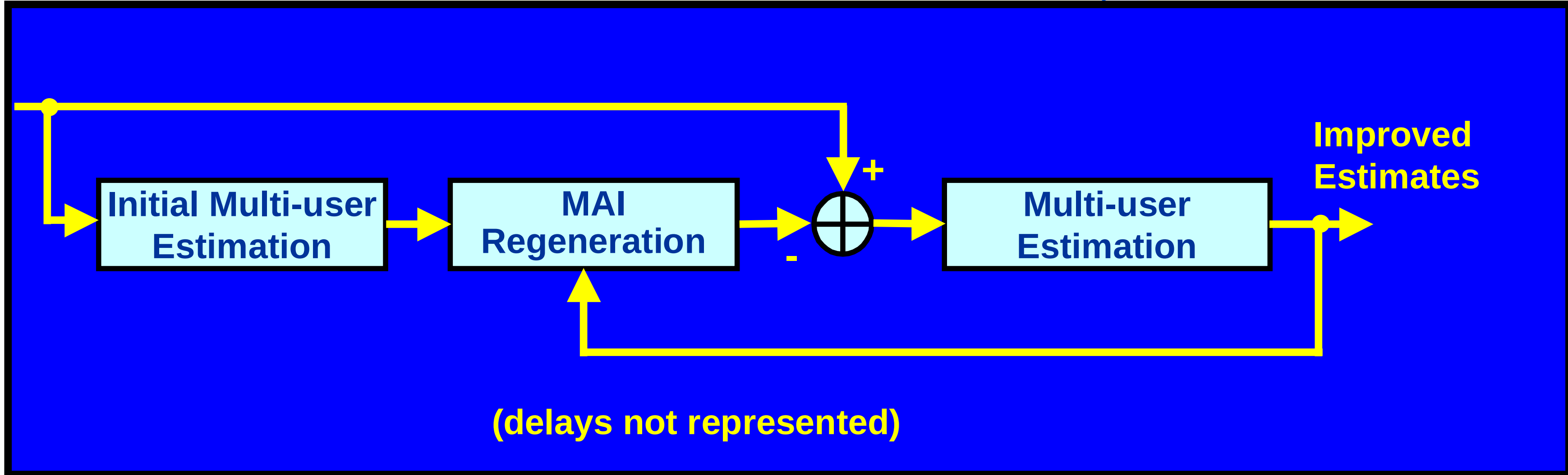
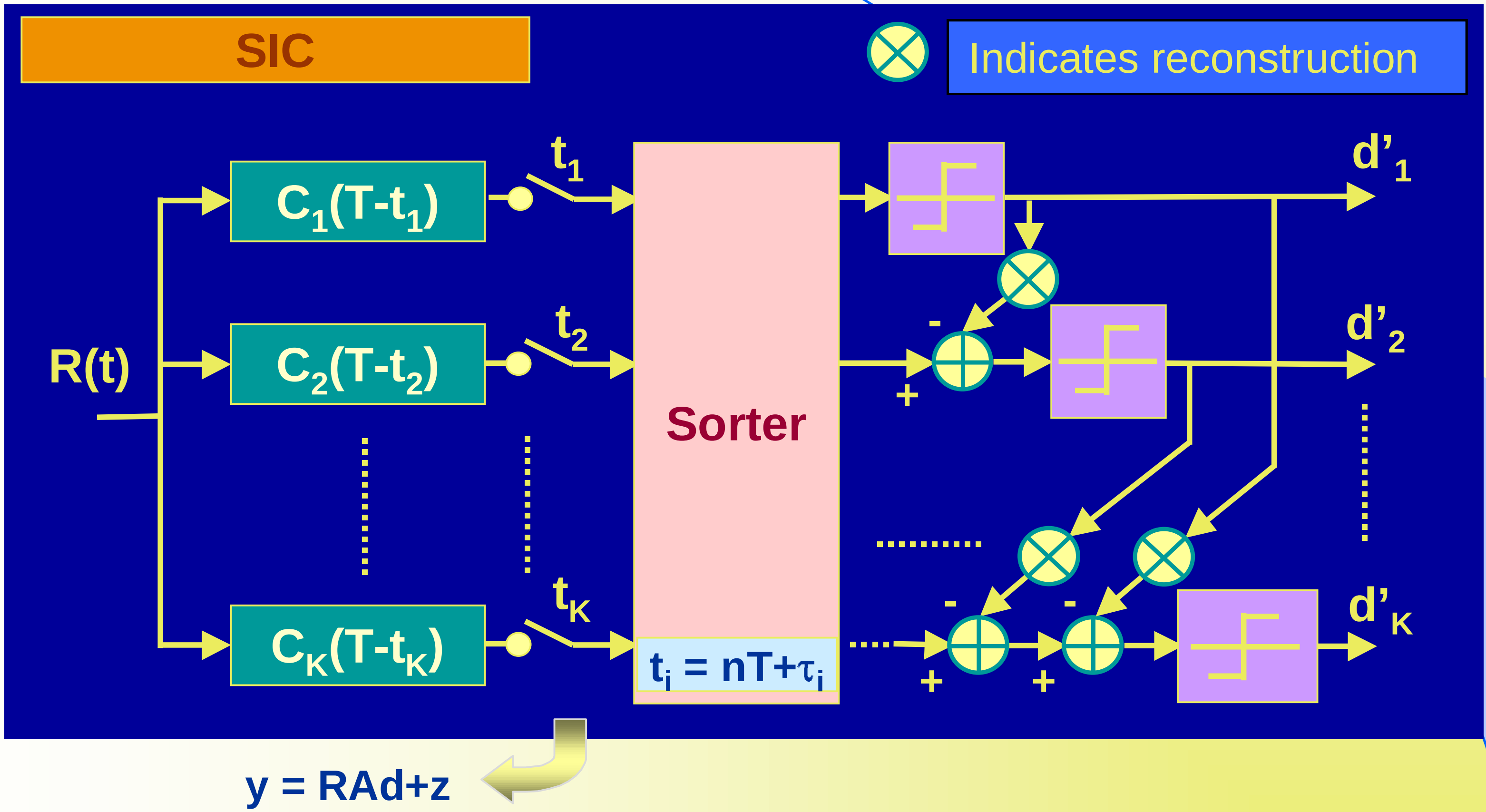
Multi-user Detection - Non Linear Approach

The non linear approach or IC (Interference Cancellation) type MUD try to subtract the influence of others users (or most of them) from the desired user and after this operation we ideally have a “cleaned” version of the input signal for subsequent detection.



