Individual Events

I1	*a*	6	I2	*P*	2	I3	*a*	–2	I4	*a*	2	IS	*P*	84
	b	$2+4\sqrt{2}$		*Q*	6		*b*	9		*b*	11		*Q*	8
	c	7		*R*	56		*c*	$\frac{1}{24}$		*c*	462		*R*	$\frac{\sqrt{3}}{4}$
	d	11		*S*	2352		*d*	$-\frac{7}{18}$		*d*	334		*S*	$1+\sqrt{2}$

Group Events

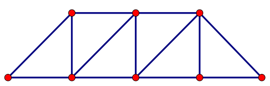
G1	*a*	47	G2	*a*	2	G3	*a*	–10	G4	*P*	500	GS	*a*	16
	b	101		*b*	4.5		*b*	0		*Q*	15		*b*	1
	c	43		*c*	15		*c*	2005		*R*	$\frac{1}{2}$		*c*	12
	d	0		*d*	1		*d*	2005		*S*	1		*d*	9

Individual Event 1

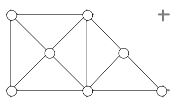
I1.1 Given that there are a positive integers less than 200 and each of them has exactly three positive factors, find the value of a.

If $x = rs$, where r and s are positive integers, then the positive factors of x may be 1, r, s and x.
In order to have exactly three positive factors, $r = s =$ a prime number.
Possible x = 4, 9, 25, 49, 121, 169. $a = 6$.

I1.2 If a copies of a right-angled isosceles triangle with hypotenuse $\sqrt{2}$ cm can be assembled to form a trapezium with perimeter equal to b cm, find the least possible value of b. (give the answer in surd form).



or



The perimeter $= 6 + 2\sqrt{2} \approx 9.4$ or $2 + 4\sqrt{2} \approx 8.9$
The least possible value of $b = 2 + 4\sqrt{2}$

I1.3 If $\sin(c^2 - 3c + 17)° = \frac{4}{b-2}$, where $0 < c^2 - 3c + 17 < 90$ and $c > 0$, find the value of c.

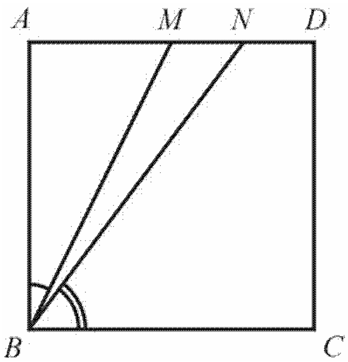
$\sin(c^2 - 3c + 17)° = \frac{4}{2+4\sqrt{2}-2} = \frac{1}{\sqrt{2}}$
$c^2 - 3c + 17 = 45$
$c^2 - 3c - 28 = 0$
$(c - 7)(c + 4) = 0$
$c = 7$ or -4 (rejected)

I1.4 Given that the difference between two 3-digit numbers $\overline{xyz}$ and $\overline{zyx}$ is $700 - c$, where $x > z$. If d is the greatest value of $x + z$, find the value of d.

$\overline{xyz} - \overline{zyx} = 700 - c$
$100x + 10y + z - (100z + 10y + x) = 700 - 7$
$99x - 99z = 693$
$x - z = 7$
Possible answers: $x = 8$, $y = 1$ or $x = 9$, $y = 2$
d is the greatest value of $x + z = 9 + 2 = 11$

Individual Event 2

I2.1 In Figure 1, *ABCD* is a square, *M* is the mid-point of *AD* and *N* is the mid-point of *MD*. If $\angle CBN : \angle MBA = P : 1$, find the value of *a*.



Let $\angle ABM = \theta$, $\angle CBM = P\theta$. Let $AB = 4$, $AM = 2$, $MN = 1 = ND$

$$\tan\theta = \frac{1}{2}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} = \frac{4}{3} = \tan P\theta,\ P = 2$$

I2.2 Given that *ABCD* is a rhombus on a Cartesian plane, and the co-ordinates of its vertices are $A(0, 0)$, $B(P, 1)$, $C(u, v)$ and $D(1, P)$ respectively. If $u + v = Q$, find the value of *Q*.

