CDMA - Code Division Multiple Access
DS/SS - Direct Sequence Spread Spectrum
and
Related Topics

by

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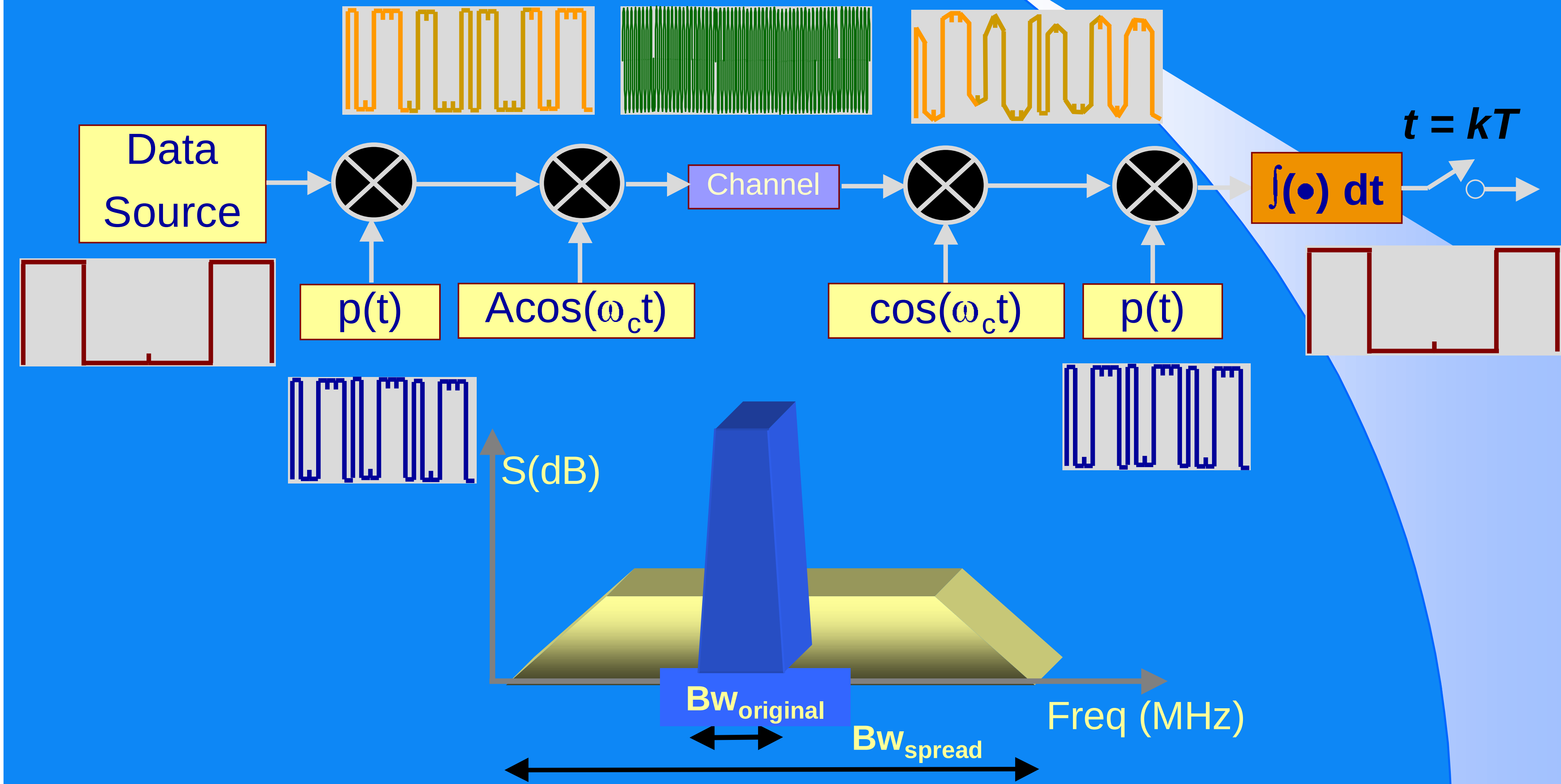
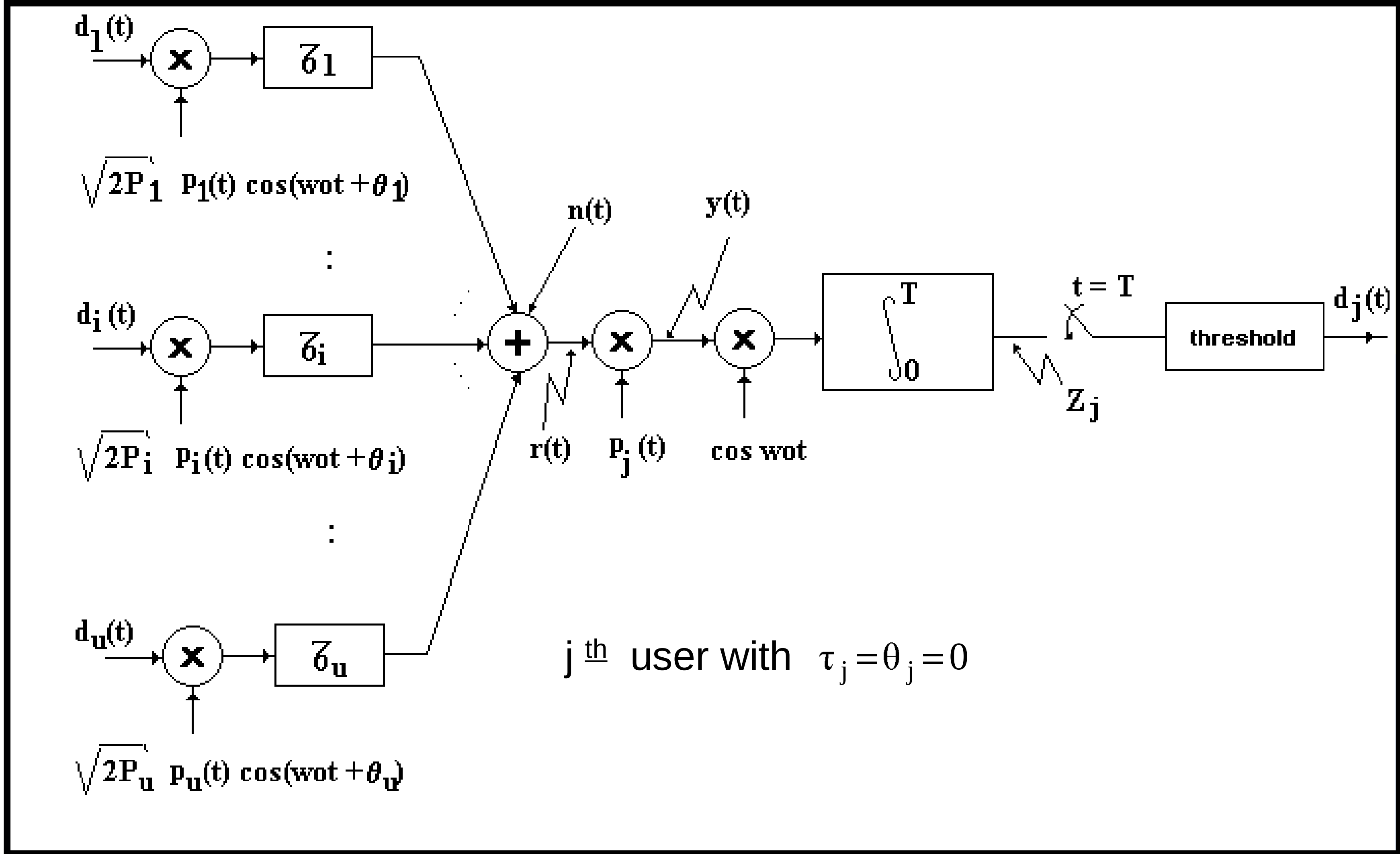
Index

DS/SS - Direct Sequence Spread Spectrum



Asynchronous CDMA System Model / Single User Receiver (Pursley's Approach)



$$r(t) = \sum_{i=1}^{U} \sqrt{2P_i}\, d_i(t-\tau_i)\, p_i(t-\tau_i) \cos(\omega_0 t + \varphi_i) + n(t)$$

Where

$p_i(.) = \pm 1$ in the interval $kT_c<t<(k+1)T_c$ with k=0;1;2;....... is the code sequence assumed with length L for all users;

$d_i(.) = \pm 1$ in the interval $jT<t<(j+1)T$ with j=0;1;2; .... is the information bit;

T_c : is the chip interval;

T : is the information bit interval;

$N=T/T_c$: is the processing gain and represents the number of chips during one bit interval (not to be confused with L, but here assumed equal to);

n(t) : is an Addictive White Gaussian Noise (AWGN) with bilateral PSD (Power Spectral Density) $S_n(f)=N_0/2$;

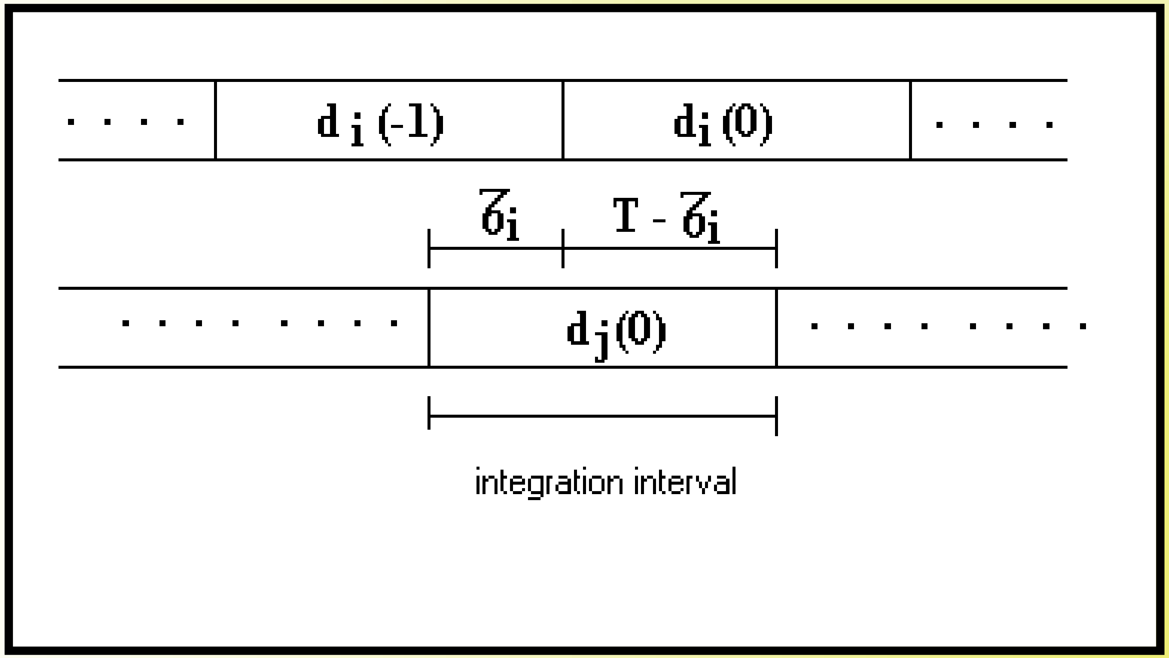
$\varphi_i = \theta_i - \omega_0 \tau_i$: is the relative phase for the i^{th} user.

