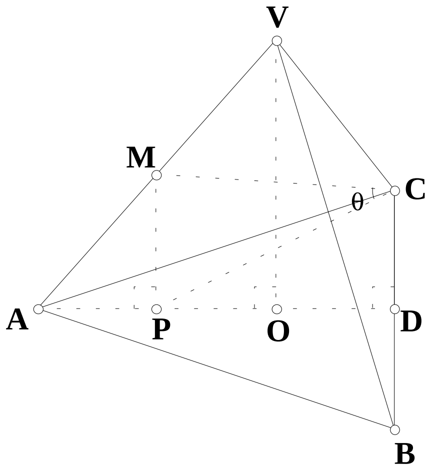
In the figure, $VABC$ is a regular tetrahedron with side = 2. M is the mid-point of VA. Find the angle between CM and the plane ABC.



Solution: Let D be the mid-point of BC.

Then $VM = MA = CD = DB = 1$. $\Delta ACD \cong \Delta ABD$ (SSS)

$\angle ADC = \angle ADB = 90°$ (corr. $\angle$s $\cong$ Δ's)

$AD = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\Delta VMC \cong \Delta AMC$ (SSS)

$\angle VMC = \angle AMC = 90°$ (corr. $\angle$s $\cong$ Δ's)

$CM = \sqrt{2^2 - 1^2} = \sqrt{3}$

O is the orthogonal projection of V on the plane ABC.

$\angle AOV = 90°$. O is the centroid of ΔABC.

$AO = \frac{2}{3} AD = \frac{2\sqrt{3}}{3}$.

P is the projection of M on the plane ABC. $\angle APM = 90°$.

$MP \parallel VO$ (corr. $\angle$s eq.)

$AP = PO = \frac{\sqrt{3}}{3}$ (intercept theorem)

In ΔAPM, $MP^2 = AM^2 - AP^2 = 1^2 - \left(\frac{\sqrt{3}}{3}\right)^2 = \frac{2}{3}$

$\Rightarrow MP = \sqrt{\frac{2}{3}}$

In ΔCPM, $\angle CPM = 90°$. required angle = $\angle PCM = \theta$

$\sin\theta = \frac{MP}{CM} = \frac{\sqrt{\frac{2}{3}}}{\sqrt{3}} = \frac{\sqrt{2}}{3}$

$\theta = 28.1°$