$\because$ *ABCD* is a rhombus, $\therefore$ It is also a parallelogram

By the property of parallelogram, the diagonals bisect each other

Mid point of *B*, *D* = mid point of *AC*

$$\left(\frac{1+2}{2}, \frac{2+1}{2}\right) = \left(\frac{0+u}{2}, \frac{0+v}{2}\right)$$

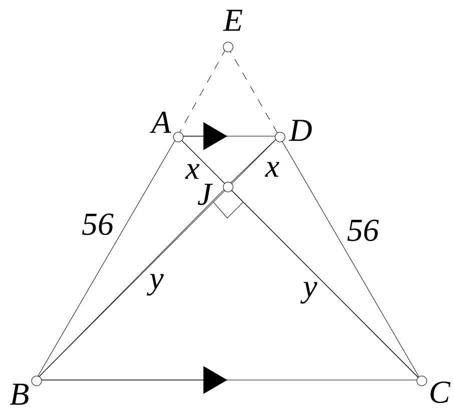
$u = 3$, $v = 3 \Rightarrow Q = u + v = 6$

I2.3 If $1 + (1 + 2) + (1 + 2 + 3) + \dots + (1 + 2 + 3 + \dots + Q) = R$, find the value of *R*.

$R = 1 + (1 + 2) + (1 + 2 + 3) + \dots + (1 + 2 + 3 + \dots + 6)$

$R = 1 + 3 + 6 + 10 + 15 + 21 = 56$

I2.4 In figure, *EBC* is an equilateral triangle, and *A*, *D* lie on *EB* and *EC* respectively. Given that *AD* // *BC*, $AB = CD = R$ and $AC \perp BD$. If the area of the trapezium *ABCD* is *S*, find the value of *S*.



$\angle ABC = \angle BCD = 60°$, *AC* intersects *BD* at *J*, $AC \perp BD$.

By symmetry, $AJ = DJ = x$, $BJ = CJ = y$.

$\angle JBC = \angle JCB = \angle JAD = \angle JDA = 45°$, $\angle ADC = 120°$

Apply sine formula on ΔACD, $\dfrac{56}{\sin 45^\circ} = \dfrac{x+y}{\sin 120^\circ}$

$x + y = 28\sqrt{6}$, area$=\dfrac{1}{2}(x+y)^2 \sin 90^\circ = \dfrac{1}{2}(784)(6) = 2352$

Individual Event 3

I3.1 Let $x\neq\pm1$ and $x\neq-3$.If a is the real root of the equation $\frac{1}{x-1}+\frac{1}{x+3}=\frac{2}{x^2-1}$,find the value of a

$$\frac{1}{x+3}=\frac{2}{x^2-1}-\frac{1}{x-1} \Rightarrow \frac{1}{x+3}=\frac{2-(x+1)}{(x-1)(x+1)}$$

$$\frac{1}{x+3}=\frac{1-x}{(x-1)(x+1)} \Rightarrow \frac{1}{x+3}=-\frac{1}{x+1}$$

$x+1=-x-3 \Rightarrow x=-2=a$

I3.2 If $b>1$, $f(b)=\frac{-a}{\log_2 b}$ and $g(b)=1+\frac{1}{\log_3 b}$. If b satisfies the equation

$|f(b)-g(b)|+f(b)+g(b)=3$, find the value of b.

$$\left|\frac{2\log 2}{\log b}-1-\frac{\log 3}{\log b}\right|+\frac{2\log 2}{\log b}+1+\frac{\log 3}{\log b}=3\Rightarrow\left|\frac{\log \frac{4}{3b}}{\log b}\right|+\frac{\log 12}{\log b}=2\Rightarrow\left|\log\frac{4}{3b}\right|=\log\frac{b^2}{12}$$

$$\log\frac{4}{3b}=\pm\log\frac{b^2}{12}$$

$$\log\frac{4}{3b}=\log\frac{b^2}{12} \text{ or } \log\frac{4}{3b}=\log\frac{12}{b^2}$$

$b^3=16$ or $b=9$

Method 2

Remark Define the maximum function of x, y as: $\text{Max}(x, y)=\frac{x+y+|x-y|}{2}$

Similarly, the minimum function of x, y is: $\text{Min}(x, y)=\frac{x+y-|x-y|}{2}$

$f(b)=\frac{-a}{\log_2 b}=\frac{4\log 2}{\log b}=\frac{\log 16}{\log b}$, $g(b)=1+\frac{1}{\log_3 b}=\frac{\log b}{\log b}+\frac{\log 3}{\log b}=\frac{\log 3b}{\log b}$, by using the change of base formula