With the assumption that we have a integer number of carrier cycles in a chip interval and perfect synchronism for desired j$^{\underline{th}}$ signal, the integrator's output for this user can be expressed as

$$Z_j = \sum_{\substack{i=1 \\ i\neq j}}^{U} \sqrt{\frac{P_i}{2}} \int_0^T p_i(t-\tau_i)\, p_j(t)\, d_i(t-\tau_i) \cos\varphi_i dt + \int_0^T n(t)\, p_j(t) \cos\omega_0 t dt + \sqrt{\frac{P_j}{2}} T d_j(0)$$

(return)

Integration interval showing the asynchronism between users



Defining the even and odd continuous partial cross-correlation functions of sequences i and j, respectively, as

$$R_{i,j}(\tau_i) = \int_0^{\tau_i} p_i(t-\tau_i)\,p_j(t)\,dt$$

and

$$\hat{R}_{i,j}(\tau_i) = \int_{\tau_i}^{T} p_i(t-\tau_i)\,p_j(t)\,dt$$

and assuming that $d_j(0)=1$, we can express Z_j as

$$Z_j = \sum_{\substack{i=1 \\ i\neq j}}^{U} \sqrt{\frac{P_i}{2}}\left[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)\right]\cos\varphi_i +$$

$$+\int_0^T n(t)\,p_j(t)\cos\omega_0 t\,dt + \sqrt{\frac{P_j T^2}{2}} = \sum_{\substack{i=1 \\ i\neq j}}^{U} \alpha_i \cos\varphi_i + N_j + S_j$$

• the first term, named MAI - Multiple Access Interference, corresponds to the interference of i^{th} user in the j^{th} desired user and has zero mean value;

• the second term corresponds to interference dues to the AWGN, with zero mean value also;

• and finally the third term corresponds to the desired signal (information bit) and its value is the mean value of Z_j.

Now in order to find the signal to noise ratio (SNR_j) we should calculate the variance of Z_j. For this purpose we will assume that

• τ_i is uniformly distributed in the interval [0,T [

• φ_i is uniformly distributed in the interval [0,2π [

• $d_i(.)$ has equal probabilities of occurrence $\Rightarrow$ P(-1)=P(+1)=0,5

The desired information is a constant so we can write

$$V_{ar}\left[Z_j\right] = V_{ar}\left[\sum_{\substack{i=1\\ i\neq j}}^{U} \alpha_i \cos\varphi_i\right] + V_{ar}\left[N_j\right]$$

For the second term we have

$$V_{ar}[N_j] = V_{ar}\left[\int_0^T n(t)\,p_j(t)\cos\omega_0 t\,dt\right] = V_{ar}\left[\sum_{k=0}^{L-1} p_j(k)\int_{kT_c}^{(k+1)T_c} n(t)\cos\omega_0 t\,dt\right]$$

It is easy to show that

$$\sum_{k=0}^{L-1} p_j(k)\int_{kT_c}^{(k+1)T_c} n(t)\cos\omega_0 t\,dt = N(0, N_0 T/4)$$

because, considering n(t) as AWGN, we can write

$$E\left\{\int_{kT_c}^{(k+1)T_c} n(t)\cos\omega_0 t\,dt\right\} = \int_{kT_c}^{(k+1)T_c} E[n(t)]\cos\omega_0 t\,dt = 0$$

and

$$E\left\{\int_{k_1T_c}^{(k_1+1)T_c} n(t)\cos\omega_0 t\,dt \int_{k_2T_c}^{(k_2+1)T_c} n(u)\cos\omega_0 u\,du\right\} = E\left\{\iint n(t)\,n(u)\cos\omega_0 t\cos\omega_0 u\,dt\,du\right\}$$

$$= \iint E\left[n(t)\,n(u)\right]\cos\omega_0 t\cos\omega_u\,dt\,du = \iint \frac{N_0}{2}\delta(t-u)\cos\omega_0 t\cos\omega_u\,dt\,du$$

$$= \frac{N_0}{2}\int_{kT_c}^{(k+1)T_c} \cos^2\omega_0\,t\,dt = N_0 T_c / 4$$

and finally

$$\sum \sigma_k^2 = \sum_{k=0}^{L-1} \frac{N_0 T_c}{4} = \frac{L T_c N_0}{4} = \frac{N_0 T}{4}$$

Now we need to calculate the variance of the first term of Z_j considering its independent random variables τ_i and φ_i and $d_i(.)$ and its distribution function as defined earlier. The calculation will be done in the three steps. The first one considering φ_i influence.

$$\mathrm{Var}\left[\sum_{\substack{i=1\\ i\neq j}}^{U}\alpha_i\cos\varphi_i\right]_1 = \frac{1}{2\pi}\int_0^{2\pi}\left\{\sum_{\substack{i=1\\ i\neq j}}^{U}\alpha_i\cos\varphi_i\right\}^2 d\varphi_i - \left\{\frac{1}{2\pi}\int_0^{2\pi}\sum_{\substack{i=1\\ i\neq j}}^{U}\alpha_i\cos\varphi_i\right\}^2$$

$$= \frac{1}{2\pi}\sum_{\substack{i=1\\ i\neq j}}^{U}\left\{\alpha_i^2\int_0^{2\pi}\cos^2\varphi_i\, d\varphi_i + \sum_{\substack{k=1\\ k\neq j}}^{U}\alpha_k\cos\varphi_k\int_0^{2\pi}\alpha_i\cos\varphi_i d\varphi_i\right\} - \frac{1}{4\pi^2}\left\{\sum_{\substack{k=1\\ k\neq j}}^{U}\alpha_i\int_0^{2\pi}\cos\varphi_i d\varphi_i\right\}^2$$

$$= \frac{1}{2\pi}\sum_{\substack{i=1\\ i\neq j}}^{U}\alpha_i^2\int_0^{2\pi}\left(\frac{1}{2}+\frac{1}{2}\cos 2\varphi_i\right)d\varphi_i = \sum_{\substack{i=1\\ i\neq j}}^{U}\frac{\alpha_i^2}{2}$$

In the second step we will consider the influence of the information bit of all other users.

$$\mathrm{Var}\left[\sum_{\substack{i=1\\i\neq j}}^{U}\alpha_i\cos\varphi_i\right]_2 = \frac{1}{4}\cdot\frac{1}{2}\left\{\sum_{\substack{i=1\\i\neq j}}^{U}P_i\left(R_{i,j}(\tau_i)+\hat{R}_{i,j}(\tau_i)\right)^2+\sum_{\substack{i=1\\i\neq j}}^{U}P_i\left(R_{i,j}(\tau_i)-\hat{R}_{i,j}(\tau_i)\right)^2\right\}$$

$$=\frac{P}{4}\left\{\sum_{\substack{i=1\\i\neq j}}^{U}\left(R_{i,j}^2(\tau_i)+\hat{R}_{i,j}^2(\tau_i)\right)\right\}$$

For this final expression we additionally have assumed that all users have the same received power (perfect power control to avoid the near far problem; this restriction will be considered later).

Finally for the third and last step, we have to calculate the delays' influence on previous variance. For this objective we need some additional definitions, references [1] and [7]. Given two sequences a_n and b_n, with its elements in [1,-1], both periodic with period p

$$\{a_n\} = \{a_0, a_1, \cdots, a_k, \cdots, a_{p-1}, a_0, \cdots\} \qquad \{b_n\} = \{b_0, b_1, \cdots, b_k, \cdots, b_{p-1}, b_0, \cdots\}$$

The periodical cross-correlation between a_n and b_n is another sequence with the same period and defined by

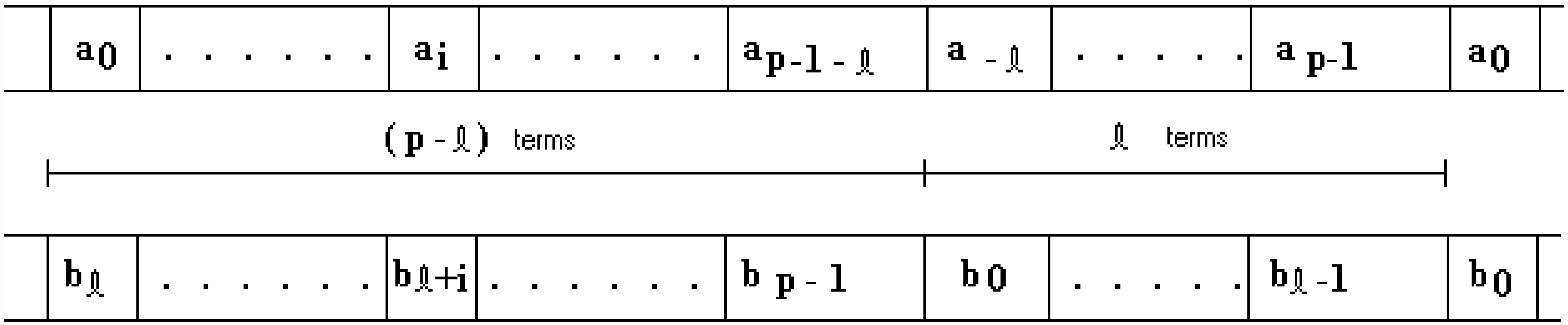
$$\theta_{a,b}(\ell) = \sum_{i=0}^{p-1} a_i b_{i+\ell} \quad \text{with } \ell \in Z$$

With this definition $\theta_{a,b}(.)$ can be interpreted as the number of agreements minus the number of disagreements between the two sequences in a complete period as a function of the relative delay between the two sequences.

The partial cross-correlation between a_n and b_n is a function defined by

$$C_{a,b}(\ell) = \begin{cases} \sum_{i=0}^{p-1-\ell} a_i b_{i+\ell} & 0 \le \ell \le p-1 \\ \sum_{i=0}^{p-1+\ell} a_{i-\ell} b_i & 1-p \le \ell < 0 \\ 0 & |\ell| \ge p \end{cases}$$

The next figure give us an interpretation for this function.



for $0 \le \ell \le p-1$ we have $(p-\ell)$ terms given by

$$\boxed{\mathbf{C_{a,b}(\ell) = a_0 b_{\ell} + a_1 b_{\ell+1} + \ldots\ldots\ldots\ldots\ldots + a_{p-1-\ell} b_{p-1}}}$$

for $1-p \le \ell < 0$ we have $(p-|\ell|)$ terms given by

$$\boxed{\mathbf{C_{a,b}(\ell) = a_{-\ell} b_0 + a_{-\ell+1} b_1 + \ldots\ldots\ldots\ldots\ldots + a_{p-1} b_{p-1+\ell}}}$$

With these definitions we can easily show that

$$\boldsymbol{\theta}_{a,b}(0) = C_{a,b}(0)$$

$$\boldsymbol{\theta}_{a,b}(\ell) = \boldsymbol{\theta}_{a,b}(\ell + p)$$

$$\boldsymbol{\theta}_{a,b}(-\ell) = \boldsymbol{\theta}_{b,a}(\ell)$$ and $$C_{a,b}(-\ell) = C_{b,a}(\ell)$$

$$\boldsymbol{\theta}_{a,b}(\ell) = C_{a,b}(\ell) + C_{a,b}(\ell - p) \text{ for } |\ell| \leq p$$

For two identical sequences we introduce the notation

$$\{a_n\} = \{b_n\} \Rightarrow \boldsymbol{\theta}_{a,a}(.) = \boldsymbol{\theta}_{a}(.) \text{ and } C_{a,a}(.) = C_{a}(.)$$

and rename the functions as periodic auto-correlation and partial auto correlation, respectively.

