

Individual Events

SI	a	2	I1	p	4	I2	a	8	I3	a	6	I4	a	23	I5	a	2
	b	770		Q	8		b	10		b	7		b	2		b	1
	c	57		R	11		c	1		c	2		c	2		c	0
	d	58		S	10		d	2000		d	9902		d	8		d	6

Group Events

SG	a	180	G6	a	1	G7	a	-1	G8	a	2	G9	a	4	G10	p	35	Spare	P	4
	b	7		b	15		b	0		b	7		b	0		Q	6		Q	6
	c	9		c	80		c	13		c	0		c	3		R	11		R	35
	d	4		d	1		d	5		d	6		d	3		s	150		S	8

Sample Individual

SI.1 If a is the smallest prime number which can divide the sum $3^{11} + 5^{13}$, find the value of a .
 3^{11} is an odd number, 5^{13} is also an odd number, so $3^{11} + 5^{13}$ is an even number, which is divisible by 2.

SI.2 For all real number x and y , $x \oplus y$ is defined as: $x \oplus y = \frac{1}{xy}$.

If $b = 4 \oplus (a \oplus 1540)$, find the value of b .

$$a \oplus 1540 = \frac{1}{2 \times 1540} = \frac{1}{3080}; b = 4 \oplus (a \oplus 1540) = \frac{3080}{4} = 770$$

SI.3 W and F are two integers which are greater than 20. If the product of W and F is b and the sum of W and F is c , find the value of c .

$$\begin{cases} WF = 770 \dots\dots(1) \\ W + F = c \dots\dots(2) \end{cases} \quad 770 = 22 \times 35; W = 22, F = 35, c = 22 + 35 = 57$$

SI.4 If $\frac{d}{114} = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{c^2}\right)$, find the value of d .

$$\begin{aligned} \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{57^2}\right) &= \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) \dots \left(1 - \frac{1}{57}\right)\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right) \dots \left(1 + \frac{1}{57}\right) \\ &= \frac{1}{2} \cdot \frac{2}{3} \dots \frac{56}{57} \times \frac{3}{2} \cdot \frac{4}{3} \dots \frac{58}{57} \\ &= \frac{1}{57} \times \frac{58}{2} = \frac{58}{114}; d = 58 \end{aligned}$$

Individual Event 1

II.1 If the interior angles of a p -sided polygon form an Arithmetic Progression and the smallest and the largest angles are 20° and 160° respectively. Find the value of p .

$$\text{Sum of all interior angles} = \frac{p}{2} (20^\circ + 160^\circ) = 180^\circ(p - 2)$$

$$90p = 180p - 360 \Rightarrow p = 4$$

II.2 In $\triangle ABC$, $AB = 5$, $AC = 6$ and $BC = p$. If $\frac{1}{Q} = \cos 2A$, find the value of Q .

(Hint: $\cos 2A = 2 \cos^2 A - 1$)

$$\cos A = \frac{6^2 + 5^2 - 4^2}{2 \times 6 \times 5} = \frac{3}{4}; \cos 2A = 2 \cos^2 A - 1 = 2 \times \left(\frac{3}{4}\right)^2 - 1 = \frac{1}{8}; Q = 8$$

11.3 If $\log_2 Q + \log_4 Q + \log_8 Q = \frac{R}{2}$, find the value of R .

$$\frac{R}{2} = \log_2 8 + \log_4 8 + \log_8 8 = 3 + \frac{3}{2} + 1 = \frac{11}{2}; R = 11$$

11.4 If the product of the numbers R and $\frac{11}{S}$ is the same as their sum, find the value of S .

$$11 \times \frac{11}{S} = 11 + \frac{11}{S} \Rightarrow \frac{110}{S} = 11, S = 10$$

Individual Event 2

12.1 If x, y and z are positive real numbers such that $\frac{x+y-z}{z} = \frac{x-y+z}{y} = \frac{-x+y+z}{x}$ and

$$a = \frac{(x+y) \cdot (y+z) \cdot (z+x)}{xyz}, \text{ find the value of } a.$$

Let $\frac{x+y-z}{z} = k, \frac{x-y+z}{y} = k, \frac{-x+y+z}{x} = k.$

$$\begin{cases} x+y-z = kz \dots\dots(1) \\ x-y+z = ky \dots\dots(2); (1) + (2) + (3): x+y+z = k(x+y+z) \Rightarrow k = 1 \\ -x+y+z = kx \dots\dots(3) \end{cases}$$

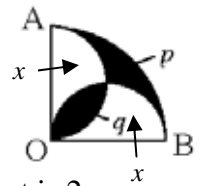
From (1), $x+y = 2z$, (2): $x+z = 2y$, (3): $y+z = 2x$; $\therefore a = \frac{(x+y) \cdot (y+z) \cdot (z+x)}{xyz} = \frac{8xyz}{xyz} = 8$

12.2 Let u and t be positive integers such that $u + t + ut = 4a + 2$. If $b = u + t$, find the value of b .

$$u + t + ut = 34 \Rightarrow 1 + u + t + ut = 35 \Rightarrow (1 + u)(1 + t) = 35 \Rightarrow 1 + u = 5, 1 + t = 7$$

$$u = 4, t = 6 \Rightarrow b = 4 + 6 = 10$$

12.3 In Figure 1, OAB is a quadrant of a circle and semi-circles are drawn on OA and OB . If p, q denotes the areas of the shaded regions, where $p = (b - 9) \text{ cm}^2$ and $q = c \text{ cm}^2$, find the value of c .
 $p = 1$, let the area of each of two unshaded regions be $x \text{ cm}^2$



Let the radius of each of the smaller semicircles be r . The radius of the quadrant is $2r$.