The first scheme considered from this class is known as SIC (Successive Interference Cancellation). In the SIC scheme initially we should order the signals in concordance of their power. The first signal (the strongest) is demodulated and reconstructed. From the delayed input version we subtract this reconstructed signal and we are ready to demodulate the second signal, and so on.

This type of detector eliminates all strongest MAI for each user. In the last step, for the last user, we have only AWGN type noise without any MAI. The next figure illustrates the process.



This type of detector has an inferior performance when compared with linear type multi-user detectors and also has a delay proportional to number of active users. Its advantage is the relative simplicity (avoids matrix inversion).

The SIC scheme is well adapted for users with some power disparities. Now, we can ask if there exists a power disparity distribution for system optimization? The answer is yes and can be exploited with the argument that we want a uniform performance for all users. With this performance objective in mind we can impose SNR=constant for all K users. From the ordering and with the k^{th} user's power nominated by P_k we can write

$$\mathbf{P_k / (P_{k+1} + P_{k+2} + \ldots + P_K + I_0)} = \gamma$$

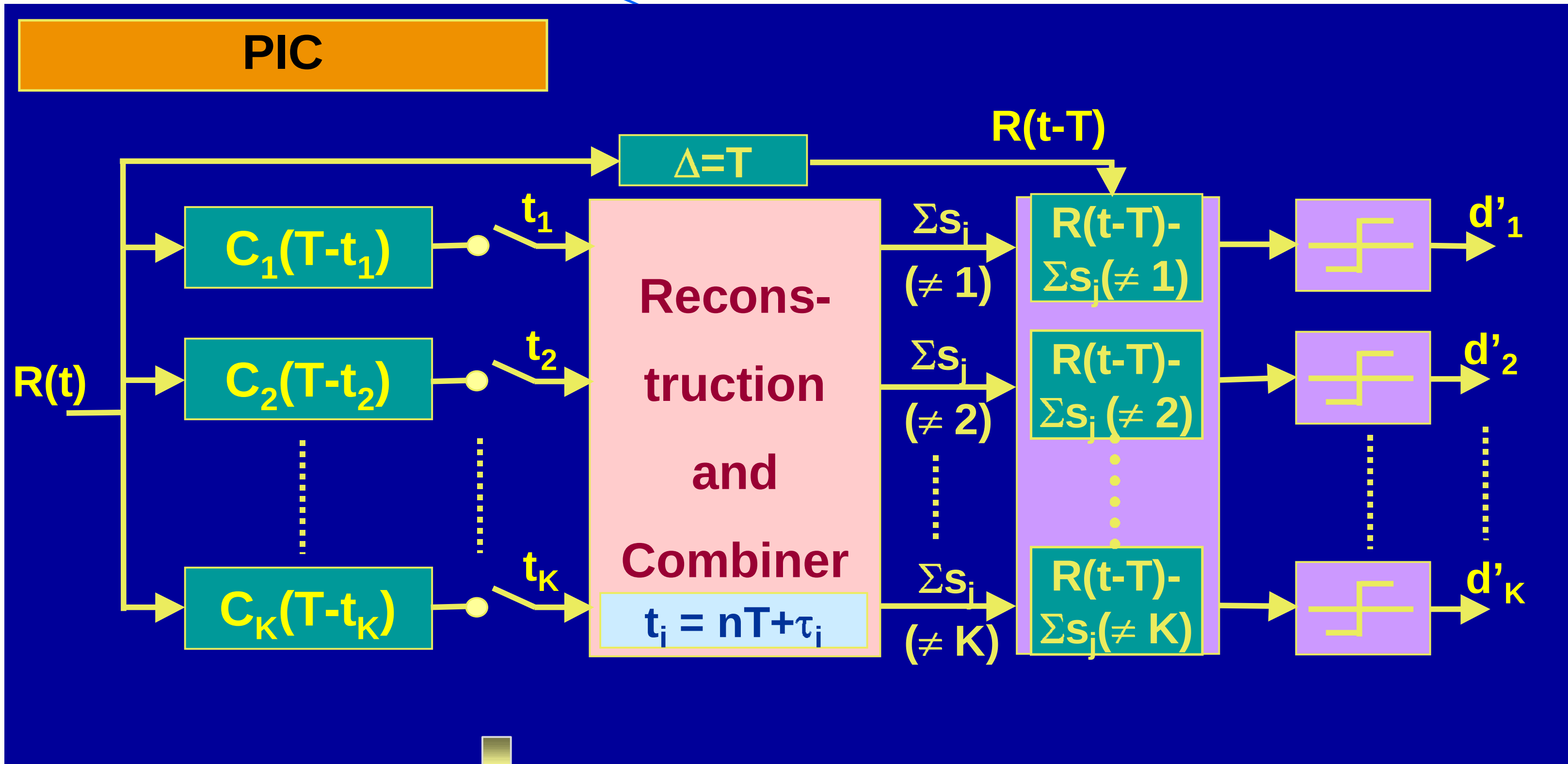
With k=1,..,K and where I_0 denotes the AWGN power.

Solving this we have $\mathbf{P_i} = \gamma(\gamma + \mathbf{1})^{\mathbf{K-i}}\,\mathbf{I_0}$ or $\mathbf{P_i / P_{i+1}} = \gamma + \mathbf{1}$

Which indicates that an exponential power distribution for the users' power is the ideal for the SIC scheme.

