Intermediate Value Theorem

Let f:$[a, b] \to \mathbb{R}$ be a continuous function, for all $x_1 < x_2 \in [a, b]$ with $f(x_1) \neq f(x_2)$.

Then f takes every value between $f(x_1)$ and $f(x_2)$ somewhere in the interval (x_1, x_2).
Proof: WLOG assume $f(x_1) < f(x_2)$. Let $f(x_1) < k < f(x_2)$.

Define g: $[x_1, x_2] \to \mathbb{R}$ by $g(x) = f(x) - k$.

$g(x_1) = f(x_1) - k < 0$
$g(x_2) = f(x_2) - k > 0$
By Bolzano's Theorem, $\exists\ t \in (x_1, x_2)$ such that $g(t) = 0$
i.e. $\exists\ t \in (x_1, x_2)$ such that $f(t) = k$