With these definitions and properties we are able to return to the original problem and for the last step we can write

$$\mathrm{Var}\left[\sum_{\substack{i=1\\ i\neq j}}^{U} \alpha_i \cos\varphi_i\right]_3 = \frac{P}{4T}\sum_{\substack{i=1\\ i\neq j}}^{U}\int_0^T\left(R_{i,j}^2(\tau_i)+\hat{R}_{i,j}^2(\tau_i)\right)d\tau_i$$

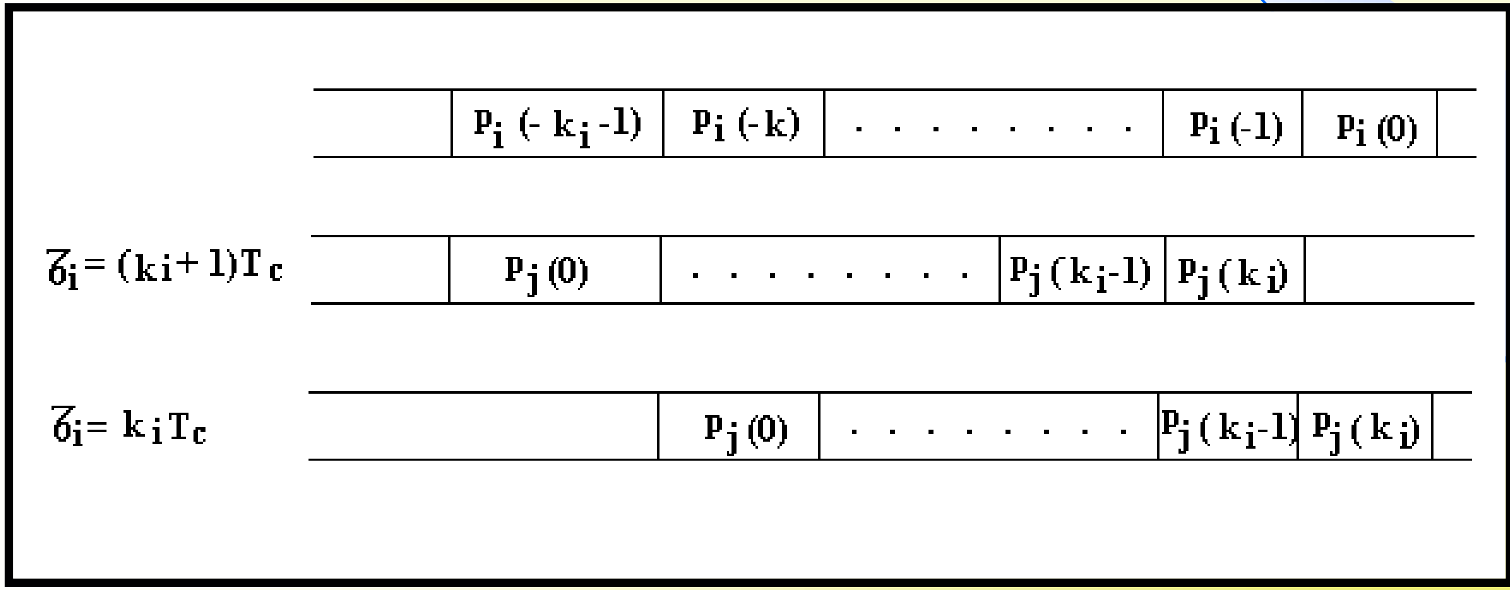
$$= \frac{P}{4T}\sum_{\substack{i=1\\ i\neq j}}^{U}\sum_{m=0}^{L-1}\int_{mT_c}^{(m+1)T_c}\left(R_{i,j}^2(\tau_i)+\hat{R}_{i,j}^2(\tau_i)\right)d\tau_i$$

Now it is necessary to establish the relation between $R_{i,j}(.)$ and $C_{i,j}(.)$

The relative delay between users i and j can be written as

$$0 \leq k_i T_c \leq \tau_i < (k_i + 1)T_c < LT_c$$

Considering two consecutive chip intervals for this two users we have



Evaluating $R_{i,j}(.)$ in these two extremes values for τ_i and considering rectangular pulses for the chips we have

$$R_{i,j}(\tau_i = k_i T_c) = \int_0^{k_i T_c} p_i(t - k_i T_c)\, p_j(t)\, dt = T_c \sum_{m=0}^{k_i - 1} p_i(m - k_i)\, p_j(m)$$

and with the substitution $k_i - 1 = L - 1 + \ell$ we have

$$R_{i,j}(\tau_i = k_i T_c) = T_c \sum_{m=0}^{L-1+\ell} p_i(m - L - \ell)\, p_j(m) = T_c \sum_{m=0}^{L-1+\ell} p_i(m - \ell)\, p_j(m)$$

and then

$$R_{i,j}(\tau_i = k_i T_c) = T_c C_{i,j}(\ell) = T_c C_{i,j}(k_i - L)$$

For the other interval's extreme $\tau_i = (k_i + 1)T_c$ and we can write

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = \int_0^{(k_i+1)T_c} p_i\left[t-(k_i+1)T_c\right]p_j(t)dt = T_c\sum_{m=0}^{k_i} p_i\left[m-(k_i+1)\right]p_j(m)$$

and now putting $k_i = L - 1 + \ell$ we have

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = T_c\sum_{m=0}^{L-1-\ell} p_i(m-L-\ell)p_j(m) = T_c\sum_{m=0}^{L-1+\ell} p_i(m-\ell)p_j(m)$$

So similar to the other extreme we obtain

$$R_{i,j}\left[\tau_i = (k_i+1)T_c\right] = T_c C_{i,j}(\ell) = T_c C_{i,j}(k_i - L + 1)$$

Now observing that the function is linear between these two points we can establish the relation

$$\mathbf{R}_{i,j}[\tau_i] = \mathbf{T_c C_{i,j}(k_i - L)} + [\mathbf{C_{i,j}(k_i - L + 1) - C_{i,j}(k_i - L)}](\tau_i - \mathbf{k_i T_c})$$

and with a completely analogous procedure we obtain

$$\hat{\mathbf{R}}_{i,j}[\tau_i] = \mathbf{T_c C_{i,j}(k_i)} + [\mathbf{C_{i,j}(k_i + 1) - C_{i,j}(k_i)}](\tau_i - \mathbf{k_i T_c})$$

For the variance expression calculation we need to evaluate integrals of $\mathbf{R}^2_{i,j}[\tau_i]$ and $\hat{\mathbf{R}}^2_{i,j}[\tau_i]$, which are linear in their variables τ_i. So we have an general expression in the form

$$R_{i,j}(\tau)=A+B\tau \Rightarrow \int_{\ell T_c}^{(\ell+1)T_c} R_{i,j}^2(\tau)\,d\tau = A^2\tau + AB\tau^2 + B^2\frac{\tau^3}{3}\Bigg|_{\ell T_c}^{(\ell+1)T_c}$$

$$=A^2T_c + AB(2\ell+1)T_c^2 + \frac{B^2}{3}(3\ell^2+3\ell+1)T_c^3$$

Now with $\ell = k_i$ we obtain the following values for A and B

$$A=(\ell+1)T_c C_{i,j}(\ell-L)-\ell T_c C_{i,j}(\ell-L+1)$$

$$B=C_{i,j}(\ell-L+1)-C_{i,j}(\ell-L)$$

We need to do the same procedure for $\hat{R}^2_{i,j}[\tau_i]$ and finally we are able to evaluate the expression

$$\sum_{m=0}^{L-1} \int_{mT_C}^{(m+1)T_C} \left(R^2_{i,j}(\tau) + \hat{R}^2_{i,j}(\tau) \right) d\tau$$

This final step is very simple but tedious; the result is

$$V_{ar}\left[Z_j\right] = \frac{P\,T^2}{12\,L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0 T}{4}$$

where

$$\beta_{i,j} = \sum_{\ell=0}^{L-1} \left\{ C^2_{i,j}(\ell-L) + C_{i,j}(\ell-L)C_{i,j}(\ell-L+1) + C^2_{i,j}(\ell-L+1) + C^2_{i,j}(\ell) + C_{i,j}(\ell)C_{i,j}(\ell+1) + C^2_{i,j}(\ell+1) \right\}$$

With this expression we are able to determine Var[Z_j] if we know the code sequences used by each user in the system. The result depends on the partial cross-correlation between sequences and can be simplified. This simplification, originally done by Pursley, references [3] and [4], is strongly based in correlation properties and its details are beyond the scope of this presentation. Complete details for this derivation can be found in [1]. The result is

$$\beta_{i,j} = 2L^2 + 4\sum_{\ell=1}^{L-1} C_i(\ell)C_j(\ell) + \sum_{\ell=1-L}^{L-1} C_i(\ell)C_j(\ell+1)$$

Where now the calculation is based only on partial auto-correlation functions instead of cross-correlation functions. This is a substantial simplification for $\beta_{i,j}$ determination. Finally with this variance determination we can express the signal to noise ratio as

$$(SNR)_j = \frac{E[Z_j]}{\sqrt{Var[Z_j]}} = \left\{ \frac{1}{6L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0}{2PT} \right\}^{-1/2} = \left\{ \frac{1}{6L^3} \sum_{\substack{i=1 \\ i \neq j}}^{U} \beta_{i,j} + \frac{N_0}{2E_b} \right\}^{-1/2}$$

and if we have a great number of users we can assume a gaussian approximation for the BER (Bit Error Rate) determination as

$$P_{e,j} = Q[(SNR)_j]$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp(-y^2/2)dy$$

This gaussian assumption causes some problems concerning Central Limit Theorem applicability, see references [5] and [6]. For practical purpose we can use it and later we will introduce a more general calculation that will overcome some eventual theoretical limitations.

