ECES 719
Wireless Communications

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Lecture 26

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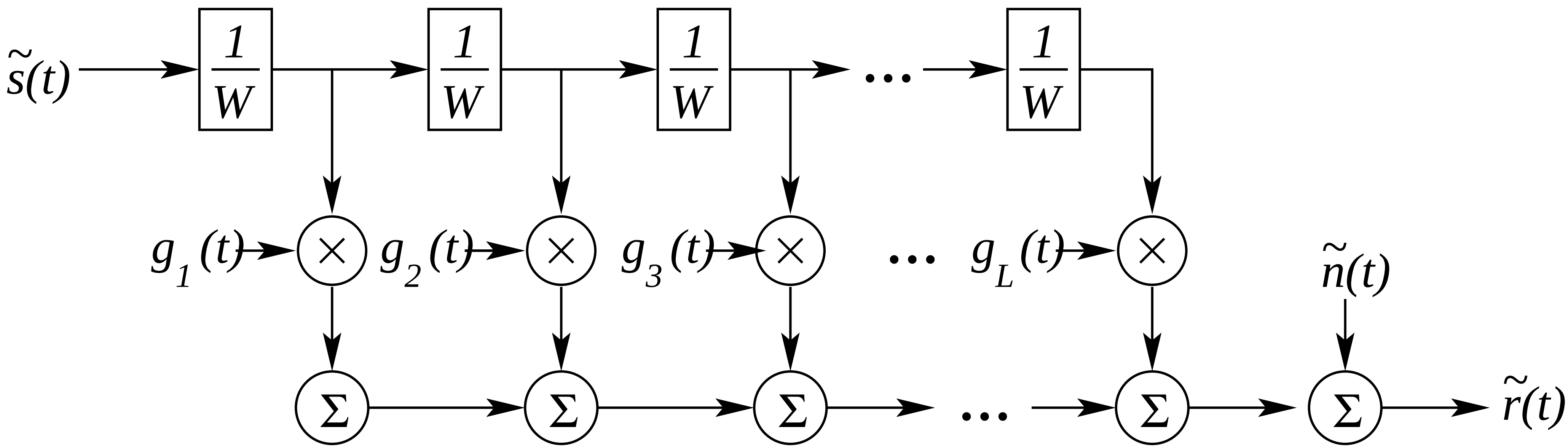
Reminder!!!

- Exam #2: Friday, 11am-1pm, in class
  - ⋆ Open book (Stüber, 1st or 2nd edition)
  - ⋆ One page (front and back, $8\frac{1}{2} \times 11$ in.) of notes
- Project:
  - ⋆ Due Wednesday, March 14, by 4:00pm.
  - ⋆ No late reports accepted!

Tapped Delay Line Model



For ideal Nyquist chip amplitude pulse shaping, then $W = 1/T_c$, so the model becomes a T_c-spaced or chip-spaced tapped delay line.

DS-SS Diversity Reception

- Receiver structures for DS-SS systems:
  - ⋆ **Conventional detector**: MF receiver, treats multiple-access interference (MAI) as additional noise.
  - ⋆ **Multiuser detector**: use knowledge of users' signals to simultaneously detect all signals.
  - ⋆ **Multipath rejection receiver**: use autocorrelation properties of spreading sequences to reject multipath interference.
  - ⋆ **Multipath diversity receiver**: use autocorrelation properties to resolve multipath components and combine for a diversity advantage.
- Suppose one of M symbols with complex envelope $\tilde{s}_m(t)$ are transmitted each baud interval.
- From the frequency-selective fading model (on previous page), the received complex envelope is

$$\tilde{r}(t) = \sum_{l=1}^{L} g_l(t)\tilde{s}_m\left(t - \frac{l}{W}\right) + \tilde{n}(t) = \hat{s}_m(t) + \tilde{n}(t)$$

RAKE Receiver

- The ML coherent receiver uses a correlator (or MF) to compute the metrics for the possible $\hat{s}_m(t)$:

$$\begin{aligned}\mu(m) &= \Re\left\{\int_0^T \tilde{r}(t)\hat{s}_m^*(t)\,dt\right\} - \hat{\mathcal{E}}_m \\ &= \Re\left\{\int_0^T \tilde{r}(t)\sum_{l=1}^{L} g_l^*(t)\tilde{s}_m^*(t-l/W)\,dt\right\} - \hat{\mathcal{E}}_m\end{aligned}$$

where $\hat{\mathcal{E}}_m$ is the energy of the received pulse $\hat{s}_m(t)$.

