B produced
Converge at this point

SS = 31.43101°
DD = 54°
R1 = 26.30904°
R4B = 50.97535°
R5B = 16.20476°
A5B = 21.01219°

ss = 40.06953°
dd = 36°
r4B = 33.07558°
r5B = 21.80141
a5B = 15.10056°

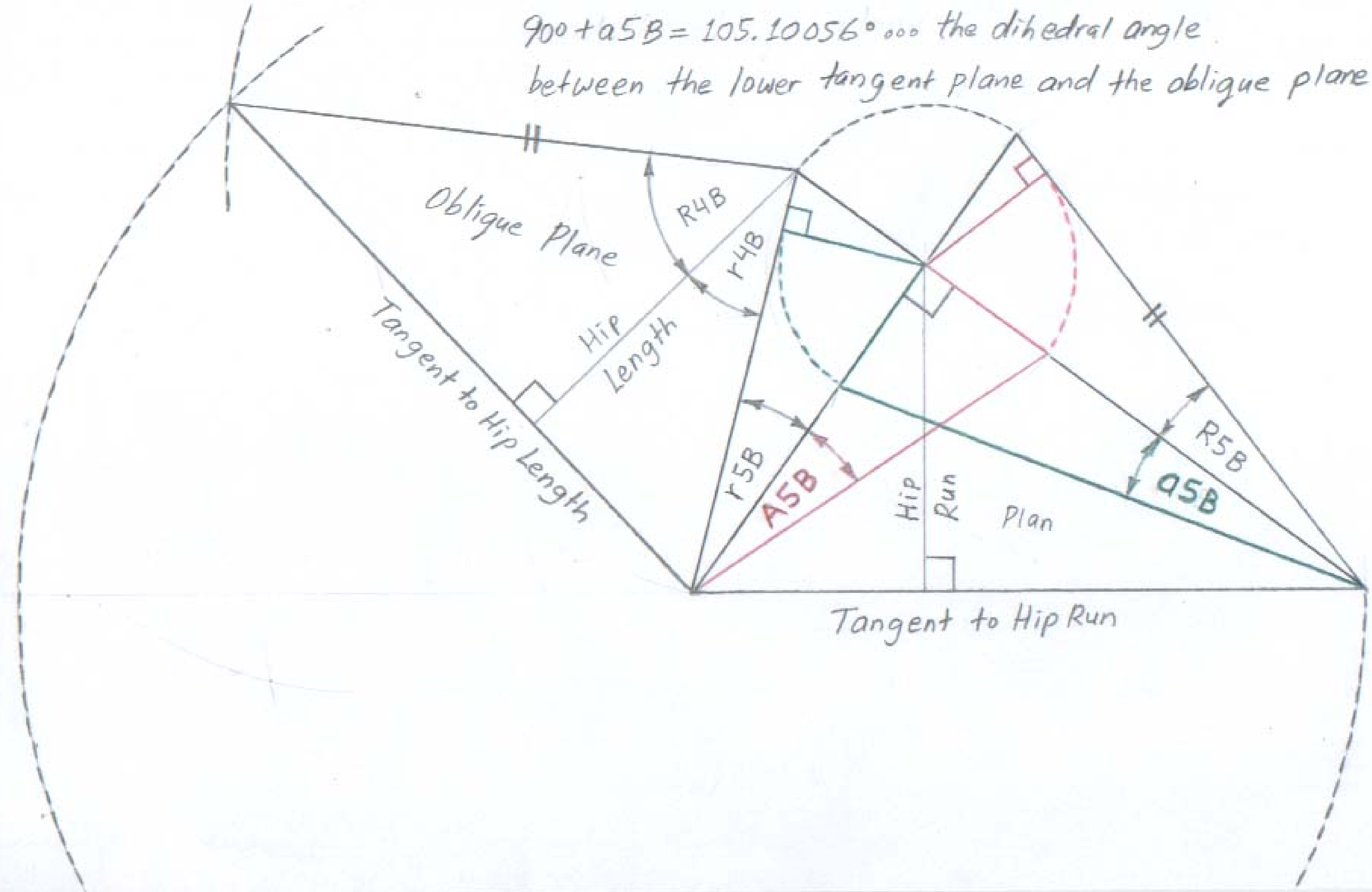
Tetrahedral Model of Tangent Handrail

Equal Slopes, Corner Angle = 108°

Triangle of r5B models both tangent planes

$90° - a5B = 74.89944°$ ... the dihedral angle between the upper tangent plane and the oblique plane

$90° + a5B = 105.10056°$ ... the dihedral angle between the lower tangent plane and the oblique plane

