The Tangent Rule

In $\triangle ABC$, $\dfrac{a+b}{a-b}=\dfrac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$

Proof: By Sine formula, $a = 2R\sin A$, $b = 2R\sin B$.

$$\frac{a+b}{a-b}=\frac{2R\sin A+2R\sin B}{2R\sin A-2R\sin B}=\frac{\sin A+\sin B}{\sin A-\sin B}$$

$$=\frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$=\frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$$ Hence the theorem is proved.

Example: $A = 65^\circ$, $b = 281.4$, $c = 208$. Use tangent rule to find $\angle B$, $\angle C$.

$b + c = 281.4 + 208 = 489.4$

$b - c = 281.4 - 208 = 73.4$

$$\frac{B+C}{2}=\frac{180^\circ-A}{2}=\frac{180^\circ-65^\circ}{2}=57.5^\circ$$

By tangent rule, $\dfrac{b+c}{b-c}=\dfrac{\tan\frac{B+C}{2}}{\tan\frac{B-C}{2}}$.

$$\frac{489.4}{73.4}=\frac{\tan 57.5^\circ}{\tan\frac{B-C}{2}}$$

$$\tan\frac{B-C}{2}=0.235420762$$

$$\frac{B-C}{2}=13.2473953^\circ$$

$B = \dfrac{B+C}{2}+\dfrac{B-C}{2} = 57.5^\circ + 13.2473953^\circ = 70.7^\circ$ (correct to 3 sig. fig.)

$C = \dfrac{B+C}{2}-\dfrac{B-C}{2} = 57.5^\circ - 13.2473953^\circ = 44.3^\circ$ (correct to 3 sig. fig.)