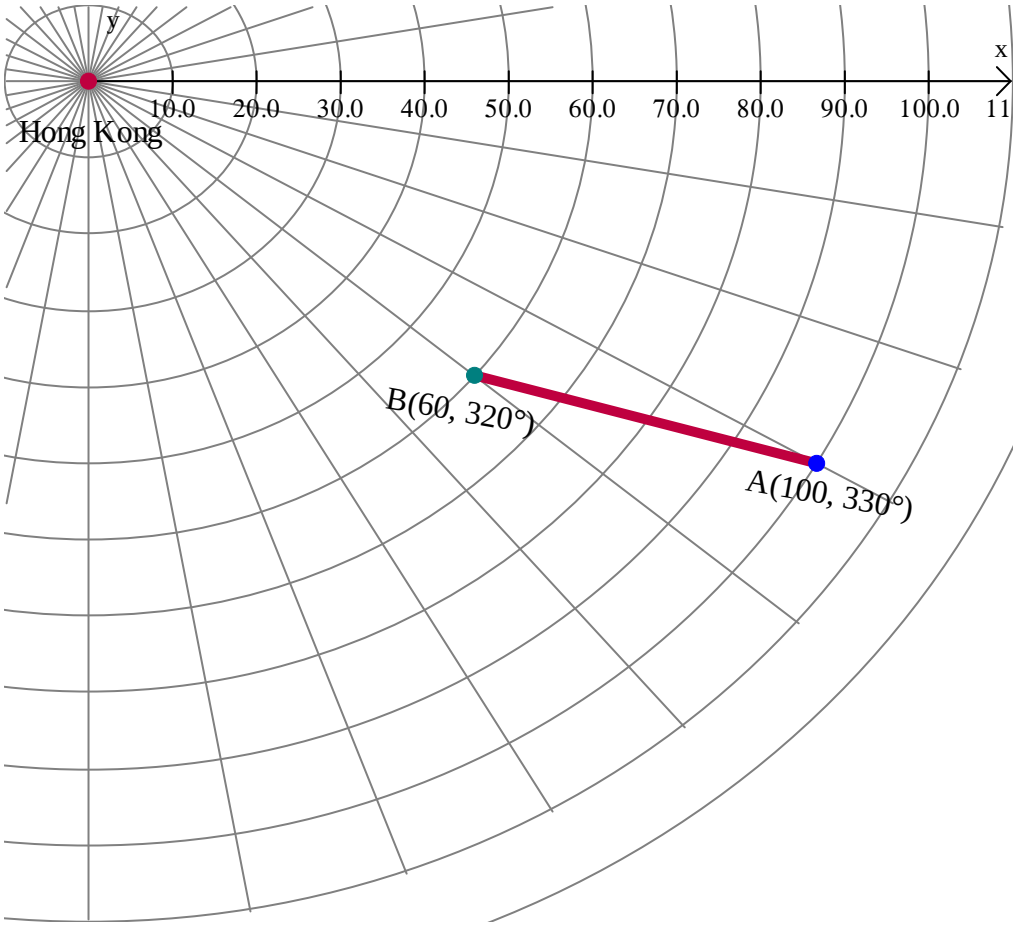
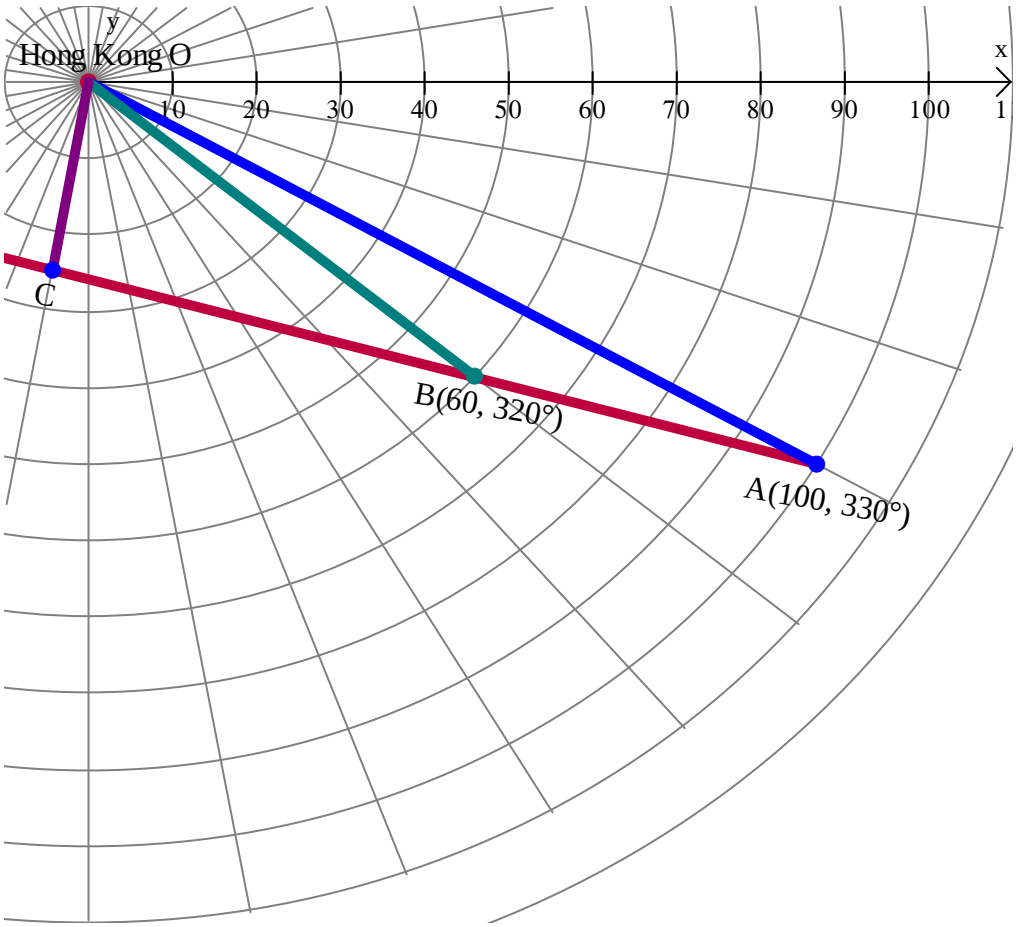
Example on polar coordinates