The second type of IC is known as PIC (Parallel Interference Cancellation).

Here after all signal reconstruction we subtract from the delayed input, R(t-T), all reconstructed signals, Σs_j, except the desired, for each branch.



$y = RAd+z$

The PIC scheme has better performance than SIC scheme. In an ideal scenario the final signal for demodulation on each branch have no MAI from other users and, additionally, the total delay is only one bit interval. As we can see in the structure and based on our former conclusions about power disparities, we can infer that the perfect power control scheme is the best suited power control algorithm for PIC detectors. Despite this fact the scheme is near far resistant.

The ZF-DF (Zero Forcing - Decision Feedback) is another IC approach and is based on Cholesky decomposition, reference [25], for matrix R

$$\mathbf{R} = \mathbf{F}^{\mathbf{T}}\mathbf{F}$$

where F is a lower triangular matrix.

Applying $(F^T)^{-1}$ to the matched filter outputs we obtain

$$\mathbf{F}^{-\mathbf{T}}\mathbf{y} = \mathbf{FAd} + \mathbf{F}^{-\mathbf{T}}\mathbf{z}$$

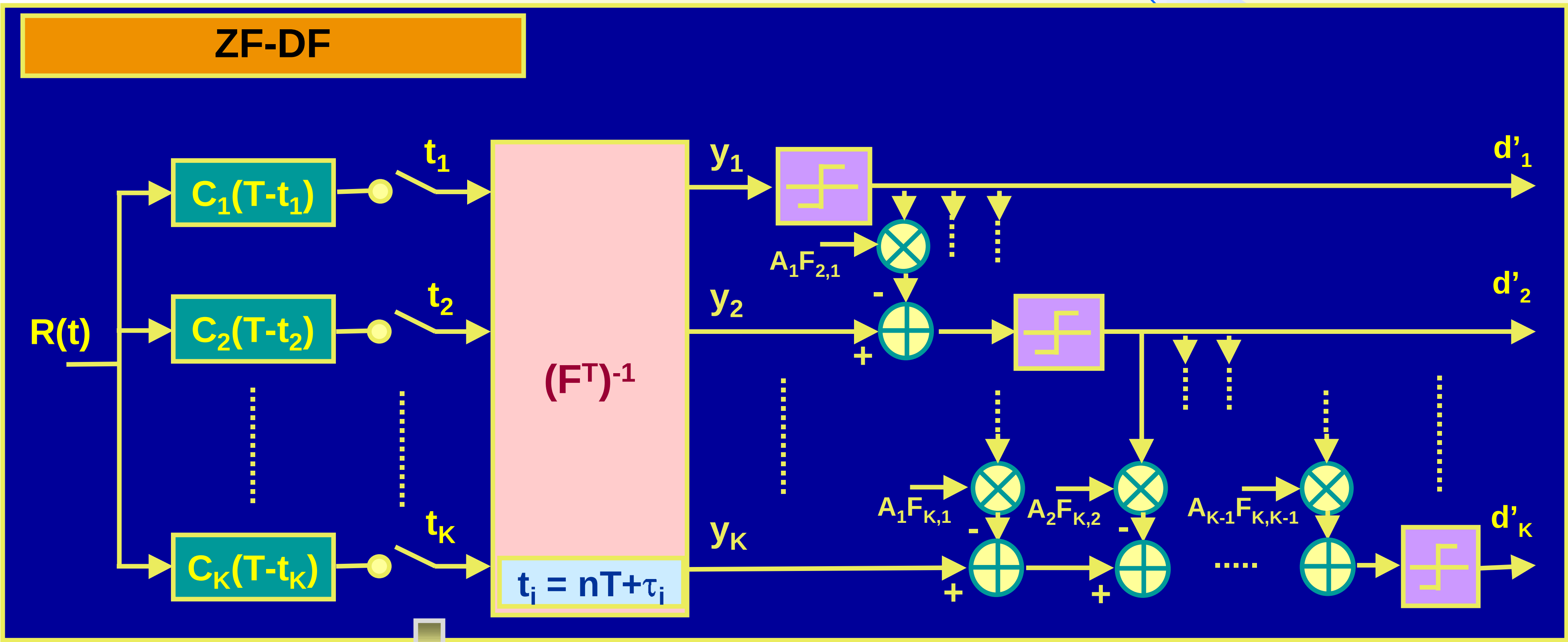
where we have used $F^{-T} = (F^T)^{-1}$ to simplify notation.

With this decomposition we can see that the signal for the first user is MAI free; the second user contains MAI only from the first; the third user contains MAI only from the first and second and so on.

Generically the i^{th} user contains MAI only from the previous users which was detected and can be subtracted.

The approach is analogous to the ZF-DF equalizer used to combat ISI, reference [25]. The next figures show us the FAd matrix and the ZF-DF implementation.

$$
FAd = \begin{bmatrix} F_{1,1} & 0 & 0 & 0 & 0 \\ F_{2,1} & F_{2,2} & 0 & 0 & 0 \\ F_{3,1} & F_{3,2} & F_{3,3} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ F_{K,1} & F_{K,2} & F_{K,3} & \cdots & F_{K,K} \end{bmatrix} \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 \\ 0 & 0 & A_3 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & 0 & A_K \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \cdots \\ d_K \end{bmatrix}
$$



$y = RAd+z$

Additional details in reference [36], in Portuguese. A final remark: As we can see the basic problem in linear multi-user detection is to solve a linear equation

$$\mathbf{y} = \mathbf{RAd} + \mathbf{z}$$

or in other form **y=Qu** (for decorrelating detector **Q=R** and for MMSE detector **Q=R+** N_o**A**$^{-2}$ where the two matrices are symmetric positive definite and block tridiagonal).

