(XV) Sketch the graph of r = 1 + cos θ ----- (1).

Find the surface area of revolution of the region bounded by the solid generated by revolving (1) about the x-axis.

Let *s* be the arc length. $x = r\cos\theta$, $y = r\sin\theta$

$dx = dr\cos\theta - r\sin\theta\, d\theta$, $dy = dr\sin\theta + r\cos\theta\, d\theta$

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{(\cos\theta dr - r\sin\theta d\theta)^2 + (\sin\theta dr + r\cos\theta d\theta)^2}$$

$$= \sqrt{\cos^2\theta (dr)^2 + r^2\sin^2\theta (d\theta)^2 + \sin^2\theta (dr)^2 + r^2\cos^2\theta (d\theta)^2}$$

$$= \sqrt{(dr)^2 + r^2(d\theta)^2} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Let *t* be the surface area of revolution. Then $dt = 2\pi y\sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$$t = \int_0^{\pi} 2\pi y\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{\pi} 2\pi r\sin\theta\sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$$

$$= 2\pi\int_0^{\pi}(1+\cos\theta)\sin\theta\sqrt{2+2\cos\theta}\, d\theta$$

$$= -2\sqrt{2}\pi\int_0^{\pi}(1+\cos\theta)^{3/2}\, d\cos\theta$$

$$= -2\sqrt{2}\pi(1+\cos\theta)^{5/2}\cdot\frac{2}{5}\Big|_0^{\pi}$$

$$= -\frac{4\sqrt{2}\pi}{5}\left[(1-1)^{5/2} - 2^{5/2}\right]$$

$$= \frac{32\pi}{5}$$