Topology in Biology

Ralph Heiner Buchholz

April 1985 - SMJ 29

The sequence of letters in Figure 1 represents the sequence of bases (which are the opposite of acids) in the DNA of a bacteriophage (a virus which only attacks bacteria) called ϕX174. It is the first complete genome (chromosomal or genetic information) ever decoded for any organism.

Each letter stands for one of four cyclic (meaning circular) bases - adenine, guanine, thymine and cytosine (see Figure 2). There are nine known genes (eight of which are marked in Figure 1) in ϕX174 - the single underline lettering represents genes A, D, J, F, G and H. Interestingly, genes B and E (doubly underlined) lie entirely inside genes A and D respectively. The bases are not directly connected to each other as they are written, rather they lie in a mesh connected by sugar, "D", and phosphate, "P", molecules (see Figure 3).

The bases are always chemically paired cytosine with guanine and thymine with adenine. Next the sugar molecules are attached either side of the so-called "base-pairs". This forms the "rungs" of a "ladder" structure which is held together by phosphate molecules (see Figure 4).

This means that one only needs to specify one side of the base pair sequence as the complementary side can be determined from the pairing rules above. If, for example, one side of the base pair sequence begins

```
cgttaca ....
```

then the other side of the strand must be

```
gcaatgt ....
```

The DNA "ladder" is actually twisted in three dimensions around the lengthwise axis to form the well known "double stranded helix". There are ten base pairs for one complete 360° twist of the ladder and the distance covered is only 3.46 nanometers (where 1 nanometer = 10^{-9} meters). For almost all living creatures (including humans) the genetic material has the form described above - but due to the simplicity of viruses they usually only contain "single stranded" rather than double stranded DNA. However during one phase of reproduction the DNA of ϕX174 is actually double stranded and furthermore the DNA helix for this and most other viruses is circular i.e. the two ends of the ladder actually meet and are chemically joined.