The given equation is equivalent to 2 Max($f(b)$, $g(b)$) = 3

If $f(b)>g(b)$, i.e. $\frac{\log 16}{\log b}>\frac{\log 3b}{\log b}\Rightarrow b<\frac{16}{3}$, then the equation is $2f(b)=3$

$\frac{2\log 16}{\log b}=3\Rightarrow \log 256=\log b^3\Rightarrow b^3=256<\left(\frac{16}{3}\right)^3\Rightarrow 1<\frac{16}{27}$

$\Rightarrow 27<16$, which is a contradiction; $\therefore$ rejected

If $f(b)\leq g(b)$, the equation is equivalent to $2g(b)=3$

i.e. $\frac{2\log 3b}{\log b}=3\Rightarrow \log 9b^2=\log b^3\Rightarrow 9b^2=b^3\Rightarrow b=3$

I3.3 Given that x_0 satisfies the equation $x^2-5x+(b-8)=0$. If $c=\frac{x_0^2}{x_0^4+x_0^2+1}$, find the value of c.

$x^2-5x+1=0$, $x^2+1=5x$, $x^4+2x^2+1=25x^2$, $x^4+x^2+1=24x^2$

$$c=\frac{x_0^2}{x_0^4+x_0^2+1}=\frac{x_0^2}{24x_0^2}=\frac{1}{24}$$

I3.4 If -2 and $216c$ are the roots of the equation $px^2+dx=1$, find the value of d.

-2 and 9 are roots of $px^2+dx-1=0$

product of roots $=-\frac{1}{p}=-2\times9\Rightarrow p=\frac{1}{18}$

sum of roots $=-\frac{d}{p}=-2+9\Rightarrow -18d=7\Rightarrow d=-\frac{7}{18}$

Individual Event 4

I4.1 Let a be a real number. If a satisfies the equation $\log_2(4^x + 4) = x + \log_2(2^{x+1} - 3)$, find the value of a.

$\log_2(4^x + 4) = \log_2 2^x + \log_2(2^{x+1} - 3)$

$4^x + 4 = 2^x\cdot(2^{x+1} - 3)$

$(2^x)^2 + 4 = 2\cdot(2^x)^2 - 3\cdot 2^x$

$0 = (2^x)^2 - 3\cdot 2^x - 4$

$(2^x - 4)(2^x + 1) = 0$

$2^x = 4$, $x = 2 = a$

I4.2 Given that n is a natural number. If $b = n^3 - 4an^2 - 12n + 144$ is a prime number, find the value of b.

Let $f(n) = n^3 - 8n^2 - 12n + 144$

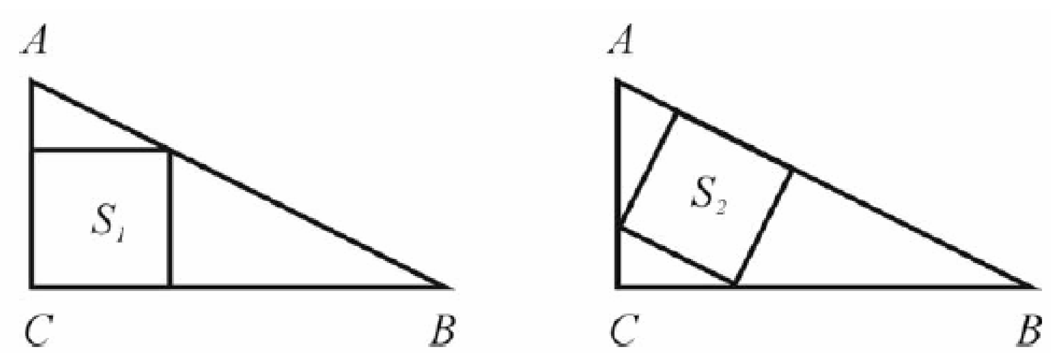
$f(6) = 6^3 - 8\cdot 6^2 - 12\cdot 6 + 144 = 216 - 288 - 72 + 144 = 0$