Instead of inverting the matrix **Q** using a direct method we can solve the linear equation using iterative methods. All iterative methods depend on a splitting of **Q**. Let **Q=D+L+U** where **D** is a diagonal matrix, **L** is strictly lower triangular and **U** is strictly upper triangular. Some of possible iterations are:

Jacobi iteration:

$$\mathbf{u}_{m+1} = \mathbf{D}^{-1}[\mathbf{y} - (\mathbf{L} + \mathbf{U})\,\mathbf{u}_m]$$

(leads to PIC)

Now the inversion is simple task because **D** is a diagonal matrix. It can be shown that Jacobi iteration converges iff the eigenvalues of **-D**$^{-1}$**(L+U)** all have magnitudes smaller than one and this fact allows us to explain why standard PIC does not converge.

Gauss-Seidel iteration:

$$\mathbf{u}_{m+1} = \mathbf{D}^{-1}[\mathbf{y} - \mathbf{L}\mathbf{u}_{m+1} - \mathbf{U}\mathbf{u}_m]$$

(leads to SIC)

Other possibilities: Jacobi Over-Relaxation iteration, Successive Over-Relaxation iteration, Conjugate Gradient iteration and so on. See reference [30] for additional details.

The main problem in applying the above iterations for detection is the size of Q which is N×K (slide 130) where $K \rightarrow \infty$ for continuous transmission. The inherent detection delay is at least K which is unacceptable even for $K < \infty$.

In fact this challenging area is open for new contributions.

Finally, concluding this multi-user section, we highlight some new possible approaches for IC type detectors, reference [13]:

1- Using the decorrelating detector as the first stage;

2- Using the already detected bits at the output of the current stage to improve detection of the remaining bits in the same stage (multi-stage principles will be explored in the next section);

3- Doing a partial MAI cancellation at each stage, with the amount of cancellation increasing for each successive stage, reference [29].

We will return to this subject in the Multi-rate CDMA Systems section where the partial MAI cancellation will be a very usual approach for multi-stage multi-rate multi-user PIC type detectors.

MPIC - Multi-stage Parallel Interference Cancellation

For this section we will assume a symbol synchronous system so that we need only consider one symbol interval i. The (chip rate) received signal for the i^{th} symbol interval is

$$\mathbf{r(i)} = \mathbf{S(i)}\,\mathbf{d(i)} + \mathbf{n(i)}$$

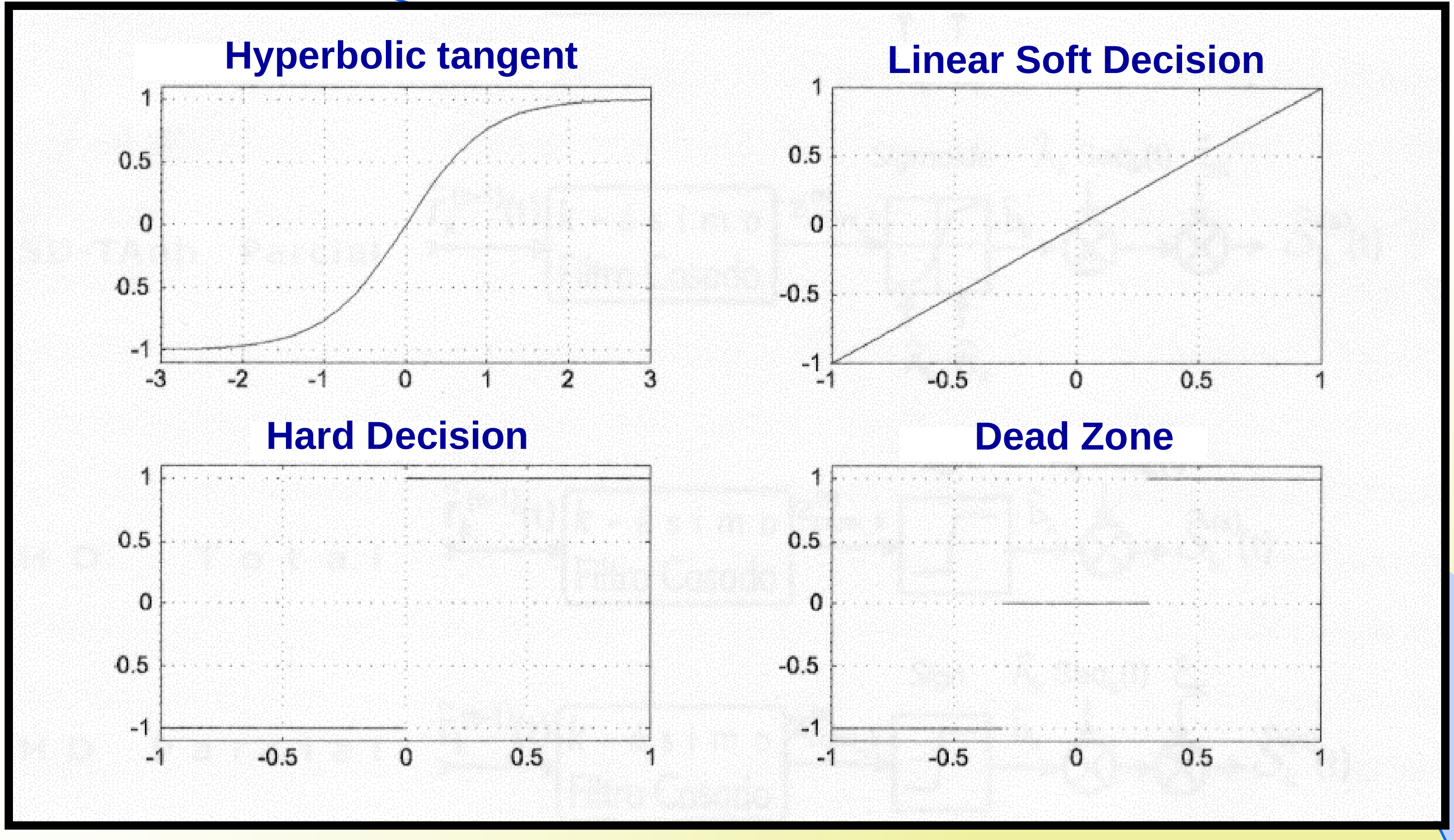
S(i) is the spreading code matrix where its k^{th} column corresponds to the k^{th} user; **d(i)** is the vector of symbols transmitted by all users and **n(i)** is an AWGN vector representing receiver's noise. To detect $d_1(i)$ (the first symbol in **d(i)**) in the first stage, we form the decision statistics

$$\mathbf{y_{1,1}(i)} = \mathbf{s_1^H(i)\,r(i)}$$

i. e., it is simply the conventional detector. **(compare)**

In our notation when there are two subscripts the first represents the stage index and the second the user index, i. e., $y_{m,k}(i)$ refers to the k^{th} user in the m^{th} stage.