- This receiver correlates $\tilde{r}(t)$ with delayed versions of the possible waveforms $\tilde{s}_m(t)$, followed by MRC (Fig. 1).

- By changing the variable of integration above, we find an alternative RAKE receiver where $\tilde{s}_m(t)$ is correlated with delayed versions of $\tilde{r}(t)$ (Fig. 2).

RAKE Receiver #1

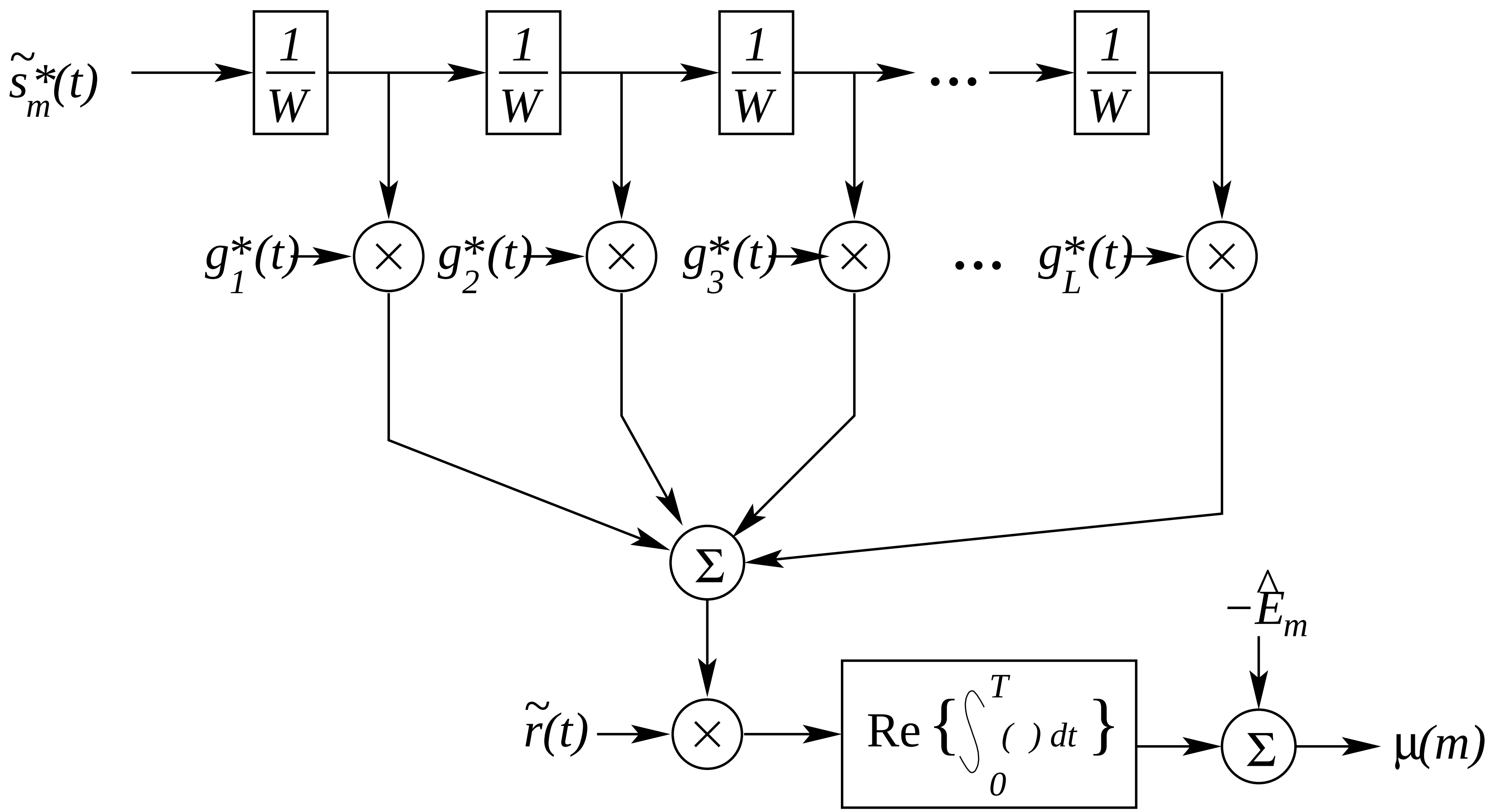


Fig. 1: *RAKE Receiver using delayed* $\tilde{s}_m(t)$.