With an additional simplification in mind we will consider two random sequences selected from the set of all 2^L binary sequences of length L, where each sequence has the same probability for its selection from the set. It is a simple matter to show that for two randomly selected sequences we have

$$E[\beta_{i,j}] = 2L^2 \quad \text{and} \quad (SNR)_j = \left[\left\{\frac{U-1}{3L} + \frac{N_0}{2E_b}\right\}^{-1/2}\right]$$

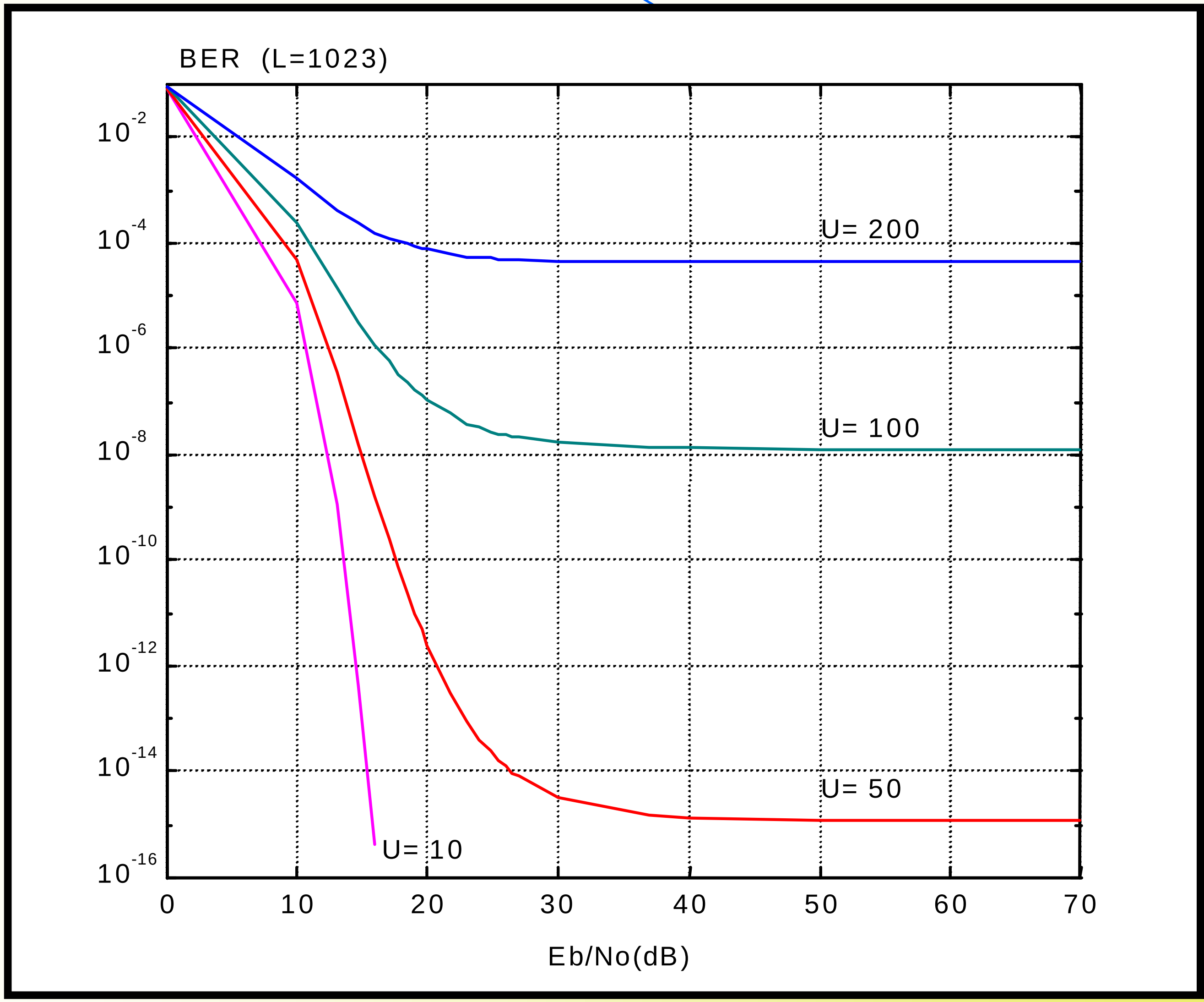
Which allows us to express the BER in an approximate, but very useful form

$$P_{e,j} = Q[(SNR)_j] = Q\left[\left(\frac{U-1}{3L} + \frac{N_0}{2E_b}\right)^{-1/2}\right]$$

(return)

obs.: with U=1 we obtain the well known result $P_e = Q(\sqrt{\frac{2E_b}{N_0}})$

In the next figure we show this final result for L=1023 and particular set of values for U.



For a given U value, the asymptotic result (when the relation E_b/N_o grows) shows us basically the MAI limitation. For this region the BER is given by

$$P_e = Q[(SNR)] = Q\left(\sqrt{\frac{3L}{U-1}}\right)$$

Numerical Example

For instance if we impose a typical desirable performance of $P_e=10^{-3}$ we should have $(E_b/N_0)_{eq}=4{,}8$ and in this case (SNR)=3,10.

If we use the value $(E_b/N_0)_{eq}=10{,}0 \Rightarrow U \approx 0{,}162L+1$, and for a system which use L=1023 we could have ≈ 166 users. So, with one user and $(E_b/N_0)_{.}=10{,}0$ we have a BER of $P_e= 4\times10^{-6}$ and with 166 users this performance degrades to $P_e=10^{-3}$.

To get some additional insight on this numerical result it is interesting to compare it with the fundamental limit given by Shannon's channel capacity theorem. As we know the channel capacity of a system is given by

$$C = B \times \log_2\left(1 + \frac{S}{N_0 B}\right) \quad \text{bits/s}$$

For B growing C approaches to $C \Rightarrow \frac{S}{N_0 \ln 2}$ bits/s showing us that we can reach C bits/s increasing the user's bandwidth. In our notation

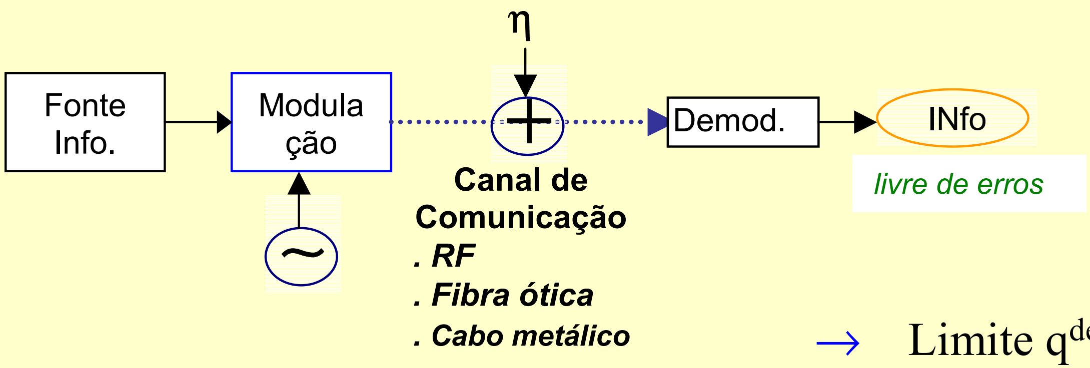
$$f_0 \ln 2 \Rightarrow \frac{E_b f_0}{(U-1) N_i + N_0} = \frac{E_b f_0}{(U-1) E_b f_0 T_c + N_0}$$

with f_0=1/T and N_i representing one user's equivalent power spectral density.

So in this case we have $U \Rightarrow \left[\frac{1}{\ln 2} - \frac{N_0}{E_b}\right] L + 1 \cong 1{,}44\, L$ where we have neglected the second term.

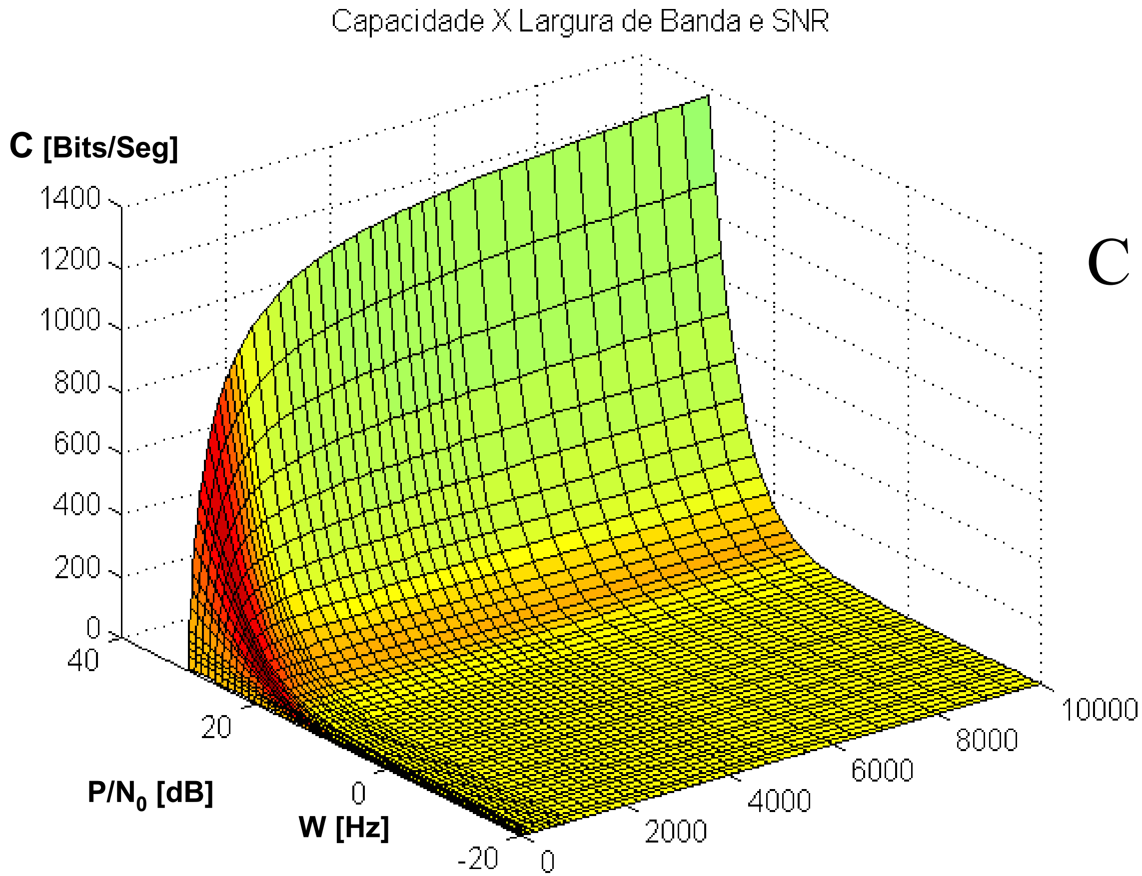
With the previous value of L now we have U≈1470 users. These numbers show us a poor performance (only 11%) if we compare our former solution with this theoretical limit.