Now $y_{m,k}(i)$, the decision signal, will be mapped with a function f(.) to obtain $z_{m,k}(i)$, the effective signal for cancellation. Some possible approach are presented in the next figure.



Using a Hyperbolic tangent decision circuit we are approaching the Hard decision circuit with strong signals and to the Linear Soft decision for weak signals (when we are not confident with the decision). On the other hand with Dead Zone type decision circuit we avoid making a decision when we are not sure (dead Zone's window can be optimized through BER minimization, see reference [34] for details).

Now in the second stage, for the k^{th} user, we want to cancel the detected interference for the K-1 users.

$$y_{2,k}(i) = s_k^H(i)[r(i) - \sum_{j=1, j\neq k}^{K} z_{1,j}(i)\, s_j(i)]$$

the signal in the brackets are used for the detection of k^{th} user in the second stage.

$$= s_k^H(i)[r(i) - \sum_{j=1}^{K} z_{1,j}(i)\, s_j(i) + z_{1,k}(i)\, s_k(i)] = s_k^H(i)\, e_1(i) + z_{1,k}(i)$$

where

$$e_1(i) = r(i) - \sum_{k=1}^{K} z_{1,k}(i)\, s_k(i)]$$

denotes the residual signal with all estimated interference from the first stage cancelled.

