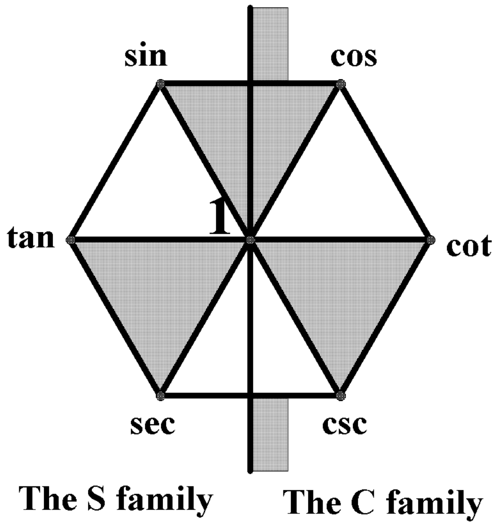
Formulae for the Trigonometric functions

Created by Francis Hung on 23 June 2008, last updated: 2009-02-06

I The magic hexagon:

Along each diagonal,



$\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

In each shaded triangle,

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

In any three adjacent vertices,

$\sin\theta = \cos\theta \cdot \tan\theta$ $\cos\theta = \sin\theta \cdot \cot\theta$

$\cot\theta = \cos\theta \cdot \csc\theta$ $\csc\theta = \cot\theta \cdot \sec\theta$

$\sec\theta = \tan\theta \cdot \csc\theta$ $\tan\theta = \sin\theta \cdot \sec\theta$

Differentiation: In each shaded triangle's edge,

$\text{DS} = + \dfrac{d\sin x}{dx} = \cos x$ $\text{DC} = - \dfrac{d\cos x}{dx} = -\sin x$

$\dfrac{d\tan x}{dx} = \sec^2 x$ $\dfrac{d\cot x}{dx} = -\csc^2 x$

$\dfrac{d\sec x}{dx} = \sec x\tan x$ $\dfrac{d\csc x}{dx} = -\csc x\cot x$

Integration: the inverse process of differentiation

$\int \cos x dx = \sin x + C$ $\int \sin x dx = -\cos x + C$

$\int \sec^2 x dx = \tan x + C$ $\int \csc^2 x dx = -\cot x + C$

$\int \sec x \tan x dx = \sec x + C$ $\int \csc x \cot x dx = -\csc x + C$

II General Solutions

$\sin\theta = \sin\alpha$ $\theta = 180°n + (-1)^n\alpha$ $\theta = n\pi + (-1)^n\alpha$, where n is an integer.

$\cos\theta = \cos\alpha$ $\theta = 360°n \pm \alpha$ $\theta = 2n\pi \pm \alpha$, where n is an integer.

$\tan\theta = \tan\alpha$ $\theta = 180°n + \alpha$ $\theta = n\pi + \alpha$, where n is an integer.

III Compound Angle Formulae

$\sin(A + B) = \sin A\cos B + \cos A\sin B$ SC + CS

$\sin(A - B) = \sin A\cos B - \cos A\sin B$ SC − CS

$\cos(A + B) = \cos A\cos B - \sin A\sin B$ CC − SS

$\cos(A - B) = \cos A\cos B + \sin A\sin B$ CC + SS

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan A\tan C}$

IV Multiple angles

$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

$\cos 3\theta = 4 \sin^3 \theta - 3 \cos \theta$

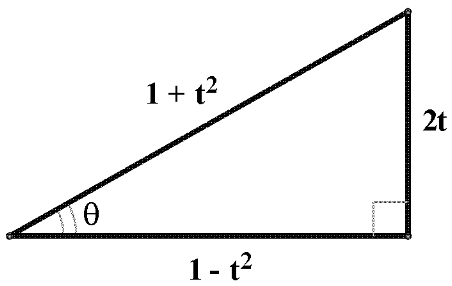
$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$ $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1-6\tan^2\theta+\tan^4\theta}$

V Half angles

Let $t = \tan\dfrac{\theta}{2}$, then $\sin \theta = \dfrac{2t}{1+t^2}$

$\cos \theta = \dfrac{1-t^2}{1+t^2}$

$\tan \theta = \dfrac{2t}{1-t^2}$



$\sin\dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{2}}$

$\cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1+\cos x}{2}}$

$\tan\dfrac{x}{2} = \pm\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$

VI Sum and Product

Sum

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

Product

$\sin X \cos Y = \dfrac{1}{2}[\sin(X+Y) + \sin(X-Y)]$

$\cos X \sin Y = \dfrac{1}{2}[\sin(X+Y) - \sin(X-Y)]$

$\cos X \cos Y = \dfrac{1}{2}[\cos(X+Y) + \cos(X-Y)]$

$\sin X \sin Y = -\dfrac{1}{2}[\cos(X+Y) - \cos(X-Y)]$