RAKE Receiver #2

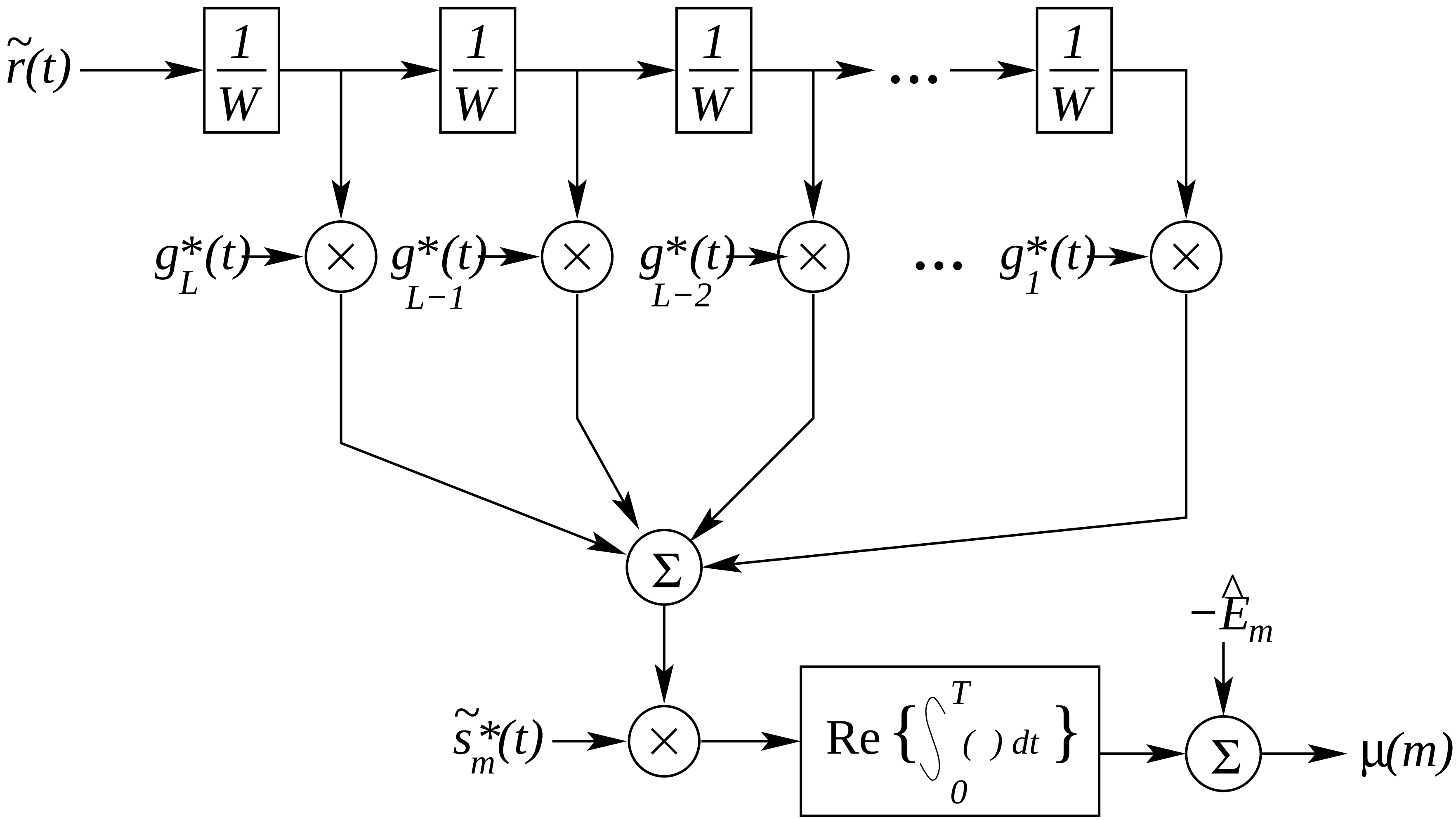


Fig. 2: *RAKE Receiver using delayed* $\tilde{r}(t)$.

Bit Error Probabilities

- Consider AWGN and fading only (no MAI)!
- For AWGN only, P_b (i.e., BER) is exactly the same for an unspread signal (no improvement for DS-SS on AWGN).
- For fading channels, consider DS/BPSK using a RAKE receiver:

  ⋆ $\tilde{s}_0(t) = -\tilde{s}_1(t) = Ah(t)$ where

$$h(t) = \sum_{k=0}^{G-1} a_k h_c(t - nT_c)$$

  ⋆ The decision variable using MRC combining of the RAKE gives

$$\begin{aligned} \mu &= \sum_{m=1}^{L} \sum_{l=1}^{L} \Re \left\{ g_m g_l^* \int_0^T \tilde{s}_0(t - m/W) \tilde{s}_0^*(t - l/W) \, dt \right\} + \tilde{n} \\ &= 2E \sum_{m=1}^{L} \alpha_m^2 + 2E \sum_{m=1}^{L} \sum_{\substack{l=1 \\ l \neq m}}^{L} \Re \{ g_m g_l^* \} \phi_{aa}(m - l) + \tilde{n} \end{aligned}$$

Ideal RAKE BER Performance

- Assume spreading sequences have an ideal autocorrelation function, i.e., $\phi_{aa}(n-m) = \delta_{nm}$.
- The BER is $P_b(\gamma_b) = Q\left(\sqrt{2\gamma_b}\right)$ where $\gamma_b = \sum_{k=1}^{L} \gamma_k$, $\gamma_k = \alpha_k^2 E/N_0$, and $g_k = \alpha_k e^{j\phi_k}$.