The process can be easily generalized with the following expressions

$$y_{m,k}(i) = s_k^H(i)\, e_{m-1}(i) + z_{m-1,k}(i)$$

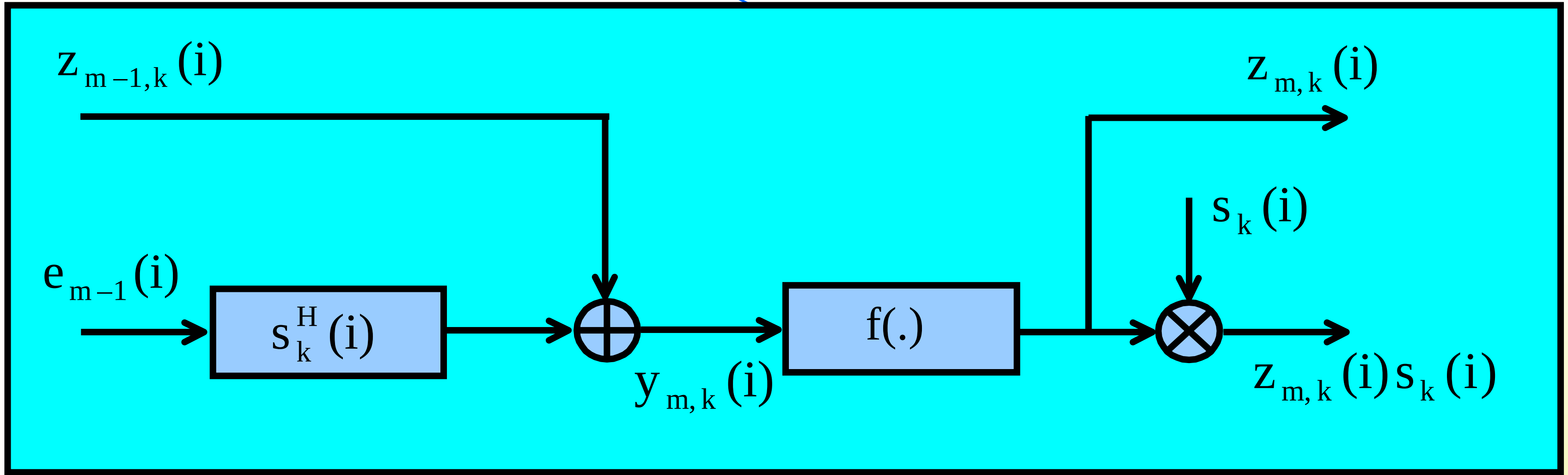
and

$$e_m(i) = r(i) - \sum_{k=1}^{K} z_{m,k}(i)\, s_k(i)$$

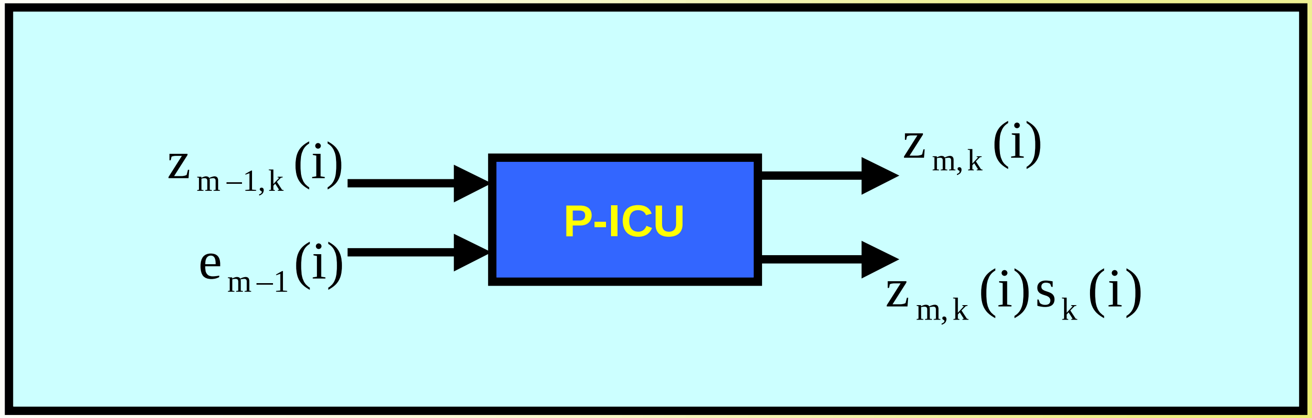
with

$$z_{m,k}(i) = f[\, y_{m,k}(i)\,]$$

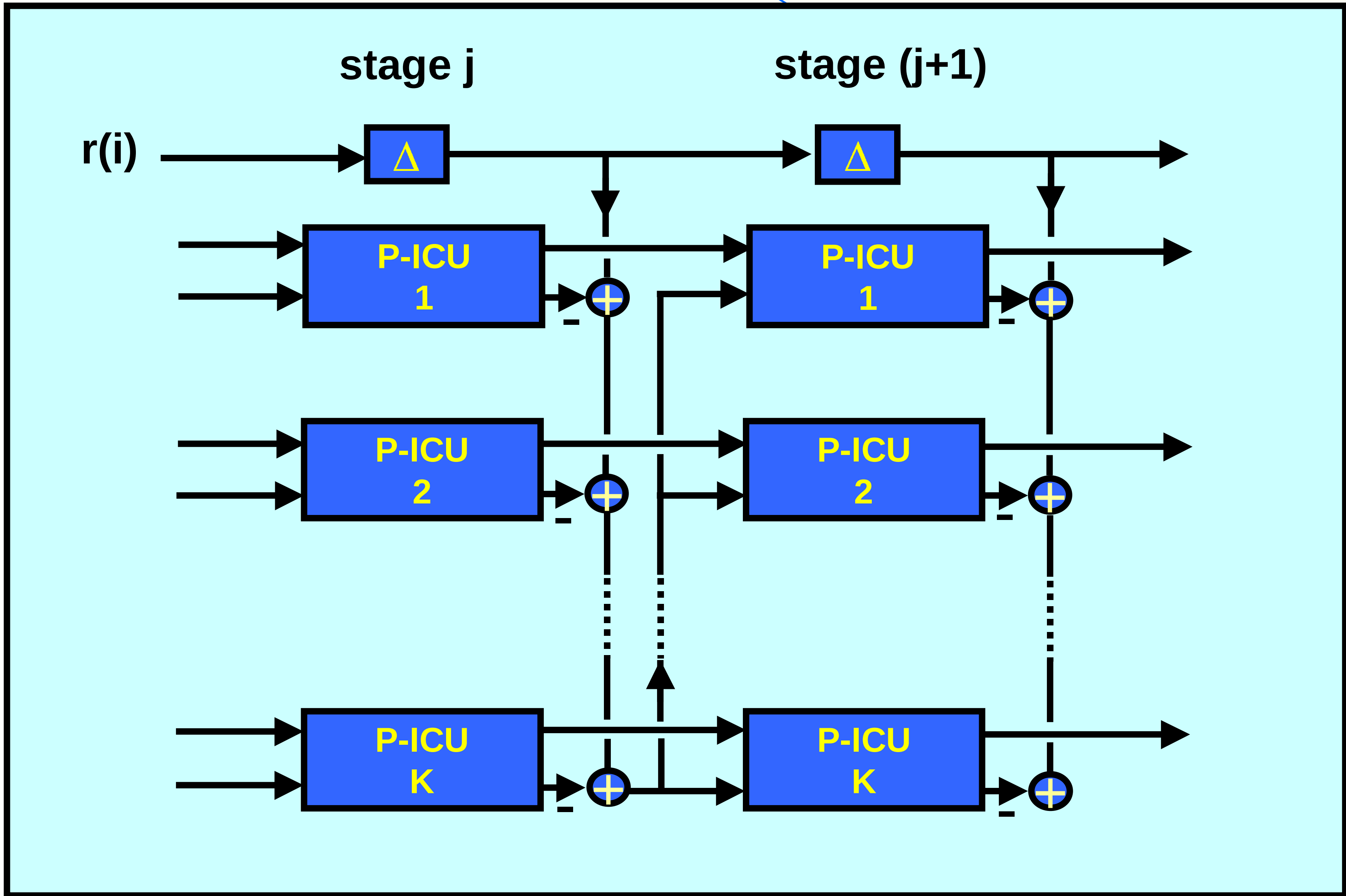
This representation allows us to implement the multi-stage PIC in a modular form with P-ICU (Parallel Interference Cancellation Unit)



For the first stage (m=1) we adopt the following initial values: $z_{m-1,k}(i)=0$ and $e_{m-1}(i)=r(i)$. This structure is represented in a modular form as



Finally with this representation we can implement a K user, multi-stage PIC detector as a set of interconnected P-ICUs (only two successive stages are represented below). Obviously, for the last stage we should use a hard decision circuit for final detection. See reference [33] for additional details.



MSIC - Multi-stage Successive Interference Cancellation

We can repeat the previous procedure for multi-stage SIC scheme. For the detection signal of first user in the first stage we can write

$$\mathbf{y_{1,1}(i) = s_1^H(i)\,r(i)}$$

For the second detection signal in the first stage we should subtract the first user estimate from the received signal

$\mathbf{r_{1,2}(i) = r(i) - z_{1,1}(i)\,s_1(i)}$ where $\mathbf{z_{m,k}(i) = f[\,y_{m,k}(i)\,]}$ is the mapping function.

Now from $\mathbf{r_{1,2}}$ we can express $\mathbf{y_{1,2}}$ as $\mathbf{y_{12}(i) = s_2^H(i)\,r_{1,2}(i)}$

And we follow with this procedure until all K (first stage) symbols have been detected. For the first symbol in the second stage we use as input signal

$$\mathbf{r_{2,1}(i) = r(i) - \sum_{k=2}^{K} z_{1,k}(i)\,s_k(i)}$$

Which is the input signal with all interference based on the estimate of first stage removed, except for the first user.