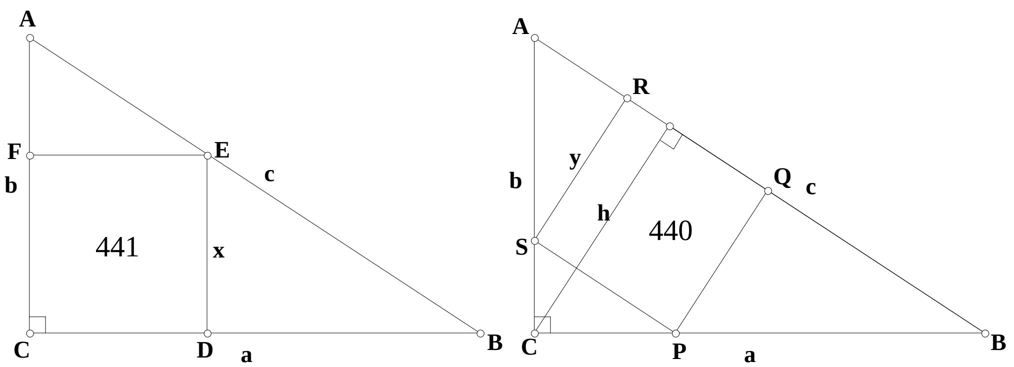
$\therefore f(6)$ is a factor

By division, $f(n) = (n - 6)(n - 6)(n + 4)$

$b = n^3 - 8n^2 - 12n + 144$, it is a prime $\Rightarrow n - 6 = 1$, $n = 7$, $b = 11$

I4.3 In Figure 1, S_1 and S_2 are two different inscribed squares of the right-angled triangle ABC. If the area of S_1 is $40b + 1$, the area of S_2 is $40b$ and $AC + CB = c$, find the value of c.





Add the label D, E, F, P, Q, R, S as shown. $CDEF$, $PQRS$ are squares.

Let $DE = x$, $SR = y$, then $x = \sqrt{441} = 21$, $y = \sqrt{440}$. Let $BC = a$, $AC = b$, $AB = c = \sqrt{a^2 + b^2}$.

Let the height of the triangle drawn from C onto AB be h, then $ab = ch$ = 2 area of Δ .....(*)

$\Delta AFE \sim \Delta ACB$: $\frac{b-x}{b}=\frac{x}{a}\Rightarrow x=\frac{ab}{a+b}=21$ ....(1)

$\Delta CSP \sim \Delta CAB$: $\frac{\text{height of }\Delta\text{CSP from C}}{SP}=\frac{h}{c}\Rightarrow\frac{h-y}{y}=\frac{h}{c}\Rightarrow y=\frac{ch}{c+h}=\sqrt{440}$

By (*), $\frac{ab}{c+\frac{ab}{c}}=\sqrt{440}\Rightarrow\sqrt{440}=\frac{ab\sqrt{a^2+b^2}}{a^2+ab+b^2}$ ......(2)

From (1) $ab = 21(a + b)$ .... (3), sub (3) into (2):

$$\sqrt{440}=\frac{21(a+b)\sqrt{(a+b)^2-2ab}}{(a+b)^2-ab}=\frac{21(a+b)\sqrt{(a+b)^2-42(a+b)}}{(a+b)^2-21(a+b)}=\frac{21\sqrt{(a+b)^2-42(a+b)}}{(a+b)-21}$$

Cross multiplying and squaring both sides:

$440[(a + b)^2 - 42(a + b) + 441] = 441[(a + b)^2 - 42(a + b)]$

$(a + b)^2 - 42(a + b) - 440\times441 = 0$

$(a + b - 462)(a + b + 420) = 0$

$AC + CB = a + b = 462$

I4.4 Given that $241c + 214 = d^2$, find the value of d.

$d^2 = 241\times462 + 214$

$d^2 = 2\times(241\times231 + 107)$

$d^2 = 2\times[(236 + 5)\times(236 - 5) + 107]$

$d^2 = 2\times(236^2 - 5^2 + 107)$

$d^2 = 2\times(236^2 + 82)$

$d^2 = 4\times(118\times236 + 41)$

$d^2 = 4\times(8\times59^2 + 41)$

$d^2 = 4\times[(3\times59)^2 - 59^2 + 41]$

$d^2 = 4\times[(177)^2 - (60 - 1)^2 + 41]$

$d^2 = 4\times(177^2 - 3600 + 120 - 1 + 41)$

$d^2 = 4\times(177^2 - 3440) = 4\times(177^2 - 3540 + 100)$

$d^2 = 4\times(177^2 - 2\times177\times10 + 10^2)$

$d^2 = 2^2\times(177 - 10)^2$

$d^2 = (2\times167)^2$

$d = 334$

Method 3

Observe the number patterns:

$34^2 = 1156$

$334^2 = 111556$

$3334^2 = 11115556$

....