- Since the γ_k's are exponentially distributed (Rayleigh fading) with the same average SNR $\bar{\gamma}_k$, the density of γ_b is

$$p(\gamma_b) = \sum_{k=1}^{L} \frac{A_k}{\bar{\gamma}_k} e^{-\gamma_b/\bar{\gamma}_k}$$

where

$$A_k = \prod_{\substack{i=1 \\ i \neq k}}^{L} \frac{\bar{\gamma}_k}{\bar{\gamma}_k - \bar{\gamma}_i}$$

- The BER is

$$\begin{aligned} P_b &= \int_0^\infty Q\left(\sqrt{2\gamma_b}\right) p(\gamma_b)\, d\gamma_b \\ &= \frac{1}{2} \sum_{k=1}^{L} A_k \left[1 - \sqrt{\frac{\bar{\gamma}_k}{1+\bar{\gamma}_k}}\right] \end{aligned}$$

Channel Model

- Assume the power delay profile of the channel exhibits an exponential decay:

$$\bar{\gamma}_k = Ce^{-k/\varepsilon}$$

where ε determines the delay spread and C is chosen so that

$$\sum_{k=1}^{L} \bar{\gamma}_k = \bar{\gamma}^b.$$

- Solving for C results in

$$\bar{\gamma}_k = \frac{\left(1 - e^{-1/\varepsilon}\right) e^{-k/\varepsilon}}{e^{-1/\varepsilon} - e^{(L+1)/\varepsilon}} \bar{\gamma}_b$$

RAKE Performance: $L = 4$