Question: N is a positive integer. When N is divided by 3, the remainder is 1. When N is divided by 5, the remainder is 2. When N is divided by 7, the remainder is 5. Find the least positive integral value of N.

Step 1: When N is divided by 3, the remainder is 1. Let $n_1 = 1$

When N is divided by 5, the remainder is 2. Let $n_2 = 2$

When N is divided by 7, the remainder is 5. Let $n_3 = 5$

$5\times7 = 35$, $3\times5 = 15$, $3\times7 = 21$, LCM of 3, 5, 7 is 105

Step 2: Solve $35m - 3k = 1$ for positive integral value of m gives $m = 2$, $k = 23$

Let $a_1 = 35\times m = 70$

Step 3: Solve $21m - 5k = 1$ for positive integral value of m gives $m = 1$, $k = 4$

Let $a_2 = 21\times m = 21$

Step 4: Solve $15m - 7k = 1$ for positive integral value of m gives $m = 1$, $k = 2$

Let $a_3 = 15\times m = 15$

Step 5: Let $N = a_1\times n_1 + a_2\times n_2 + a_3\times n_3 = 70\times1 + 21\times2 + 15\times5 = 187$

The minimum positive integral value of N is the remainder when 187 divided by 105, which is 82.

Please refer to the following website:

http://episte.math.ntu.edu.tw/articles/sm/sm_01_01_2/page6.html

有一組連續的四個正整數，從小到大依次排列，第一個是 5 的倍數；第二個是 7 的倍數；第三個是 9 的倍數；第四個是 11 的倍數。試求最小的可能整數。

Step 1:

Let the four numbers be n, $n + 1$, $n + 2$, $n + 3$

When n is divided by 5, the remainder is 0. Let $n_1 = 0$

$n + 1$ is divisible by 7; when n is divided by 7, the remainder is 6. Let $n_2 = 6$

$n + 2$ is divisible by 9; when n is divided by 9, the remainder is 7. Let $n_3 = 7$

$n + 3$ is divisible by 11, when n is divided by 11, the remainder is 8. Let $n_4 = 8$

$7\times9\times11 = 693$, $5\times9\times11 = 495$, $5\times7\times11 = 385$, $5\times7\times9 = 315$, LCM of 5, 7, 9,11 is 3465

Step 2: Solve $5m + 693k = 1$ for positive integral value gives $m = -277$, $k = 2$

Let $a_1 = 693\times2 = 1386$

Step 3: Solve $7m + 495k = 1$ for positive integral value gives $m = -212$, $k = 3$

Let $a_2 = 495\times3 = 1485$

Step 4: Solve $9m + 385k = 1$ for positive integral value gives $m = -171$, $k = 4$

Let $a_3 = 385\times4 = 1540$

Step 5: Solve $11m + 315k = 1$ for positive integral value gives $m = 84$, $k = -3$

Let $a_4 = 315\times(-3) = -945$

Step 6: Let $N = a_1\times n_1 + a_2\times n_2 + a_3\times n_3 + a_4\times n_4 = 1386\times0+1485\times6+1540\times7-945\times8=12130$

The minimum positive integral value of N is the remainder when 12130 divided by 3465, which is 1735.