$\therefore d = 334$

Also, $33^2 = 1089$

$333^2 = 110889$

$3333^2 = 11108889$

....

Method 2: $d^2 = 241\times462 + 214 = 111556$

$300^2 = 90000 < 111556 < 160000 = 400^2 \Rightarrow 300 < d < 400$

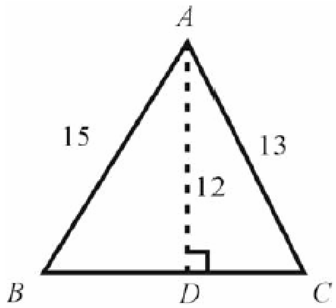
$330^2 = 108900 < 111556 < 115600 = 340^2 \Rightarrow 330 < d < 340$

The unit digit of d^2 is 6 $\Rightarrow$ the unit digit of d is 4 or 6

$335^2 = 112225 \Rightarrow d = 334$

Individual Event (Spare)

IS.1 In figure 1, ΔABC is an acute triangle, $AB = 15$, $AC = 13$, and its altitude $AD = 12$. If the area of the ΔABC is P, find the value of P.



$BD = \sqrt{15^2 - 12^2} = 9$

$CD = \sqrt{13^2 - 12^2} = 5$

P = area of $\Delta = \frac{1}{2}(9+5)\times 12 = 84$

IS.2 Given that x and y are positive integers. If $x^4 + y^4$ is divided by $x + y$, the quotient is $P + 13$ and the remainder is Q, find the value of Q.

$x^4 + y^4 = 97(x + y) + Q$, $0 \le Q < x + y$

Without loss of generality, assume $x \ge y$,

$x^4 \le x^4 + y^4 = 97(x + y) + Q < 98(x + y) \le 98(2x) = 196x$

$x^3 < 196$

$x \le 5$

On the other hand, $x^4 + y^4 = 97(x + y) + Q = x^3(x + y) - y(x^3 - y^3)$

$\Rightarrow (x^3 - 97)(x + y) = y(x^3 - y^3) + Q$

RHS $\ge 0 \Rightarrow$ LHS $\ge 0 \Rightarrow x^3 \ge 97$

$x \ge 5$

$\therefore x = 5$

Next, $x^4 + y^4 = 97(x + y) + Q = y^3(x + y) + x(x^3 - y^3)$

$\Rightarrow (97 - y^3)(x + y) = x(x^3 - y^3) - Q$

$\Rightarrow (97 - y^3)(x + y) > x(x^3 - y^3) - (x + y)$

$\Rightarrow (98 - y^3)(x + y) > 0$

$\Rightarrow 98 > y^3$

$\Rightarrow 4 \ge y$ .... (1)

$0 \le Q \Rightarrow 0 \le x^4 + y^4 - 97(x + y)$

$\Rightarrow 97(5 + y) \le 625 + y^4$

$\Rightarrow 97y \le 140 + y^4$ .... (2)

By trial and error, $y = 1$ and $y = 4$ satisfies (2)

So there are two possible pairs $(x, y) = (5, 1)$ or $(5,4)$.

$5^4 + 1^4 = 626 = (5 + 1)\times 104 + 2$, the quotient is not 97.

$5^4 + 4^4 = 881 = (5 + 4)\times 97 + 8$, $Q = 8$

IS.3 Given that the perimeter of an equilateral triangle equals to that of a circle with radius $\frac{12}{Q}$ cm. If the area of the triangle is $R\pi^2$ cm^2, find the value of R.

Radius of circle $= \frac{12}{8}$ cm = 1.5 cm $\Rightarrow$ circumference $= 2\times\pi\times 1.5$ cm $= 3\pi$ cm

Side of the equilateral triangle = π cm

Area of the triangle $= \frac{1}{2}\pi^2 \sin 60^\circ \text{ cm}^2 = \frac{\sqrt{3}}{4}\pi^2 \text{ cm}^2 \Rightarrow R = \frac{\sqrt{3}}{4}$

IS.4 Let $W = \frac{\sqrt{3}}{2R}$, $S = W + \cfrac{1}{W + \cfrac{1}{W + \cfrac{1}{W + \cdots}}}$, find the value of S.