Capacidade Canal (Shannon)



→ Limite q^{de} Info em um **canal** limitado em:[-W, W] e Pot, P

→ Canal com Ruído AWGN: DEP=$N_0/2$ e Variância = σ^2



$$C = W.\mathrm{Log}_2\left(1 + \frac{P}{N_0.W}\right) \qquad \left[{}^{\mathrm{bits}}\!/_{\mathrm{seg}}\right]$$

$$\lim_{W \to \infty} C = 1.443 \frac{P}{N_0}$$

Exemplo: *Cálculo da interferência Multiusuário em sistema CDMA a partir das SMC*

- *Sejam **seqs SMC** de grau **n=5** (portanto, 6 pol. primitivos, e U= 6 usuários).*
- Sejam os coeficientes dos 6 polinômios de grau n=5 que geram as SMC:

1	*x*	*x2*	*x3*	*x4*	*x5*
1	0	1	0	0	1
1	0	1	1	1	1
1	1	1	0	1	1
1	1	0	1	1	1
1	1	0	0	0	1
1	0	0	1	0	1

Comprimento das sequências:

$L = 2^n - 1 = 31$

- As sequências geradas por estes 6 polinômios, com condições iniciais dada por g(x)= 0 0 0 0 1, respectivamente, são (bipolarizando)

- -1 1 -1 1 -1 -1 -1 1 -1 -1 1 1 1 -1 -1 -1 -1 -1 1 1 -1 -1 1 -1 1 1 -1 1 1 1 1

- -1 1 -1 -1 1 -1 1 -1 1 1 1 -1 -1 -1 1 -1 -1 -1 -1 -1 1 1 -1 1 1 -1 -1 1 1 1 1

- -1 -1 1 -1 1 -1 1 1 -1 1 1 1 -1 1 -1 -1 -1 -1 -1 1 -1 -1 1 1 -1 -1 -1 1 1 1 1

- -1 -1 -1 1 1 -1 -1 1 -1 -1 -1 -1 -1 1 -1 1 1 1 -1 1 1 -1 1 -1 1 -1 -1 1 1 1 1

- -1 -1 -1 -1 -1 1 -1 1 -1 1 1 -1 -1 1 1 1 -1 1 1 1 1 -1 -1 -1 -1 -1 1 -1 1 -1 1

- -1 1 1 -1 1 -1 -1 1 1 -1 -1 -1 -1 -1 1 1 1 -1 -1 1 -1 -1 -1 1 -1 1 -1 1 1 1 1

AutoCorrelações Cruzadas Aperiódicas, C_{ii}, para os 6 usuários,

$$Ci(l) = \sum_{i=0}^{L-1-l} a_i + a_{i+l}, \qquad 0 \le l \le L-1$$

C_i =

0 -1 0 -1 -2 3 0 1 2 -3 2 1 -4 -5 2 -3 4 3
-2 -3 2 -3 -2 -1 -4 1 0 -1 0 -1

0 -1 0 1 -2 1 0 -3 -2 1 -4 -5 2 1 2 -3 -2 -3
4 3 -2 1 2 -1 -2 1 -2 -1 0 -1

0 1 0 1 -2 -3 -2 -3 2 -5 -4 -1 -2 -5 -2 1 4 1
0 3 4 -3 2 1 2 1 -2 -1 -2 -1

0 1 2 1 -2 -3 -2 -1 4 -1 0 5 4 -1 0 -1 0 -5
-6 -1 0 -5 0 1 2 1 -2 -3 -2 -1

0 7 -4 3 0 -3 -8 1 -4 1 0 -3 2 -7 -2 -3 -2 -3
2 -1 10 -1 6 -1 2 -1 0 -1 0 -1

0 -1 -2 -1 -4 1 0 -1 0 -5 2 3 2 1 4 -5 -2 -3
-4 -3 4 -1 0 -1 -2 3 0 1 0 -1

A matriz B_{ij} , que determina a influência da interferência multiusuário em um sistema CDMA:

$$B_{i,j} = 2L^2 + 4.\sum_{l=1}^{L-1} C_i(l).C_j(l) + \sum_{l=0}^{L-2} C_j(l).C_i(l+1) + \sum_{l=0}^{L-1} C_i(l).C_j(l+1)$$

onde $b_{i,j}$ deve ser interpretado como a influência do usuario i sobre o usuario j (i = 1.. U, i ≠ j), com $B_{i,i}$ = 0. Além disto, a matriz $B_{i,j}$ é simétrica, ($B_{i,j} = B_{j,i}$).

O valor médio dos elementos de uma coluna ou linha desta matriz repesentará a interferência de todos os demais usuários sobre o usuario considerado (j).

Como valem as propriedades: $C_a(l) = C_a(-l)$, $C_a(0) = L$, para cada usuário a expressão acima pode ser modificada:

$$B_{i,j} = 2L^2 + 4.\sum_{l=1}^{L-1} C_i(l).C_j(l) + 2.\sum_{l=0}^{L-1} C_i(l).C_j(l+1)$$

que pode ser ainda escrita como:

$$B_{i,j} \simeq 2L^2 + 4.\sum_{l=1}^{L-1} C_i(l).C_j(l) + \sum_{l=1-L}^{L-1} C_i(l).C_j(l+1)$$

⇒ Pode-se ainda demonstrar que se os polinômios a e b são recíprocos, então: $C_a(l) = C_b(l)$

Calculando-se a matriz $B_{i,j}$ através da última expressão resulta

	0	**1694**	**2070**	**1894**	**1790**	**2046**
	1694	**0**	**1966**	**1758**	**2054**	**1918**
	2070	**1966**	**0**	**2022**	**2406**	**1782**
B_{ij} =	**1894**	**1758**	**2022**	**0**	**2014**	**2262**
	1790	**2054**	**2406**	**2014**	**0**	**1974**
	2046	**1918**	**1782**	**2262**	**1974**	**0**
MAI_j =	**9494**	**9390**	**10246**	**9950**	**10238**	**9982**

A somatória dos elementos de uma coluna (ou linha) representa a influência dos outros usuários sobre o usuário correspondente à coluna (ou linha) analisada.

⇒ Valor médio de uma coluna corresponderá à *interferência média devida aos usuários (i)* sobre o usuário associado à coluna. Os **valores médios** e respectivos **desvios padrões** são

1.898,8 ±159,3; 1.878,0 ±156,1; 2.049,2 ±252,9 1.990,0 ±185,8
2.047,6 ±243,9 1.996,4 ±176,9

⇒ estes valores médios estão próximos ao limite $\mathbf{2L^2} = 2.\ 31^2 = \mathbf{1922}$

⇒A BER em um sistema DS/CDMA Assíncrono Convencional em canal AWGN quando um grande número de sinais (usuários) interferentes estiver presentes pode ser aproximada a uma gaussiana:

$$P_{e,j} = Q\left[\left(\frac{S}{N}\right)_j\right] \quad , \text{ com } \quad \left(\frac{S}{N}\right)_j = \left(\frac{1}{6L^3}.\sum_{\substack{i=1 \\ i\neq j}}^{U} B_{i,j} + \frac{N_0}{2.E_b}\right)^{-1/2}$$

Se as sequências empregadas forem randômica, o segundo e terceiro termos da expressão original de B_{ij} resultam iguais a 0 e portanto, **$B_{ij} = 2L^2$**.

Para este exemplo, embora as sequências empregadas sejam deterministicas (SMC), observa-se que os valores médios das interferências multiusuários estão próximas do limite **$2L^2 = 1922$**.

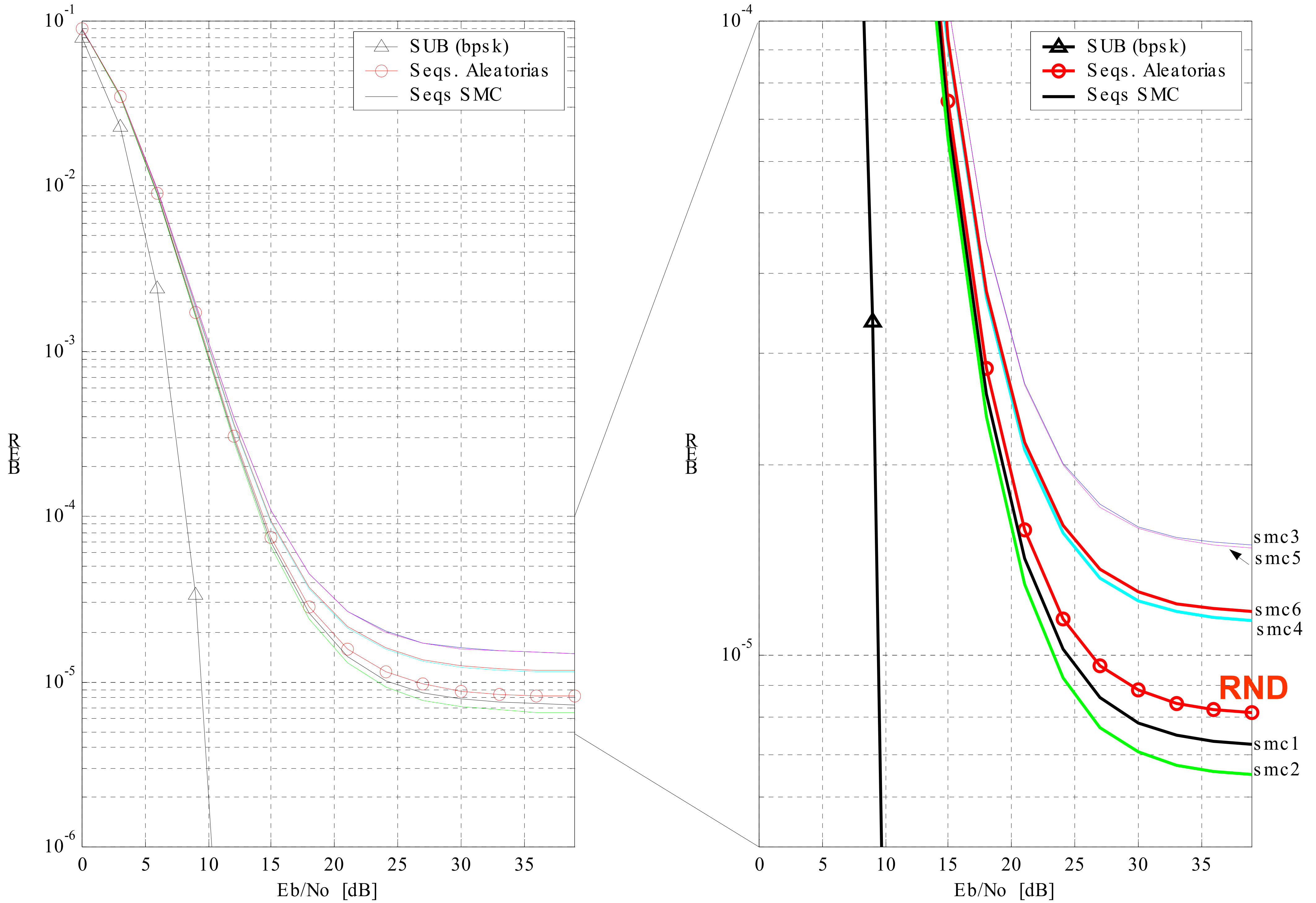
Assim, pode-se aplicar a aproximação:

$$\left(\frac{S}{N}\right)_j \cong \left(\frac{U-1}{3L} + \frac{N_0}{2.E_b}\right)^{-1/2}$$

Ainda: o **valor assintótico da probabilidade de erro de bit** para o sistema DS/CDMA Convencional assíncrono em canal AWGN será obtido quando a relação sinal ruído for muito alta; assim, o termo $N_0/2.E_b$ pode ser desprezado.

$$\left(\frac{S}{N}\right)_j \cong \left(\frac{U-1}{3L}\right)^{-1/2} = 4,31$$

$$\Rightarrow P_{e,j}^{as\,\text{sint}} = Q(4,31) \simeq 8,1 \times 10^{-6}$$



MAI_j (SMC) = 9494 9390_{min} 10246_{max} 9950 10238 9982

Asynchronous CDMA System Model / Single User Receiver (Worst Case Approach)

Returning to the previous complete expression for Z_j (compare) and considering again the equal powers case we can rewrite it as

$$Z_j = \sqrt{\frac{PT^2}{2}}[d_j(0) + \gamma_j(\underline{d},\underline{\tau},\underline{\varphi})] + N_j$$

with

$$\gamma_j(\underline{d},\underline{\tau},\underline{\varphi}) = \sum_{\substack{i=1 \\ i \neq j}}^{U} I_{i,j}(\underline{d}_i,\tau_i,\varphi_i)$$

Where the normalized interference from user i on user j can be expressed as

$$I_{i,j}(\underline{d}_i,\tau_i,\varphi_i) = T^{-1}[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

Considering equal probabilities for the information bits

$$P_e = \frac{1}{2}\text{Prob}[d_j(0) = 1 \text{ and } Z_j < 0] + \frac{1}{2}\text{Prob}[d_j(0) = -1 \text{ and } Z_j > 0]$$

Considering the symmetry

$$P_e = \text{Prob}[d_j(0) = 1 \text{ and } Z_j < 0] = \text{Prob}[\sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})] + N_j < 0]$$

$$P_e = \text{Prob}\{N_j < -\sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\} = \text{Prob}\{N_j > \sqrt{\frac{PT^2}{2}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\}$$

We have

$$P_e = Q\{\sqrt{\frac{2PT}{N_0}}[1+\gamma_j(\underline{b},\underline{\tau},\underline{\varphi})]\}$$

The function Q(x) varies monotonically with x. So, for its maximization (worst case) we need to minimize the argument.