At noon, a typhoon was located at 100 km, 120° (with polar coordinates $A(100, 330°)$) from Hong Kong. One hour later, the typhoon moved to 60 km, 130° (with polar coordinates $B(60, 320°)$). If the speed and the direction of the typhoon remained unchanged, how far will the typhoon be nearest to Hong Kong? When will it be nearest to Hong Kong? What is the bearing of the typhoon at that time?





$OA = 100$, $OB = 60$, $\angle AOB = 10°$

$AB^2 = 100^2 + 60^2 - 2\times100\times60 \cos 10°$

$AB = 42.21737751$ km

$$\frac{\sin\angle BAO}{60} = \frac{\sin 10^\circ}{AB} \Rightarrow \sin\angle BAO = \frac{60\sin 10^\circ}{42.21737751} = 0.246791517,\ \angle BAO = 14.28773159°$$

Produce AB to C so that $OC \perp AC$.

$AC = OA \cos \angle BAO = 100 \cos 14.28773159° = 96.90685977$ km

$$\text{Time} = \frac{96.90685977}{42.21737751} = 2.295425853 \text{ hours (at 2:17:44 p.m.)}$$

$OC = OA \sin \angle BAO = 100\times0.246791517 = 24.67915175$ km

$\angle AOC = 180° - \angle OCA - \angle OAC = 180° - 90° - 14.28773159° = 75.71226841°$

$330° - 75.71226841° = 254.2877316°$

The polar coordinates of C is (24.7, 254°).

The true bearing of C from O is $90° + 360° - 254° = 196°$

$\therefore$ At 2: 18 p.m., the typhoon will be nearest to Hong Kong.

At that time, the typhoon is 24.7 km from Hong Kong.

It is at a direction of true bearing 196° from Hong Kong.

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