$W = 2$, $S = 2 + \frac{1}{S} \Rightarrow S^2 - 2S - 1 = 0$, $S = \frac{1 \pm \sqrt{2}}{2}$, $S > 0$, $\therefore S = 1 + \sqrt{2}$ only

Group Event 1

G1.1 Given that a is an integer. If 50! is divisible by 2^a, find the largest possible value of a.

2, 4, 6, 8, …, 50 are divisible by 2, there are 25 even integers.

4, 8, …, 48 are divisible by 4, there are 12 multiples of 4.

8, …, 48 are divisible by 8, there are 6 multiples of 8.

16, …, 48 are divisible by 16, there are 3 multiples of 16.

32 is the only multiple of 32.

$a = 25 + 12 + 6 + 3 + 1 = 47$

G1.2 Let $[x]$ be the largest integer not greater than x. For example, $[2.5] = 2$. If $b = \left[100\times\frac{11\times 77+12\times 78+13\times 79+14\times 80}{11\times 76+12\times 77+13\times 78+14\times 79}\right]$, find the value of b.

$$100\times\frac{11\times 77+12\times 78+13\times 79+14\times 80}{11\times 76+12\times 77+13\times 78+14\times 79}$$

$$=100\times\frac{11\times 76+12\times 77+13\times 78+14\times 79+11+12+13+14}{11\times 76+12\times 77+13\times 78+14\times 79}$$

$$=100\times\left(1+\frac{11+12+13+14}{11\times 76+12\times 77+13\times 78+14\times 79}\right)$$

$$= 100 + \frac{11\times 100+12\times 100+13\times 100+14\times 100}{11\times 76+12\times 77+13\times 78+14\times 79}$$

$11\times76+12\times77+13\times78+14\times79 < 11\times100+12\times100+13\times100+14\times100 < 2(11\times76+12\times77+13\times78+14\times79)$

$$1<\frac{11\times 77+12\times 78+13\times 79+14\times 80}{11\times 76+12\times 77+13\times 78+14\times 79}<2$$

$$101<100\times\frac{11\times 77+12\times 78+13\times 79+14\times 80}{11\times 76+12\times 77+13\times 78+14\times 79}<102,\ b = 101$$

G1.3 If there are c multiples of 7 between 200 and 500, find the value of c.

$\frac{200}{7} = 28.6$, the least multiple of 7 is $7\times29 = 203$

$\frac{500}{7} = 71.4$, the greatest multiple of 7 is $7\times71 = 497$

$497 = a + (c - 1)d = 203 + (n - 1)\cdot 7$; $c = 43$

G1.4 Given that $0 \le x_0 \le \frac{\pi}{2}$ and x_0 satisfies the equation $\sqrt{\sin x+1}-\sqrt{1-\sin x}=\sin\frac{x}{2}$. If $d = \tan x_0$, find the value of d.

$$\left(\sqrt{\sin x+1}-\sqrt{1-\sin x}\right)^2=\left(\sin\frac{x}{2}\right)^2$$

$$1 + \sin x + 1 - \sin x - 2\sqrt{1-\sin^2 x}=\frac{1-\cos x}{2}$$

$2(2 - 2\cos x) = 1 - \cos x$

$\cos x = 1$

$x_0 = 0$

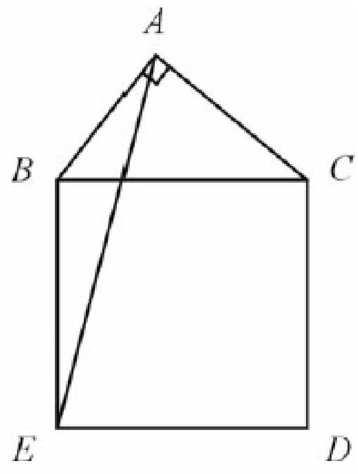
$d = \tan x_0 = 0$

Group Event 2

G2.1 If the tenth-place digit of 5^{5^5} is *a*, find the value of *a*.

5^{5^5} = …125, the tenth-place digit = *a* = 2

G2.2 In Figure 1, ΔABC is a right-angled triangle, $AB = 3$ cm, $AC = 4$ cm and $BC = 5$ cm. If *BCDE* is a square and the area of ΔABE is *b* cm^2, find the value of *b*.