The symmetry of $\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})$ allow us to write

$$\min[\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})] = -\max[\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})] = -\max[|\gamma_i(\underline{d},\underline{\tau},\underline{\varphi})|] = -\sum_{\substack{i=1\\ i\neq j}}^{U}\max[|I_{i,j}(\underline{d_i},\tau_i,\varphi_i)|]$$

Therefore we should maximize

$$I_{i,j}(\underline{d_i},\tau_i,\varphi_i) = T^{-1}.[d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)].\cos\varphi_i$$

considering φ_i's influence and observing that

$$\max[\cos\varphi_i] = 1 \quad \text{and} \quad \min[\cos\varphi_i] = -1$$

we can write

$$-T^{-1}\max[|d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)|] < I_{i,j}(\underline{d_i},\tau,\varphi) < T^{-1}\max[|d_i(-1)R_{i,j}(\tau_i) + d_i(0)\hat{R}_{i,j}(\tau_i)|]$$

Next considering all possible combinations for the interfering bit

$$T^{-1}\max[|d_i(-1)R_{i,j}(\tau_i)+d_i(0)\hat{R}_{i,j}(\tau_i)|]=T^{-1}\max[|R_{i,j}(\tau_i)|+|\hat{R}_{i,j}(\tau_i)|]$$

Therefore we have the intermediate result

$$-T^{-1}.\max[|R_{i,j}(\tau_i)|+|\hat{R}_{i,j}(\tau_i)|]<I_{i,j}(\underline{d_i},\tau,\varphi)<T^{-1}.\max[|R_{i,j}(\tau_i)|+|\hat{R}_{i,j}(\tau_i)|]$$

Finally the function $|R_{i,j}(\tau_i)|+|\hat{R}_{i,j}(\tau_i)|$ has local extremes in multiples of T_c

(remember that this function is linear between points in which the arguments are successive multiples of T_c). If we denote by $\varepsilon_{i,j}$ the largest value of the above function calculated in multiples of T_c, i. e. ,

$$\tau_i = 0,\ T_c,\ 2T_c,\ldots,\ (N-1)T_c$$

Therefore we can bound each normalized interference to the interval

$$-T^{-1}\varepsilon_{i,j} < I_{i,j}(\underline{d}_i,\tau,\varphi) < T^{-1}\varepsilon_{i,j}$$

And finally considering all users

$$P_{e,j} = Q\{\sqrt{\frac{2PT}{N_0}}[1 - T^{-1}\sum_{\substack{i=1 \\ i\neq j}}^{U}\varepsilon_{i,j}]\}$$

Which is the worst case performance, see reference [2]. Obviously the best case performance is given by

$$P_{e,j} = Q\{\sqrt{\frac{2PT}{N_0}}[1 + T^{-1}\sum_{\substack{i=1 \\ i\neq j}}^{U}\varepsilon_{i,j}]\}$$

Asynchronous CDMA System Model / Single User Receiver (Weber's Approach)

In this approach, reference [6], all users are considered independent and degrade other users performance through an equivalent PSD. This approach is based on the following equation

$$(SNR)_U = \frac{2E_b}{N_o + \sum_{i=1}^{U-1} N_i}$$

Where

$(SNR)_U$: is desired user's SNR in presence of other (U-1) users

N_o : is the unilateral PSD of AWGN

N_i : is the equivalent PSD for one user's interference which can be obtained by

$$N_i = \frac{P_i}{R_c} = P_i T_c = \alpha_i P_o T_c$$

where

P_i : absolute power for $i^{\underline{th}}$ user

R_c : chip rate ($R_c = 1/T_c$) common to all users

α_i : power factor with P_0 as reference

and from the relation $(SNR)_U$ we can determine the bit error probability with a gaussian assumption

$$P_e = Q[(\sqrt{(SNR)_U}]$$

From previous equations and a perfect power control assumption (α_i=1 for all users) we can derive

$$(SNR)_U = \frac{2E_b}{N_o + \sum_{i=1}^{U-1} N_i} = \frac{1}{\frac{N_0}{2E_b} + \frac{(U-1)P_i T_c}{2E_b}} = \frac{1}{\frac{N_0}{2E_b} + \frac{(U-1)P_i T}{2E_b L}}$$

Therefore

$$P_e = Q\left[\left(\frac{U-1}{2L} + \frac{N_0}{2E_b}\right)^{-1/2}\right]$$

Comparing this result with the Pursley's approach we can see that Weber's approach underestimate the CDMA performance when compared with Pursley's approach.

(A)synchronous CDMA System Model / Single User Receiver (Morrow and Lehnert 's Results)

Considering random sequences for user's spreading codes Morrow and Lehnert establish four additional results as presented below (see reference [15] for developments' details).

1 - synchronous carrier's phase and chip's delay.

$$P_e = Q[(\frac{U-1}{N} + \frac{N_o}{2E_b})^{-1/2}]$$

2 - synchronous carrier's phase and chip's delay uniformly distributed in the interval [0,T].

$$P_e = Q[(\frac{2(U-1)}{3N} + \frac{N_o}{2E_b})^{-1/2}]$$

3 - synchronous chip's delay and carrier's phase uniformly distributed in the interval [0,2π]. This result coincides with Weber's approach.

$$P_e = Q[(\frac{U-1}{2N} + \frac{N_o}{2E_b})^{-1/2}]$$

4 - chip's delay and carrier's phase uniformly distributed in the interval [0,T] and [0,2π], respectively. Obviously, this result is identical to Pursley's approach with random codes.

$$P_e = Q[(\frac{U-1}{3N} + \frac{N_o}{2E_b})^{-1/2}]$$

Asynchronous CDMA System Model / Single User Receiver (Nazari and Ziemer's Approach)

In this approach, reference [8], we return to the general normalized interference expression

$$\mathbf{I}_{i,j}(\underline{\mathbf{d}_i},\tau_i,\boldsymbol{\varphi}_i) = \mathbf{T}^{-1}[\mathbf{d}_i(-1)\mathbf{R}_{i,j}(\tau_i) + \mathbf{d}_i(0)\hat{\mathbf{R}}_{i,j}(\tau_i)]\cos\boldsymbol{\varphi}_i$$

(return)

and try to find the exact distribution for each interference. After this individual determination we will obtain the final interference distribution by convolution of various contributions. So

1) if $d_i(-1)=d_i(0)$

$$\mathbf{I}_{i,j}(\underline{\mathbf{d}_i},\tau_i,\boldsymbol{\varphi}_i) = \mathbf{T}^{-1}\mathbf{d}_i(0)[\mathbf{R}_{i,j}(\tau_i) + \hat{\mathbf{R}}_{i,j}(\tau_i)]\cos\boldsymbol{\varphi}_i$$

Exactly in a similar form as we have obtained in the Pursley's approach it can be shown that

$$\Re_{i,j}(\tau_i) = R_{i,j}(\tau_i) + \hat{R}_{i,j}(\tau_i) = (T_c - \tau + \ell_i T_c)\theta_{i,j}(\ell_i) + (\tau - \ell_i T_c)\theta_{i,j}(\ell_i + 1)$$

Where ℓ_i is the integer part of the relation τ_i/T_c and

$$\theta_{i,j}(\ell) = C_{i,j}(\ell) + C_{i,j}(\ell - N)$$

2) if $d_i(-1) \neq d_i(0)$

$$I_{i,j}(\underline{d_i}, \tau_i, \varphi_i) = T^{-1} d_i(0)[R_{i,j}(\tau_i) - \hat{R}_{i,j}(\tau_i)]\cos\varphi_i$$

$$\hat{\Re}_{i,j}(\tau) = R_{i,j}(\tau_i) - \hat{R}_{i,j}(\tau_i) = (T_c - \tau + \ell_i . T_c)\hat{\theta}_{i,j}(\ell_i) + (\tau - \ell_i . T_c)\hat{\theta}_{i,j}(\ell_i + 1)$$

where now

$$\hat{\theta}_{i,j}(\ell) = C_{i,j}(\ell) - C_{i,j}(\ell - N)$$

Next we will find the distribution probability for $d_i(-1)=d_i(0)$. First of all we define

$$\mathbf{m_{i,j}} = \{\boldsymbol{\theta}_{i,j}(\ell_i) + [\boldsymbol{\theta}_{i,j}(\ell_i + 1) - \boldsymbol{\theta}_{i,j}(\ell_i)]\mathbf{u}\}\cos\boldsymbol{\varphi}_i$$

where

$$\mathbf{u} = \frac{\tau_i - \ell_i \mathbf{T_c}}{\mathbf{T_c}}$$

is uniformly distributed in the interval [0,1[with the assumption that all τ_i is uniformly distributed in [0,T]. And φ_i is also uniformly distributed in the interval [0,2π[. Now we should consider two cases

a) $$\boldsymbol{\theta}_{i,j}(\ell_i) = \boldsymbol{\theta}_{i,j}(\ell_i + 1)$$

b) $$\boldsymbol{\theta}_{i,j}(\ell_i) \neq \boldsymbol{\theta}_{i,j}(\ell_i + 1)$$

For a) case

$$\mathbf{m_{i,j}} = \boldsymbol{\theta}_{i,j}(\ell_i)\cos\boldsymbol{\varphi}_i$$

We have a constant multiplied by a cos(.) function with an argument that is uniformly distributed. We know, reference [9], that in this case the probability distribution for the random variable $m_{i,j}$ is given by

$$f_{m_{i,j}}(d) = \frac{1}{\pi \left|\theta_{i,j}(\ell_i)\right| [1 - (\frac{d}{\theta_{i,j}(\ell_i)})^2]^{1/2}}$$

for

$$|d| < \left|\theta_{i,j}(\ell_i)\right|$$

and 0 otherwise. Note that this function has a singular point for $d=\theta_{i,j}(l_i)$

For b) case

$$m_{i,j} = Y \cos \varphi_i$$

where

$$Y = \theta_{i,j}(\ell_i) + [\theta_{i,j}(\ell_i + 1) - \theta_{i,j}(\ell_i)]u)$$

and now Y is uniformly distributed in the interval [A,B] where A and B are given by

$$\mathbf{A} = \min[\theta_{i,j}(\ell_i), \theta_{i,j}(\ell_i + 1)]$$ and $$\mathbf{B} = \max[\theta_{i,j}(\ell_i), \theta_{i,j}(\ell_i + 1)]$$

For this case cos φ_i is multiplying Y so we can change the polarity of A and B without any change in the probability density function of m_{ij}. With this observation we can consider B>0 and $|B| > |A|$ and obtain, reference [9]

$$f_{m_{i,j}}(d) = \frac{1}{\pi(B-A)} \ln\left|\frac{B + (B^2 - d^2)^{1/2}}{A + (A^2 - d^2)^{1/2}}\right|$$ for $$|d| < |A|$$

$$f_{m_{i,j}}(d) = \frac{1}{\pi(B-A)} \ln\left|\frac{B + (B^2 - d^2)^{1/2}}{d}\right|$$ for $$|A| < |d| < |B|$$

and zero otherwise.

