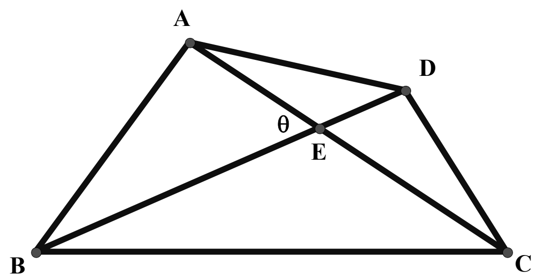
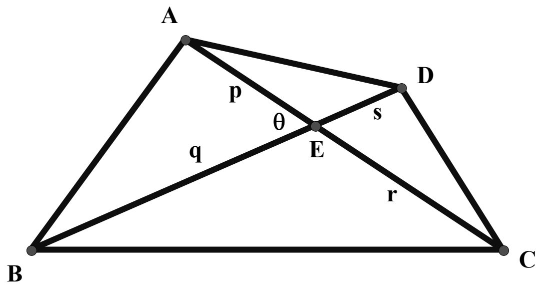
In a quadrilateral $ABCD$. Given $AC = x$, $BD = y$ and $\angle AEB = \theta$, find the area of a quadrilateral.





Suppose the diagonals AC and BD intersect at E.

Let $AE = p$, $BE = q$, $CE = r$, $DE = s$.

Then $\angle CEB = 180° - \theta$ (adj. $\angle$s on st. line)

$\angle CED = \theta$ (vert. opp. $\angle$s)

$\angle AED = 180° - \theta$ (adj. $\angle$s on st. line)

Area of $ABCD$ = area of ΔABE + area of ΔBCE + area of ΔCDE + area of ΔADE

$$= \frac{1}{2}pq\sin\theta + \frac{1}{2}qr\sin(180^\circ - \theta) + \frac{1}{2}rs\sin\theta + \frac{1}{2}ps\sin(180^\circ - \theta)$$

$$= \frac{1}{2}pq\sin\theta + \frac{1}{2}qr\sin\theta + \frac{1}{2}rs\sin\theta + \frac{1}{2}ps\sin\theta$$

$$= \frac{1}{2}\sin\theta(pq + qr + rs + ps)$$

$$= \frac{1}{2}\sin\theta[p(q+s) + r(q+s)]$$

$$= \frac{1}{2}\sin\theta(p+r)(q+s)$$

$$= \frac{1}{2}xy\sin\theta$$

Example 1: If $AC = x = 8$, $BD = y = 6$ and $\angle AEB = \theta = 60°$

Area of $ABCD$ = $\frac{1}{2}\cdot 8\cdot 6\sin 60^\circ = 24\times\frac{\sqrt{3}}{2} = 12\sqrt{3}$

Example 2 If $AC = 10$, $BD = 8$ and $AC \perp BD$

Area of $ABCD$ = $\frac{1}{2}\cdot 8\cdot 10\sin 90^\circ = 40$