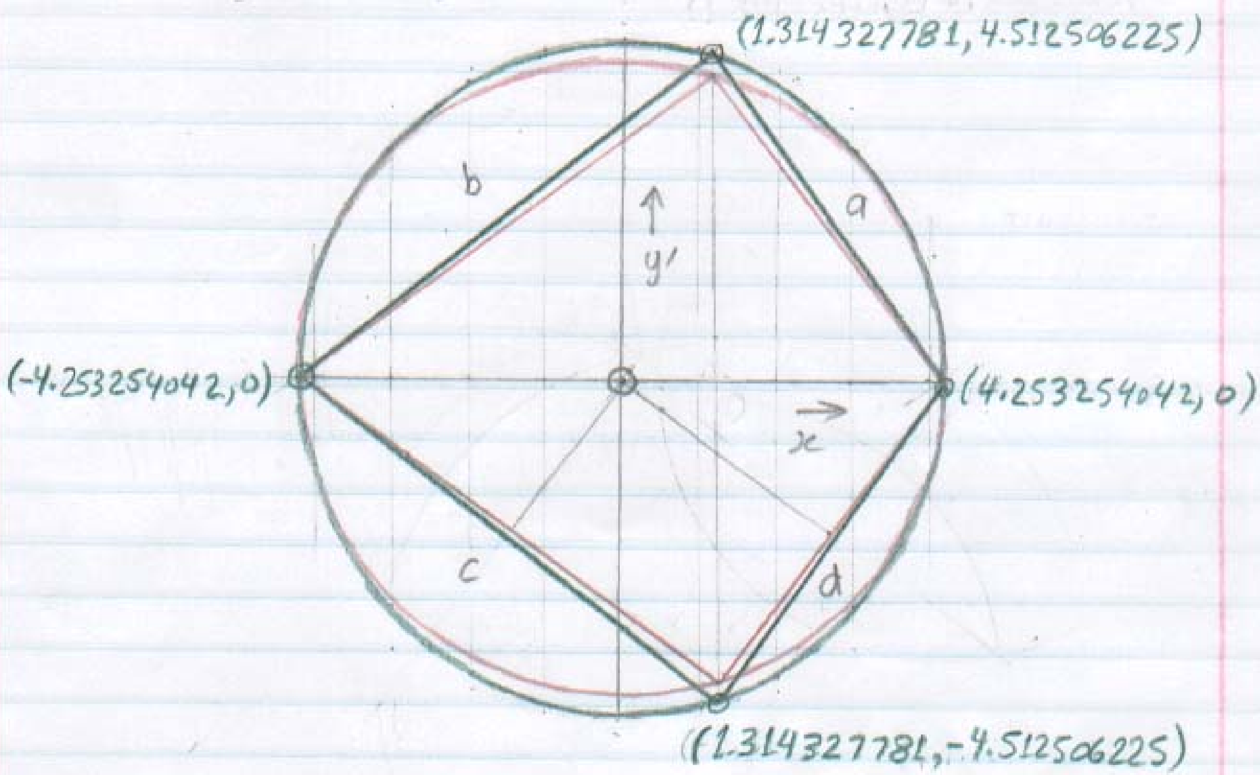


Cyclic Quadrilateral mapped onto Ellipse on Oblique Plane

Cutting plane angle = 26.30904346° = θ

Minor axis (on x-axis) = 8.506508083 = 5/cos54°

Major axis (on y'-axis) = minor axis/cos θ = 9.489459664



Angle between a, b = 84.050933°

Angle between c, d = 84.050933°

Angle between b, c = 78.049299°

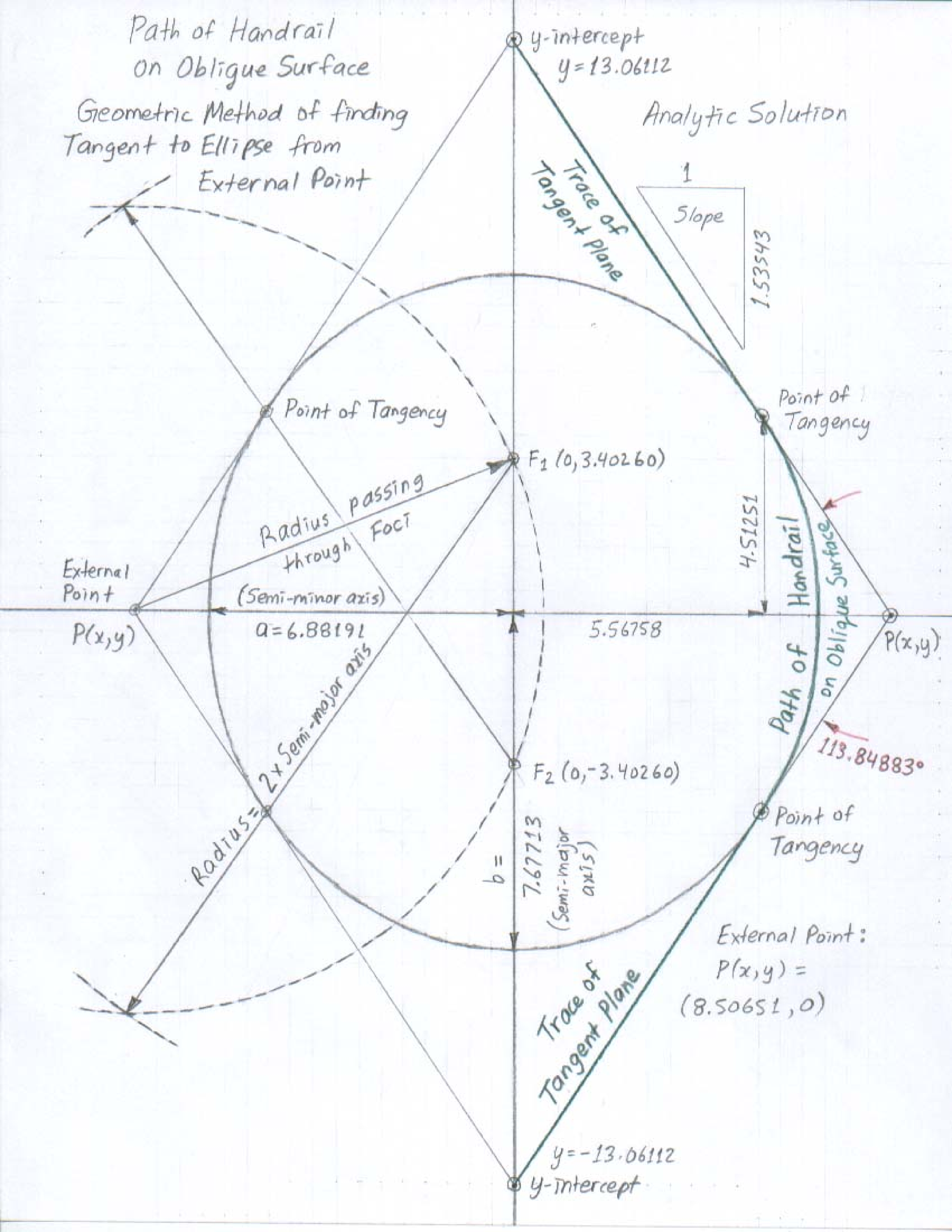
Angle between a, d = 113.848835°

$$\text{Angle between Lines} = \arccos\left[\frac{x_a x_b + y_a y_b}{\sqrt{x_a^2+y_a^2}\ \sqrt{x_b^2+y_b^2}}\right]$$

Path of Handrail on Oblique Surface

Geometric Method of finding Tangent to Ellipse from External Point

Analytic Solution



ANALYTIC SOLUTION

Handrail following Surface of Oblique Plane

Plan View Radius $= 5 \tan 54° = 6.881909602$
$= $ Semi-minor axis (a)

Hip Slope Angle $= \arctan(4/(2 \times 5 \sin 54°))$
$= 26.30904346°$

Semi-major axis $= 5 \tan 54° / \cos 26.30904346°$
$= 7.677134136\ (b)$

External Point $P(x,y)$:

$x = 5 \tan 54° \cos 36° + 5 \cos 54° = 8.506508084,\ y = 0$

Slope of Tangent $(m) = \left(xy \pm \sqrt{a^2y^2 + b^2(x^2 - a^2)}\right)/(x^2 - a^2)$
$= \pm 1.535426826$

Equations of Tangents: $y = mx \pm \sqrt{b^2 + a^2m^2}$
$= \pm 1.535426826 \pm \underbrace{13.06112071}_{y\text{-intercept}}$

Point of Tangency with Ellipse:

x (Plan View) $= 5 \tan 54° \cos 36° = 5.567581822$

y (Plan View) $= 5 \sin 54° = 4.045084972$

y' (on Oblique Plane) $= 5 \sin 54° / \cos 26.30904346°$
$= 4.512506225$

or y' (Ellipse Equation) $= b\sqrt{a^2 - x^2}/a = 4.512506222$

Angle between Tangents $= 2 \times \arctan(1.535426826)$

Compare to trig and vector $\longrightarrow$ $= 113.8488347°$

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