So we can obtain the probability density function of normalized interference of i$\underline{th}$ user on j$\underline{th}$ desired user for $d_i(-1)=d_i(0)$ case as

$$\mathbf{p1_{m\ i,j}(d)} = \frac{1}{N}\sum_{j=1}^{N} \mathbf{f_{m\ i,j/\ell_j}(d/\ell_j)}$$

Defining now $p2_{m_{i,j}}(d)$ as the probability density function of normalized

interference of i$\underline{th}$ user on j$\underline{th}$ desired user for $d_i(-1)\neq d_i(0)$ we can obtain it

with a simple change: $\boldsymbol{\theta_{i,j}(\ell_i)}$ by $\boldsymbol{\hat{\theta}_{i,j}(\ell)}$. Finally considering equal

probabilities for the interference data bit we can write:

$$\mathbf{p_{m\ i,j}(d)} = \frac{1}{2}\mathbf{p1_{mi,j}(d)} + \frac{1}{2}\mathbf{p2_{mi,j}(d)}$$

Remembering that with our definition we have

$$I_{i,j}(\underline{d}_i, \tau_i, \varphi_i) = T^{-1} T_c d_i(0) m_{i,j} = \frac{1}{N} d_i(0) m_{i,j}$$

and that the desired j^{th} user's total interference is given by

$$\gamma_j(\underline{d}, \underline{\tau}, \underline{\varphi}) = \sum_{\substack{i=1 \\ i \neq j}}^{U} I_{i,j}(\underline{d}_i, \tau_i, \varphi_i)$$

$\Rightarrow$ its distribution can be determined by

$$f_{\gamma_j(\underline{d}, \tau, \underline{\varphi})} = \bigotimes_{\substack{i=1 \\ i \neq j}}^{U} f_{I_{i,j}}$$

where $\otimes$ denotes the convolution operation (valid with the assumed independence among various interference). Observe also that we have constants to consider in this evaluation (normalization). Finally knowing the MAI distribution we can find the BER easily by

$$P_e = \text{Prob}\left(\left[\sqrt{\frac{PT^2}{2}} \gamma_j(\underline{b}, \underline{\tau}, \underline{\varphi}) + \eta_j \right] > \sqrt{\frac{PT^2}{2}} \right)$$

This last step involves a new convolution and an integral that should be numerically evaluated.

Sequences Selection for QS - Quasi-Synchronous CDMA Systems / Single User Receiver

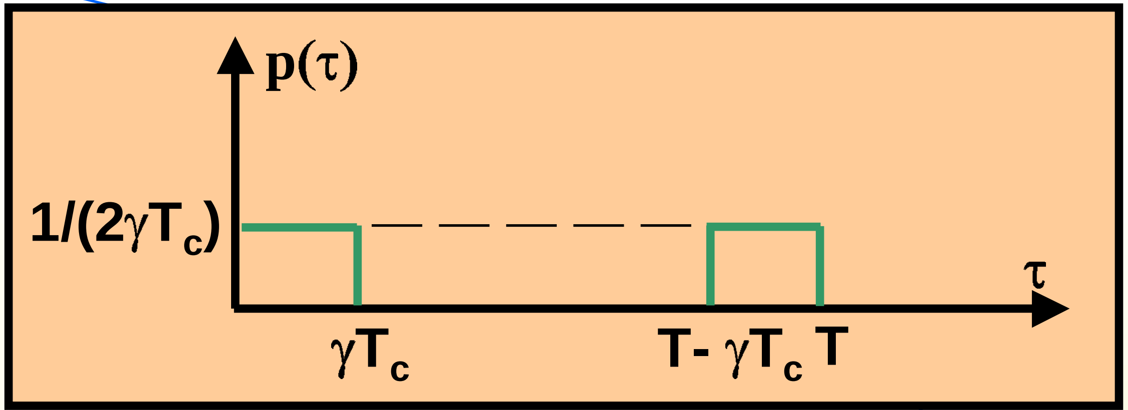
For QS systems the deviations of various random variables are assumed to be constrained within small intervals.

Considering one chip interval (T_c) of the code sequence it is difficult to maintain synchronism at carrier level. However with distribution of one pilot signal it is possible to maintain the chip dispersion within predefined boundaries.

The BER calculation for QS systems will be done with the chip delay of various subscribers maintained in the range:

$$\left|\tau\right| \leq \gamma \, \mathbf{T_c} \quad \text{with} \quad \mathbf{0} \leq \gamma \leq \mathbf{1}$$

and with uniform distribution given by



$$p(\tau) = \begin{cases} (2\gamma T_c)^{-1} & \text{for} \quad 0 \le \tau < \gamma T_c \quad \text{and} \quad T - \gamma T_c \le \tau < T \\ 0 & \text{for} \quad \gamma T_c \le \tau < T - \gamma T_c \end{cases}$$

Now following exactly the same procedure as we have developed for the Pursley's approach we obtain for the $j^{\underline{th}}$ user in a QS system

$$P_e = Q[(SNR)_j] = Q[(\frac{1}{4L^2}\sum_{\substack{i=1 \\ i\neq j}}^{U} \rho_{i,j} + \frac{N_0}{2E_b})^{-1/2}]$$

where

$$\rho_{i,j} = 2\left(1-\gamma+\frac{\gamma^2}{3}\right)C^2_{i,j}(0)+\gamma\left(1-\frac{2\gamma}{3}\right)C_{i,j}(0)\left[C_{i,j}(1)+C_{i,j}(-1)\right] + \frac{\gamma^2}{3}\left[C^2_{i,j}(1)+C^2_{i,j}(-1)+C^2_{i,j}(N-1)+C^2_{i,j}(1-N)\right]$$

Using this equation it's possible to establish objective rules for sequences selection to be used in a given QS system, based on a few and simple parameters (more details in reference [2]).

<u>Given U sequences a sub-optimum strategy for selection could be:</u>

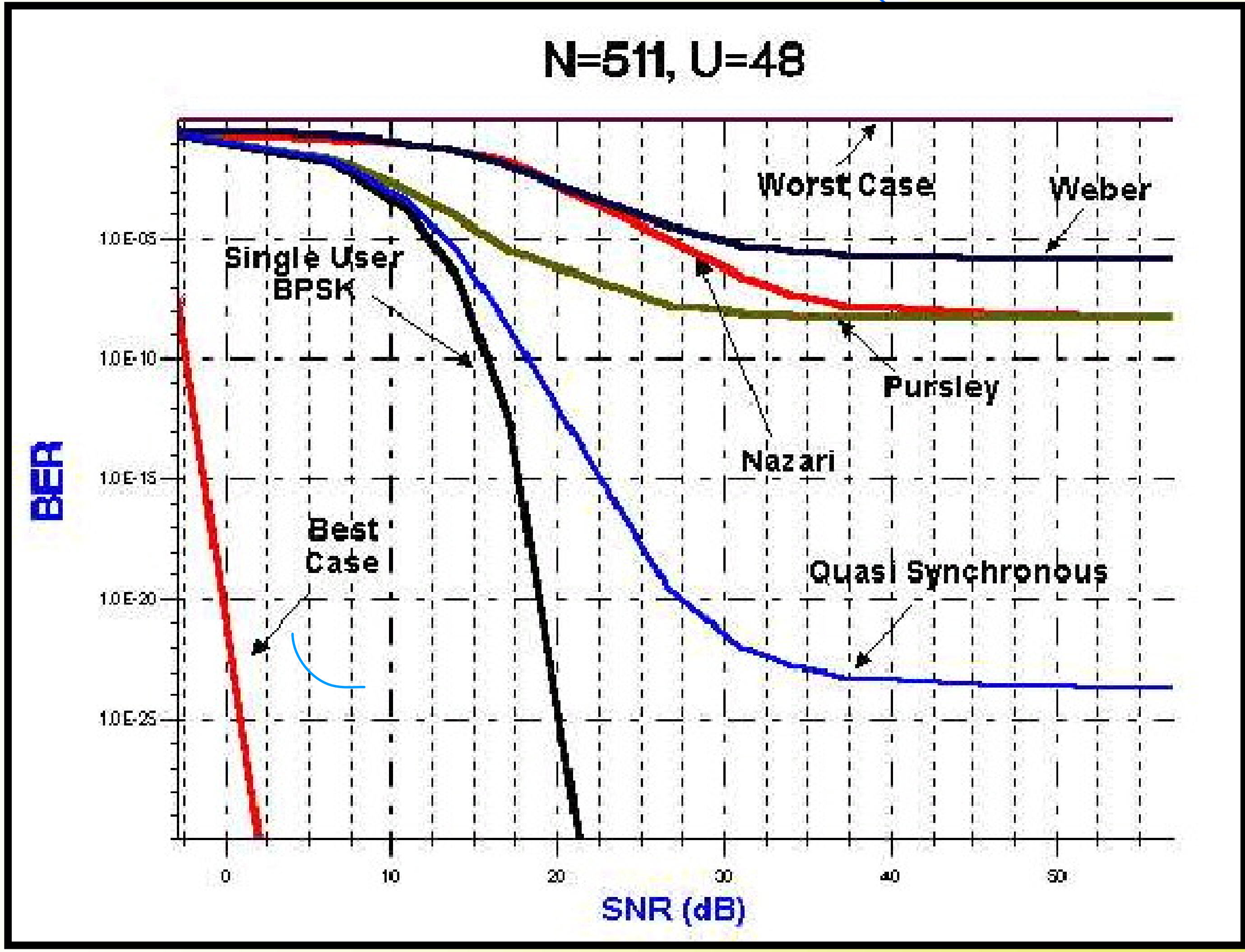
- The first sequence is selected as the first in the list with arbitrary phase;

- The second sequence is selected as the second of the list and its relative phase is calculated considering the minimization of its influence in the first;

- The $i^{\underline{th}}$ sequence is selected as the $i^{\underline{th}}$ of the list and its relative phase is calculated considering the minimization of its influence in the previous (i-1).