$\cos B = \frac{3}{5}$

Height of ΔABE from $A = AB \cos B = 3\times\frac{3}{5}=\frac{9}{5}$

$b = \frac{1}{2}\cdot 5\times\frac{9}{5}=\frac{9}{2}$

G2.3 Given that there are *c* prime numbers less than 100 such that their unit digits are not square numbers, find the values of *c*.

The prime are: {2, 3, 5, 7, 13, 17, 23, 37, 43, 47, 53, 67, 73, 83, 97}

$c = 15$

G2.4 If the lines $y = x + d$ and $x = -y + d$ intersect at the point $(d - 1, d)$, find the value of *d*.

$x = -(x + d) + d$

$x = 0 = d - 1$

$d = 1$

Group Event 3

G3.1 If a is the smallest real root of the equation $\sqrt{x(x+1)(x+2)(x+3)+1}=71$, find the value of a.

Let $t = x + 1.5$, then the equation becomes $\sqrt{(t-1.5)(t-0.5)(t+0.5)(t+1.5)+1}=71$

$$\sqrt{\left(t^2-\frac{9}{4}\right)\left(t^2-\frac{1}{4}\right)+1}=71$$

$$\sqrt{t^4-\frac{5}{2}t^2+\frac{9}{16}+1}=71\Rightarrow\sqrt{t^4-\frac{5}{2}t^2+\frac{25}{16}}=71\Rightarrow\sqrt{\left(t^2-\frac{5}{4}\right)^2}=71$$

$$t^2-\frac{5}{4}=71\Rightarrow t^2=\frac{289}{4}\Rightarrow t=\frac{17}{2}\quad\text{or}\quad t=-\frac{17}{2}$$

$x = t - 1.5 = \pm\frac{17}{2}-\frac{3}{2}$ = 7 or –10, a = the smallest root = –10

G3.2 Given that p and q are prime numbers satisfying the equation $18p + 30q = 186$. If $\log_8\frac{p}{3q+1} = b \geq 0$, find the value of b.

$18p + 30q = 186 \Rightarrow 3p + 5q = 31$

Note that the number "2" is the only prime number which is even.

$3p + 5q = 31$ = odd number $\Rightarrow$ either $p = 2$ or $q = 2$

If $p = 2$, then $q = 5$; $b=\log_8\frac{p}{3q+1}=\log_8\frac{2}{3\times5+1}=\log_8\frac{1}{16}<0$ (rejected)

If $q = 2$, then $p = 7$; $b=\log_8\frac{p}{3q+1}=\log_8\frac{7}{3\times2+1}=0$

G3.3 Given that for any real numbers x, y and z, $\oplus$ is an operation satisfying

(i) $x \oplus 0 = 1$, and

(ii) $(x \oplus y) \oplus z = (z \oplus xy) + z$.

If $1\oplus2004 = c$, find the value of c.

$c = 1 \oplus 2004 = (1\oplus0) \oplus2004 = (2004\oplus0)+ 2004 = 1 + 2004 = 2005$

G3.4 Given that $f(x) = (x^4 + 2x^3 + 4x - 5)^{2004} + 2004$. If $f\left(\sqrt{3}-1\right) = d$, find the value of d.

$x=\sqrt{3}-1$

$(x + 1)^2 = \left(\sqrt{3}\right)^2$

$x^2 + 2x - 2 = 0$

By division, $x^4 + 2x^3 + 4x - 5 = (x^2 + 2x - 2)(x^2 + 2) - 1 = -1$

$d = f\left(\sqrt{3}-1\right) = f(x) = (x^4 + 2x^3 + 4x - 5)^{2004} + 2004 = (-1)^{2004} + 2004 = 2005$

Group Event 4

G4.1 If $f(x)=\dfrac{4^x}{4^x+2}$ 及 $P=f\left(\dfrac{1}{1001}\right)+f\left(\dfrac{2}{1001}\right)+\cdots+f\left(\dfrac{1000}{1001}\right)$, find the value of P.

$$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}=\frac{4+4+2\cdot 4^x+2\cdot 4^{1-x}}{4+4+2\cdot 4^x+2\cdot 4^{1-x}}=1$$

$$P=f\left(\frac{1}{1001}\right)+f\left(\frac{2}{1001}\right)+\cdots+f\left(\frac{1000}{1001}\right)$$

$$=f\left(\frac{1}{1001}\right)+f\left(\frac{1000}{1001}\right)+f\left(\frac{2}{1001}\right)+f\left(\frac{999}{1001}\right)+\cdots+f\left(\frac{500}{1001}\right)+f\left(\frac{501}{1001}\right)=500$$

G4.2 Let $f(x)=|x-a|+|x-15|+|x-a-15|$, where $a\le x\le 15$ and $0<a<15$. If Q is the smallest value of $f(x)$, find the value of Q.

