$y = |\sin x|$ is periodic with period $= \pi$

$G(x) = \int_0^x |\sin t| dt$

If $0 \le x \le \pi$, then for all t such that $0 \le t \le x$, $|\sin t| = \sin t$.

$G(x) = \int_0^x |\sin t| dt = \int_0^x \sin t dt = -\cos x + \cos 0 = 1 - \cos x$

$G(\pi) = \int_0^\pi |\sin t| dt = 1 - \cos \pi = 2$

$G(2\pi) = \int_0^\pi |\sin t| dt + \int_\pi^{2\pi} |\sin t| dt = 2 + 2 = 4$

$G(3\pi) = 6$, $G(4\pi) = 8, \ldots, G(n\pi) = 2n$ where n is any integer.

If $G(x)$ is a periodic function, then there is a positive constant T such that

$$G(x + T) = G(x)$$

If $T > \pi$, let m be the greatest positive integer such that $T = m\pi + a$, where $0 \le a < \pi$

Let $x = \pi - a$, $G(x + T) = G(x) \Rightarrow G(\pi - a + m\pi + a) = G(\pi - a)$

$G((m + 1)\pi) = G(\pi - a)$

$2(m + 1) = 1 - \cos(\pi - a)$

$2m + 2 = 1 + \cos a$

$2m + 1 = \cos a$

contradict to the fact that $-1 \le \cos a \le 1$

If the period $T < \pi$

However, let $x = \pi - T$, $G(x + T) = G(x) \Rightarrow G(\pi - T + T) = G(\pi - T)$

$G(\pi) = G(\pi - T)$

$2 = 1 - \cos T$

$\cos T = -1$

$T = \pi$

contradict to the fact that $T < \pi$

If $T = \pi$, let $x = \pi$, then $G(x + T) = G(x) \Rightarrow G(\pi + \pi) = G(\pi) \Rightarrow 4 = 2$, contradiction

Therefore G cannot be a periodic function.