This strategy corresponds to a real scenario where the subscribers are introduced one by one (more details in reference [21]).

The next figure illustrates the various capacity determination processes until this point (reference [2]) for a specific set of values for N and U (Gold codes was used for deterministic sequences). These results confirm the superior performance of QS as expected and this advantage is 10dB for BER=10^{-7} in comparison with an asynchronous system with the same parameters.



Comparação dos Métodos de Cálculo de BER Sistema DS/CDMA Convencional (MF) Assíncrono em canal AWGN

- **Método de Weber**
  - mais conservador
  - deve ser utilizado em um primeira abordagem (segurança e simplicidade dos cálculos)

- **Método de NAZARI**
  - mais preciso (ponto de vista teórico)
  - **mais complexo** (cálculo computacional)
    - ✓ devido necessidade amostrar grande quantidade de **PDF´s** com descontinuidades , cuja região de existência aumenta proporcionalmente a N ⇒ introduz **erros** na área destas PDFs (idealmente deveria = 1)
    - ✓ erros se propagam à medida que as convoluções (proporcionais a U) ocorrem.
    - ✓ Cálculos numéricos sugerem que dentro de certos limites, os resultados do método podem ser desnormalizados, levando-se em conta esta integridade final resultante.

- **Método de Pursley**
  - não é exato para um pequeno número de usuários:
    - ✓ modela a MAI como sendo uma variável aleatória Gaussiana
    - ✓ os resultados de Pursley e Nazari se aproximam à medida que **U** ↑ (qdo a aproximação Gaussiana é mais apropriada)
- **Método Worst Case**
  - resultados corretos para pequenos **U**.
    - ✓ Por ex: para U=2 e N=31; U=4 e N=127
    - ✓ nestes casos, método alcança precisão teórica (entre os limites lower e upper bound)

- **Método de Cálculo - Sistemas QS-CDMA**
  - segue literalmente a abordagem de Pursley
  - resulta melhor desempenho (menor BER @Eb/No), devido à condição QS-CDMA
    - ✓ devido pequenos atrasos (controlados) das seqs. entre si e;
    - ✓ conjunto de seqs com fases (sub-) otimizadas

Relação da Pe Assintótica: Pursley X Weber
Seqs Aleatórias

- **Pursley**

$$\lim_{E_b\to\infty} Pe = \lim_{E_b\to\infty} Q\left[\left(\frac{U-1}{3L}+\frac{N_0}{2E_b}\right)^{-1/2}\right] = Q\left(\sqrt{\frac{3L}{U-1}}\right)$$

- **Weber**

$$\lim_{E_b\to\infty} Pe = \lim_{E_b\to\infty} Q\left[\left(\frac{U-1}{2L}+\frac{N_0}{2E_b}\right)^{-1/2}\right] = Q\left(\sqrt{\frac{2L}{U-1}}\right)$$

$$\therefore \frac{\text{argumento Q(x) Pursley}}{\text{argumento Q(x) Weber}} = \sqrt{\frac{3}{2}}$$

Relação da Pe Assintótica: Pursley X Weber

Seqs Aleatórias

- **Seja as curvas desempenho do slide 50:**
  - $(Pe^{Pursley})_{Assintót} \cong 10^{-8}$
  - Consultando tabela de valores da função Q(x):

$\Rightarrow x_{Pursley} \cong 5{,}612 \quad \therefore x_{Weber} = x_{Pursley} \sqrt{(2/3)}$

$\Rightarrow \therefore x_{Weber} = 4{,}5822$

- **Weber**

$\Rightarrow \therefore \mathbf{Q\ (x_{Weber}) \cong 2{,}3 \times 10^{-6}}$

$\Rightarrow$ Resultado bastante próximo ao observado no gráfico da pg. 50.

References and Bibliography

[1] P. J. E. Jeszensky, **Notas de Aula do Curso de Comunicação por Espalhamento Espectral: Uma motivação para o estudo de seqüências de códigos**, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia Eletrônica, 1992, 19 p.

[2] J. R. Fitzgerald Jr., **Tópicos sobre a Caracterização de Desempenho de Sistemas DS-CDMA** , M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia Eletrônica, São Paulo, 1996, 134 p.

[3] M. B. Pursley, **Performance Evaluation for Phase-Coded Spread Spectrum Multiple-Access Communication - Part I: System Analysis**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 795-799.

[4] M. B. Pursley and D. V. Sarwate, **Performance Evaluation for Phase-Coded Spread Spectrum Multiple-Access Communication - Part II: Code Sequence Analysis**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 800-803.

[5] K. Yao, **Error Probability of Asynchronous Spread Spectrum Multiple Access Communication Systems**, IEEE Transactions on Communications, Vol. 25, Nº 8, August 1977, pp. 803-809.

[6] C. L. Weber, K. H. Gaylord and B. H. Batson, **Performance Considerations of Code Division Multiple-Access Systems**, IEEE Transactions on Vehicular Technology, Vol. 30, Nº 1, February 1981, pp. 3-10.

[7] D. V. Sarwate and M. B. Pursley, **Crosscorrelation Properties of Pseudorandom and Related Sequences**, Proceedings of the IEEE, Vol. 68, Nº 5, May 1980, pp. 593-619.

[8] N. Nazari and R. E. Ziemer, **Computationally Efficient Bounds for the Performance of Direct-Sequence Spread Spectrum Multiple-Access Communications Systems in Jamming Environments**, IEEE Transactions on Communications, Vol. 36, Nº 5, May 1988, pp. 577-587.

[9] A. Papoulis, **Probability, Random Variables, and Stochastic Processes**, 3rd Edition , McGraw-Hill, 1991, 666 p.

[10] R. Cameron and B. Woerner, **Performance Analysis of CDMA with Imperfect Power Control**, IEEE Transactions on Communications, Vol. 44, Nº 7, July 1996, pp. 777-781.

[11] K. I. Kim, **CDMA Cellular Engineering Issues**, IEEE Transactions on Vehicular Technology, Vol. 42, Nº 3, August 1993, pp. 345-350.

[12] L. B. Milstein and T. Rappaport, **Performance Evaluation for Cellular CDMA**, IEEE Journal on Selected Areas in Communications, Vol. 10, Nº 4, May 1992, pp. 680-689.

[13] S. Moshavi, **Multiuser Detection for DS-CDMA Communications**, IEEE Communications Magazine, Vol. 34, Nº 10, October 1996, pp. 124-136.

[14] F. D. Priscoli and F. Sestini, **Effects of Imperfect Power Control and User Mobility on a CDMA Cellular Network**, IEEE Journal on Selected Areas in Communications, Vol. 14, Nº 9, December 1996, pp. 1809-1816.

[15] R. K. Morrow Jr. and J. S. Lehnert, **Bit-to-Bit Dependence in Slotted DS/SSMA Packet Systems with Random Signature Sequences**, IEEE Transactions on Communications, Vol. 37, Nº 10, October 1989, pp. 1052-1061.

[16] C. S. Hemsi, **Análise de Sistemas DS/CDMA de Múltipla Taxa**, M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2000, 247 p.

[17] M. W. D. Rolim, **Tópicos sobre a Determinação da Capacidade de Sistemas DS/CDMA**, M. Sc. Dissertation, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2000,173 p.

[18] T. Abrão, **Canceladores de Interferência Multiusuário Aplicados a Sistemas DS/CDMA de Múltipla Taxa**, Ph. D. Thesis, Escola Politécnica da Universidade de São Paulo, Departamento de Engenharia de Telecomunicações e Controle, 2001, 378 p.

[19] A. J. Viterbi, **CDMA: Principles of Spread Spectrum Communication**, Addison Wesley Publishing Co., 1995, 245 p.

[20] W. C. Y. Lee, **Mobile Communications Engineering**, McGraw-Hill Book Co., 2nd Edition, 1998, 689 p.

[21] P. J. E. Jeszensky and J. R. Fitzgerald Jr., **Sequences Selection for Quasi-Synchronous CDMA Systems**, ISSSTA'98-Fifth IEEE International Symposium on Spread Spectrum Techniques and Applications, Sun City, South Africa, 1998, pp. 706-708.

[22] J. K. Holmes, **Coherent Spread Spectrum Systems**, John Wiley & Sons Inc., 1982, 624 p.

[23] S. Verdú, **Minimum Probability of Error for Asynchronous Gaussian Multiple Access Channels**, IEEE Transactions on Information Theory, Vol. 32, Nº 1, January 1986, pp. 85-96.

[24] S. Verdú, **Multiuser Detection**, Cambridge University Press, 1998, 451 p.

[25] J. G. Proakis, **Digital Communications**, McGraw-Hill Book Co., 2nd Edition, 1989, 905 p.

[26] C. S. Wijting, T. Ojanpera, M. J. Juntti, K. Kansanen and R. Prasad, **Groupwise Serial Multiuser Detectors for Multirate DS-CDMA**, Proceedings of IEEE Vehicular Technology Conference (VTC'99), Houston, USA, May 16-20, 1999.

[27] M. J. Juntti, **Performance of Multiuser Detection in Multirate CDMA Systems**, Wireless Personal Communications, Kluver Academic Publishers, 1999.

[28] C. A. Balanis, **Antenna theory: analysis and design**, John Wiley & Sons, 2nd Edition, 1996, 960 p.

[29] D. Divsalar and M. Simon, **Improved CDMA Performance Using Parallel Interference Cancellation**, JPL (Jet Propulsion Laboratory) Publication 95-21, October 1995.

[30] G. H. Golub and C. F. V. Loan, **Matrix Computations**, The John Hopkins University Press, 3rd Edition, 1996, 694 p.

[31] J. C. Liberti and T. S. Rappaport, **Smart Antennas for Wireless Communications : IS-95 and Third Generation CDMA Applications**, Prentice Hall, 1999, 528 p.

[32] L. K. Rasmussen, T. J. Lim and A. L. Johansson, **A Matrix-Algebraic Approach to Successive Interference Cancellation in CDMA**, IEEE Transactions on Communications, Vol. 48, Nº 1, January 2000, pp. 145-151.

[33] D. Guo, L. K. Rasmussen, S. Sun and T. J. Lim, **A Matrix-Algebraic Approach to Linear Parallel Interference Cancellation in CDMA**, IEEE Transactions on Communications, Vol. 48, Nº 1, January 2000, pp. 152-161.

[34] A. L. C. Hui and K. B. Letaief, **Successive Interference Cancellation for Multiuser Asynchronous DS/CDMA Detectors in Multipath Fading Links**, IEEE Transactions on Communications, Vol. 46, Nº 3, March 1998, pp. 384-391.

[35] T. Abrão and P. J. E. Jeszensky, **Detectores Multiusuários para DS/CDMA - Lineares Fixos**, will appear in the Brazilian Telecommunication Society Magazine.

[36] T. Abrão and P. J. E. Jeszensky, **Detectores Multiusários para DS/CDMA - Canceladores de Interferência**, will appear in the Brazilian Telecommunication Society Magazine.