Given $\sin^2 x + \sin^2 y = \sin(x + y)$, where x and y are acute angles. Prove that $x + y = 90°$

Proof: Since $\sin^2 x + \sin^2 y = (\sin x + \sin y)^2 - 2 \sin x \sin y$

$$= \left(2\sin\frac{x+y}{2}\cos\frac{x-y}{2}\right)^2 - 2\sin x\sin y$$

$$= 4\sin^2\frac{x+y}{2}\cos^2\frac{x-y}{2} + \cos(x+y) - \cos(x-y)$$

$$= [1 - \cos(x + y)][1 + \cos(x - y)] + \cos(x + y) - \cos(x - y)$$

$$= 1 - \cos(x + y)\cos(x - y)$$

Hence $\sin^2 x + \sin^2 y = \sin(x + y)$ becomes

$1 - \cos(x + y)\cos(x - y) = \sin(x + y)$

$\cos(x + y)\cos(x - y) = 1 - \sin(x + y)$ ……….(*)

Suppose $x + y \neq 90°$

If $x + y > 90°$, by (*): LHS < 0, RHS > 0 impossible.

If $x + y < 90°$, without loss of generality let $x > y > 0$.

Then $\cos(x - y) > \cos(x + y) > 0$

By (*) $1 - \sin(x + y) = \cos(x + y) \cos(x - y) > \cos^2(x + y)$

$1 - \sin(x + y) > 1 - \sin^2(x + y)$

$\sin^2(x + y) - \sin(x + y) > 0$

$\sin(x + y)[\sin(x + y) - 1] > 0$

$\sin(x + y) < 0$ or $\sin(x + y) > 1$ which is impossible.

Therefore $x + y = 90°$

The question is proved.