Of course, this signal should be "cleaner" than the signal in the first stage which did not benefit from canceling interference from users 2 to K.

Therefore

$$y_{2,1}(i) = s_1^H(i)\,r_{2,1}(i) = s_1^H[r(i) - \sum_{k=1}^{K} z_{1,k}(i)\,s_k(i) + z_{1,1}(i)\,s_1(i)] = z_{1,1}(i) + s_1^H(i)\,r_{1,K+1}(i)$$

Where $\mathbf{r}_{1,K+1}(\mathbf{i})$ was defined as

$$r_{1,K+1}(i) = r(i) - \sum_{k=1}^{K} z_{1,k}(i)\,s_k(i)$$

and represents the "cleaned" input signal after the first stage, i. e., a signal with all estimated interference in the first stage removed. For the second symbol in the second stage we form

$$r_{2,2}(i) = r_{2,1}(i) - z_{2,1}(i)\,s_1(i) = r(i) - \sum_{k=2}^{K} z_{1,k}(i)\,s_k(i) - z_{2,1}(i)\,s_1(i)$$

Which is the input signal with all interference based on the estimate of the first stage removed, except for the first user which was removed with the second estimate. From this expression we can write the next decision signal as

$$y_{2,2}(i) = s_2^H(i)[r(i) - z_{2,1}(i)\,s_1(i) - \sum_{k=3}^{K} z_{1,k}(i)\,s_k(i)] = s_2^H(i)[r_{2,2}(i) + z_{1,2}(i)\,s_2(i)]$$

Therefore $$y_{2,2}(i) = s_2^H(i)\,r_{2,2}(i) + z_{1,2}(i)$$

Now we are able to generalize this procedure to other users in the second stage with the following expressions

$$y_{2,k}(i) = s_k^H(i) r_{2,k}(i) + z_{1,k}(i)$$

$$r_{2,k}(i) = r_{2,k-1}(i) - z_{2,k-1}(i)\, s_{k-1}(i)$$

With the definition $r_{2,1}(i) = r_{1,K+1}(i)$

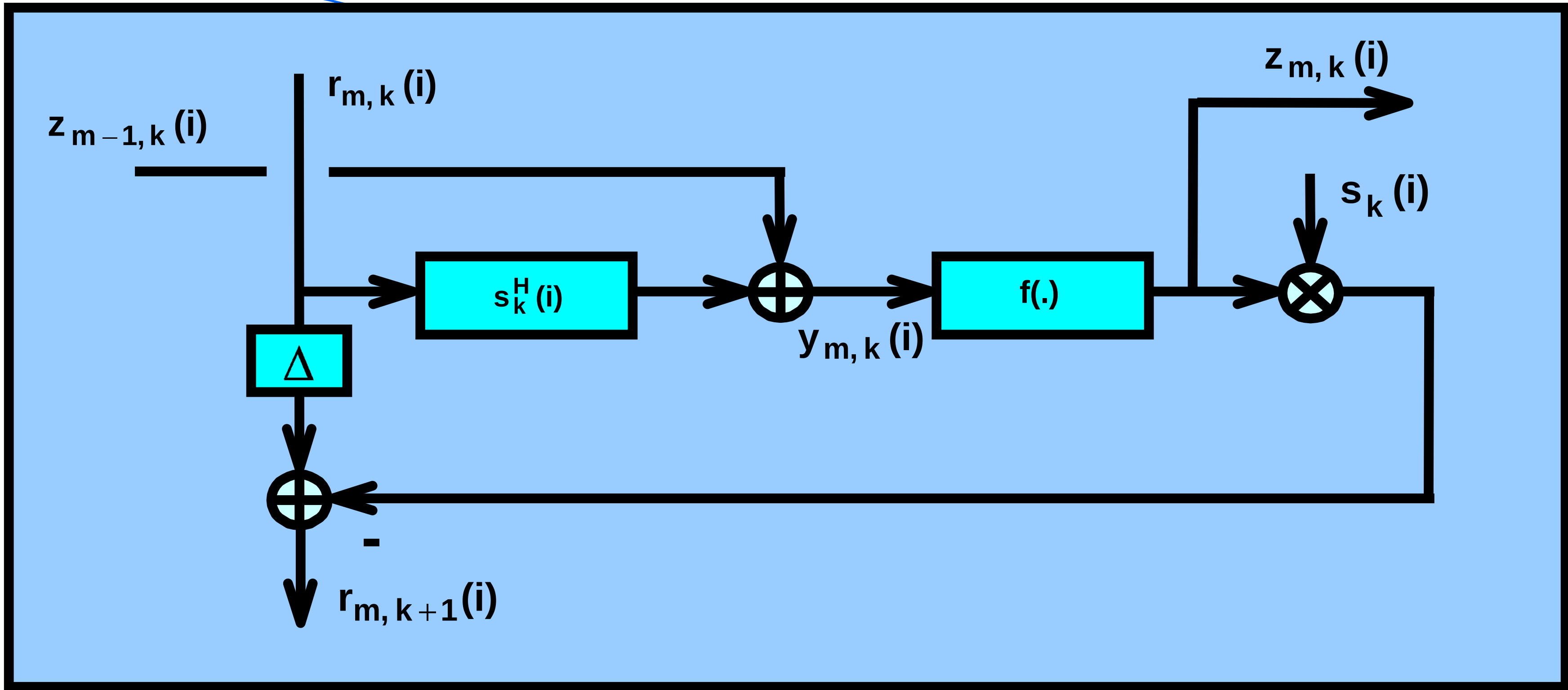
And for a generic user k in the m-th stage

