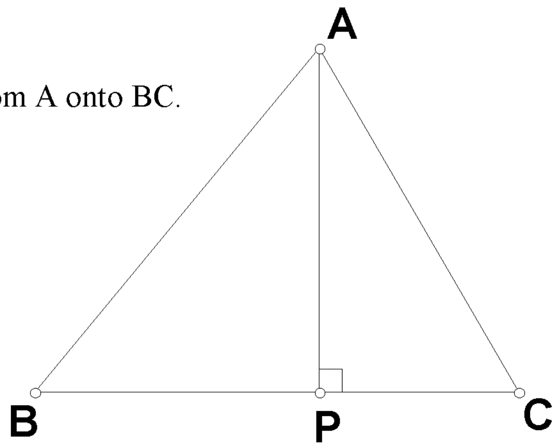
The **Cosine formula**



In $\triangle ABC$, let P be the foot of perpendicular from A onto BC.

BC = BP + PC

$a = c \cos B + b \cos C$ ……(1)

Similarly, $b = c \cos A + a \cos C$ ……(2)

$c = a \cos B + b \cos A$ ……(3)

From (1) $\cos B = \dfrac{a - b\cos C}{c}$ ……(4)

From (2) $\cos A = \dfrac{b - a\cos C}{c}$ ……(5)

Substitute (4) and (5) into (3),

$c = a\dfrac{a - b\cos C}{c} + b\dfrac{b - a\cos C}{c}$

$c^2 = a^2 + b^2 - 2ab\cos C$.

Since a, b and c are symmetric variables, we can derive the similar formulae:

$b^2 = a^2 + c^2 - 2ac\cos B$ and

$a^2 = b^2 + c^2 - 2bc\cos A$.

We can also change cosine as the subject of the formula:

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$;

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$ and

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Example (SAS) Given $a = 6$, $b = 5$, $C = 60°$, find c.

$c^2 = 6^2 + 5^2 - 2\times 6\times 5\times \cos 60°$

$c = \sqrt{31}$

Example (SSS) Given that $a = 6$, $b = 5$, $c = 7$, find C.

$\cos C = \dfrac{6^2 + 5^2 - 7^2}{2\times 6\times 5} = \dfrac{1}{5}$

$C = 78.5°$

Example (SSA) Given that $a = 8$, $b = 5$, $B = 30°$, find c.

$5^2 = 8^2 + c^2 - 2\times 8\times c\cos 30°$

$c^2 - 8\sqrt{3}c + 39 = 0$, a quadratic equation in c.

$c = 4\sqrt{3} \pm 3$

Please see the right figure for reference.