$f(x)=x-a+15-x+15-x+a=30-x\ge 30-15=15=Q$

G4.3 If $2^m=3^n=36$ and $R=\dfrac{1}{m}+\dfrac{1}{n}$, find the value of R.

$\log 2^m=\log 3^n=\log 36$

$m\log 2=n\log 3=\log 36$

$m=\dfrac{\log 36}{\log 2}$; $n=\dfrac{\log 36}{\log 3}$

$$\frac{1}{m}+\frac{1}{n}=\frac{\log 2}{\log 36}+\frac{\log 3}{\log 36}=\frac{\log 6}{\log 36}=\frac{\log 6}{2\log 6}=\frac{1}{2}$$

G4.4 Let $[x]$ be the largest integer not greater than x, for example, $[2.5]=2$. If $a=1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\cdots+\dfrac{1}{2004^2}$ and $S=[a]$, find the value of a.

$$1<a=1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots+\frac{1}{2004^2}<1+\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{2003\times 2004}$$

$$=1+1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{2003}-\frac{1}{2004}$$

$$=2-\frac{1}{2004}<2$$

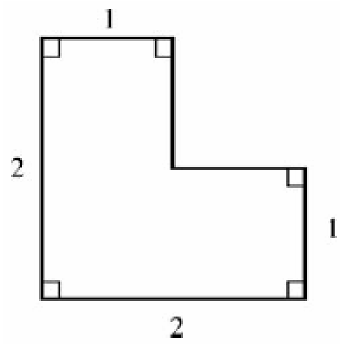
$S=[a]=1$

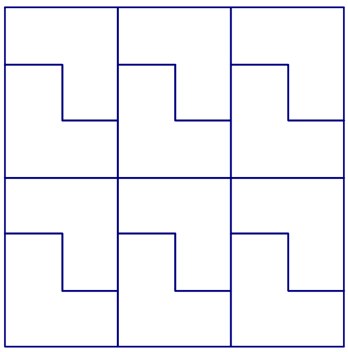
Group Event (Spare)

GS.1 For all integers n, F_n is defined by $F_n = F_{n-1} + F_{n-2}$, $F_0 = 0$ and $F_1 = 1$.
If $a = F_{-5} + F_{-4} + \dots + F_4 + F_5$, find the value of a.
$F_2 = 0 + 1 = 1$, $F_3 = 1 + 1 = 2$, $F_4 = 1 + 2 = 3$, $F_5 = 2 + 3 = 5$
$F_{-1} + 0 = 1 \Rightarrow F_{-1} = 1$, $F_{-2} + 1 = 0 \Rightarrow F_{-2} = -1$, $F_{-3} + (-1) = 1 \Rightarrow F_{-3} = 2$,
$F_{-4} + 2 = -1 \Rightarrow F_{-4} = -3$, $F_{-5} + (-3) = 2 \Rightarrow F_{-5} = 5$
$a = F_{-5} + F_{-4} + \dots + F_4 + F_5 = 5 - 3 + 2 - 1 + 1 + 0 + 1 + 1 + 2 + 3 + 5 = 16$

GS.2 Given that x_0 satisfies the equation $x^2 + x + 2 = 0$. If $b = x_0^4 + 2x_0^3 + 3x_0^2 + 2x_0 + 1$, find the value of b.
By division, $b = x_0^4 + 2x_0^3 + 3x_0^2 + 2x_0 + 1$
$= (x_0^2 + x_0 + 2)(x_0^2 + x_0) + 1 = 1$

GS.3 Figure 1 shows a tile. If C is the minimum number of tiles required to tile a square, find the value of C.





$C = 12$

GS.4 If the line $5x + 2y - 100 = 0$ has d points whose x and y coordinates are both positive integers, find the value of d.
$(x, y) = (18, 5), (16, 10), (14, 15), (12, 20), (10, 25), (8, 30), (6, 35), (4, 40), (2, 45)$
$d = 9$