$$y_{m,k}(i) = s_k^H(i) r_{m,k}(i) + z_{m-1,k}(i)$$

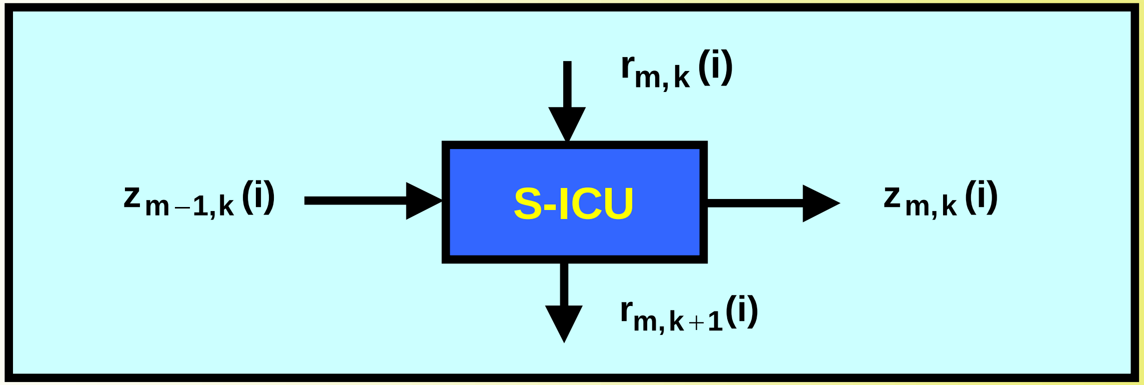
$$r_{m,k+1}(i) = r_{m,k}(i) - z_{m,k}(i)\, s_k(i)$$

With the definition $r_{m,1}(i) = r_{m-1,K+1}(i)$

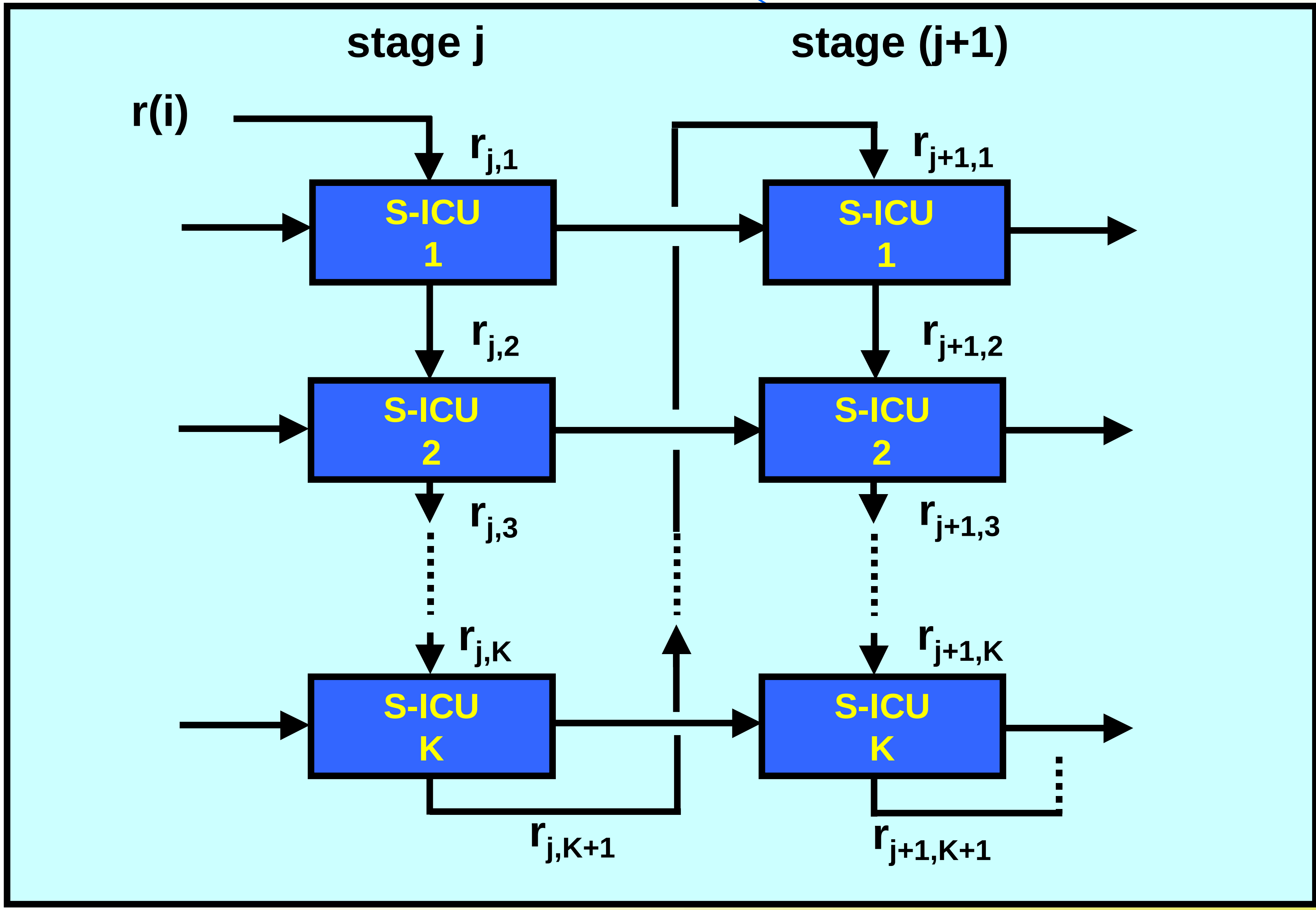
This representation allows us to implement the multi-stage SIC in a modular form with S-ICU (Serial Interference Cancellation Unit)



For the first stage (m=1) we adopt the following initial values: $z_{m-1,k}(i)=0$ and $r_{m,1}(i)=r(i)$. This structure is represented in a modular form as



Finally with this representation we can implement a K user, multi-stage SIC detector as a set of interconnected S-ICUs (only two successive stages are represented below). Obviously, for the last stage we should use a hard decision circuit for final detection. See reference [32] for additional details.



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