

The **Heron's formula** ---- the area of a triangle, given 3 sides.

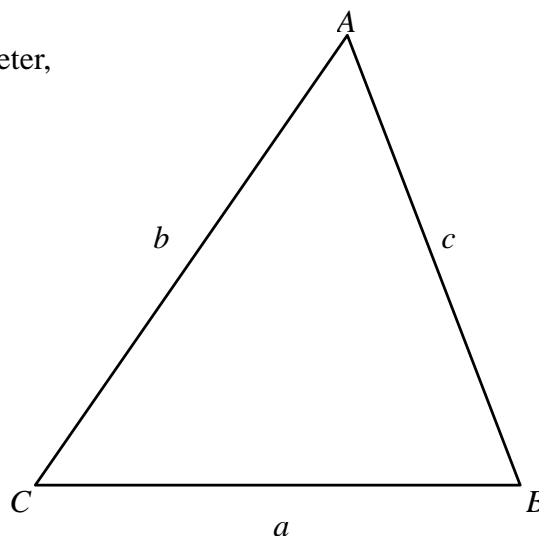
In $\triangle ABC$, let $s = \frac{1}{2}(a + b + c)$, half of a perimeter,

then the area $= \sqrt{s(s-a)(s-b)(s-c)}$.

Proof: By cosine rule $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$1 - \cos^2 C = 1 - \left(\frac{a^2 + b^2 - c^2}{2ab} \right)^2$$

$$\begin{aligned} \sin^2 C &= \frac{4a^2b^2 - (a^2 + b^2 - c^2)^2}{4a^2b^2} \\ &= \frac{(2ab + a^2 + b^2 - c^2)(2ab - a^2 - b^2 + c^2)}{4a^2b^2} \\ &= \frac{[(a+b)^2 - c^2][c^2 - (a-b)^2]}{4a^2b^2} \\ &= \frac{(a+b+c)(a+b-c)(a-b+c)(-a+b+c)}{4a^2b^2} \\ &= \frac{(a+b+c)(a+b+c-2c)(a+b+c-2b)(a+b+c-2a)}{4a^2b^2} \\ &= \frac{2s(2s-2c)(2s-2b)(2s-2a)}{4a^2b^2} = \frac{4s(s-a)(s-b)(s-c)}{a^2b^2} \end{aligned}$$



$$\text{area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} ab \sqrt{\frac{4s(s-a)(s-b)(s-c)}{a^2b^2}} = \sqrt{s(s-a)(s-b)(s-c)}$$

As an example, let $a = 5$, $b = 6$, $c = 7$. then $s = \frac{1}{2}(5 + 6 + 7) = 9$

$$s - a = 9 - 5 = 4, s - b = 9 - 6 = 3, s - c = 9 - 7 = 2$$

$$\text{area} = \sqrt{9 \times 4 \times 3 \times 2} = 6\sqrt{6}$$