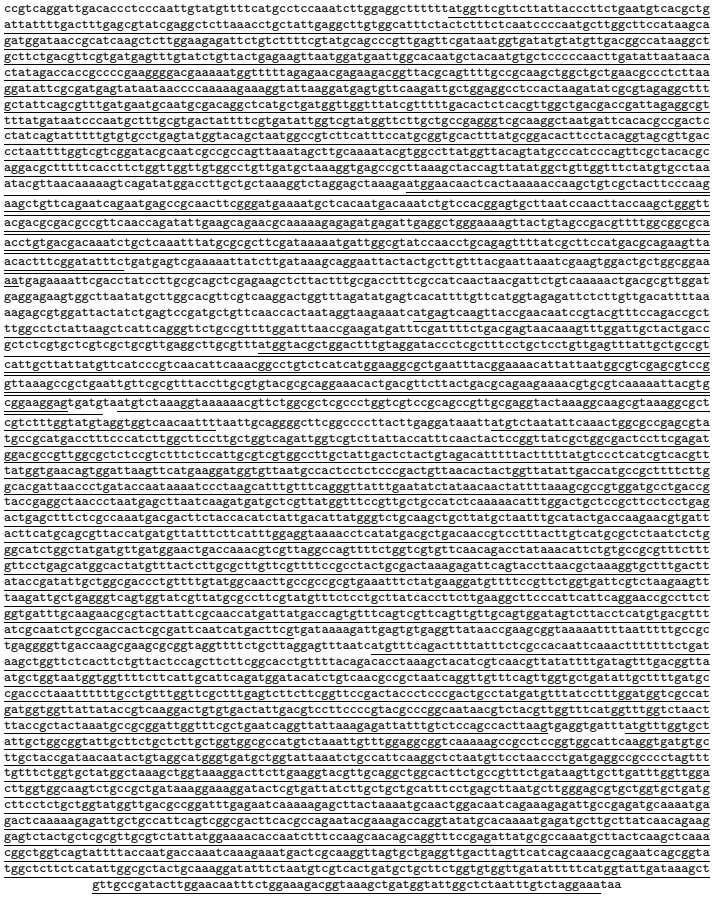

Figure 1: ϕX174 base sequence

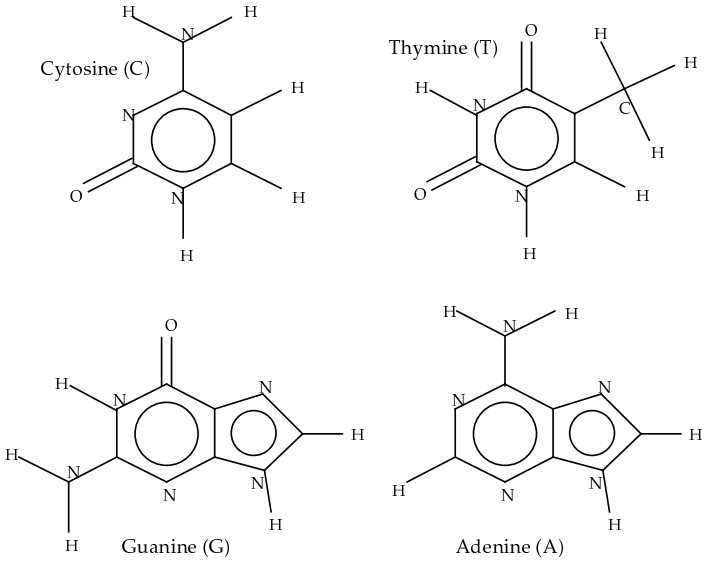

Figure 2: The bases of life

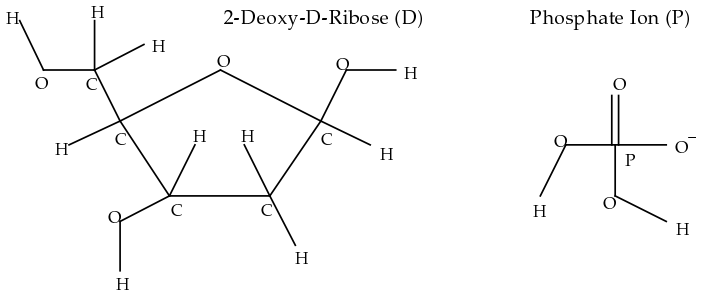

Figure 3: Sugar and phosphate molecules

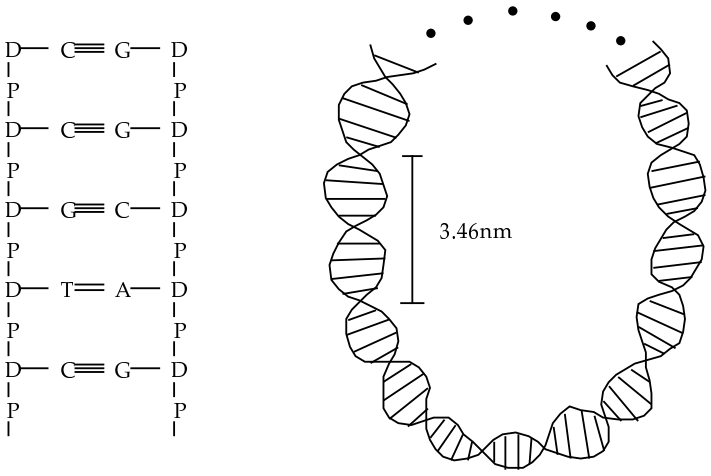

Figure 4: Ladder and Helix

This raises an interesting question. Is the DNA ring of ϕX174, during reproduction, topologically equivalent to a cylinder or a Möbius band?

Now the number of letters in Figure 1 is easily found to be 5375 and so the number of base pairs in the reproductive phase DNA of ϕX174 is 5375. Recalling that the helix has ten base pairs for every 360° twist or equivalently five base pairs for every 180° twist (or half-turn) we see that there must be 1075 half-turns before the ends of the ladder are joined.

If we give a strip of paper an even number (including zero) of half-turns and then glue the ends together the resulting object will be "cylinder-like" i.e. have two sides. However, if we give it an odd number of half-turns and glue the ends it will be "Möbius-like" i.e. have only one side.

So from the above we can say that the DNA ring of reproductive ϕX174 is in fact Möbius-like.

References

1. Principles of Genetics Irwin h. Herskowitz Pub. Macmillan

2. Gödel, Escher, Bach : An Eternal Golden Braid Douglas R. Hofstadter Pub. Harvester Press