$$x + q = \text{area of one semi-circle} = \frac{\pi r^2}{2}; 2x + p + q = \text{area of the quadrant} = \frac{1}{4} \pi (2r)^2 = \pi r^2$$

$$2 \times (1) = (2), 2x + 2q = 2x + p + q \Rightarrow q = p; c = 1$$

12.4 Let $f_0(x) = \frac{1}{c-x}$ and $f_n(x) = f_0(f_{n-1}(x)), n = 1, 2, 3, \dots$. If $f_{2000}(2000) = d$, find the value of d .

$$f_0(x) = \frac{1}{1-x}, f_1(x) = f_0\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$f_2(x) = f_0\left(1 - \frac{1}{x}\right) = \frac{1}{1-\left(1-\frac{1}{x}\right)} = x, \text{ which is an identity function.}$$

So $f_5(x) = f_2(x) = x, \dots, f_{2000}(x) = x; f_{2000}(2000) = 2000 = d$

Individual Event 3

13.1 For all integers m and n , $m \otimes n$ is defined as: $m \otimes n = m^n + n^m$. If $2 \otimes a = 100$, find the value of a .

$$2^a + a^2 = 100 ; 64 + 36 = 2^6 + 6^2 = 100, a = 6$$

13.2 If $\sqrt[3]{13b+6a+1} - \sqrt[3]{13b-6a-1} = \sqrt[3]{2}$, where $b > 0$, find the value of b .

$$\left(\sqrt[3]{13b+37} - \sqrt[3]{13b-37}\right)^3 = 2$$

$$13b + 37 - 3\sqrt[3]{(13b+37)^2\sqrt[3]{13b-37}} + 3\sqrt[3]{(13b-37)^2\sqrt[3]{13b+37}} - (13b - 37) = 2$$

$$24 = \sqrt[3]{(13b)^2 - 37^2}\sqrt[3]{13b+37} - \sqrt[3]{(13b)^2 - 37^2}\sqrt[3]{13b-37}$$

$$24 = \sqrt[3]{(13b)^2 - 37^2} \sqrt[3]{2}; \quad (\because \sqrt[3]{13b+37} - \sqrt[3]{13b-37} = \sqrt[3]{2})$$

$$13824 = [(13b)^2 - 1369] \times 2$$

$$6912 + 1369 = 169 b^2$$

$$b^2 = 49 \Rightarrow b = 7$$

13.3 In figure 2, $AB = AC$ and $KL = LM$. If $LC = b - 6$ cm and $KB = c$ cm, find the value of c .

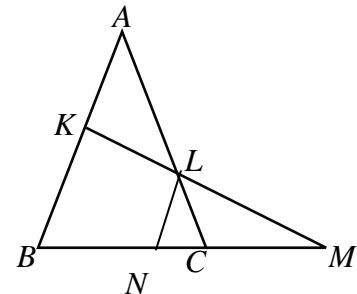
Draw $LN \parallel AB$ on BM .

$BN = NM$ intercept theorem

$\angle LNC = \angle ABC = \angle LCN$ (corr. \angle s, $AB \parallel LN$, base \angle s, isos. Δ)

$LN = LC = b - 6$ cm = 1 cm (sides opp. eq. \angle s)

c cm = $KB = 2 LN = 2$ cm (mid point theorem)



13.4 The sequence $\{a_n\}$ is defined as $a_1 = c$, $a_{n+1} = a_n + 2n$ ($n \geq 1$). If $a_{100} = d$, find the value of d .

$$a_1 = 2, a_2 = 2 + 2, a_3 = 2 + 2 + 4, \dots, a_{100} = 2 + 2 + 4 + \dots + 198 = 2 + \frac{1}{2}(2 + 198) \cdot 99 = 9902 = d$$

Individual Event 4

14.1 Mr. Lee is a years old, $a < 100$. If the product of a and his month of birth is 253, find the value of a .

$$253 = 11 \times 23; 11 = \text{his month of birth and } a = 23$$

14.2 Mr. Lee has $a + b$ sweets. If he divides them equally among 10 persons, 5 sweets will be remained. If he divides them equally among 7 persons, 3 more sweets are needed. Find the minimum value of b .

$$10m + 5 = 7n - 3 = 23 + b$$

$$7n - 10m = 8; n = 4, m = 2, 23 + b = 7 \times 4 - 3 = 25, b = 2$$

14.3 Let c be a positive real number. If $x^2 + 2\sqrt{c}x + b = 0$ has one real root only, find the value of c

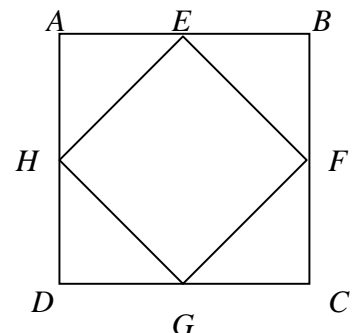
$$x^2 + 2\sqrt{c}x + 2 = 0, \Delta = 4(c - 2) = 0 \Rightarrow c = 2$$

14.4 In figure 3, the area of the square $ABCD$ is equal to d . If E, F, G, H are the mid-points of AB, BC, CD and DA respectively and $EF = c$, find the value of d .

$$\text{Area of } EFGH = c^2 = 2^2 = 4$$

$$\text{Area of } ABCD = 2 \times \text{area of } EFGH = 8$$

$$d = 8$$



Individual Event 5

I5.1 If $144^p = 10$, $1728^q = 5$ and $a = 12^{2p-3q}$, find the value of a .

$$a = 12^{2p-3q} = 144^p \div 1728^q = 10 \div 5 = 2$$

I5.2 If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$, $b = \frac{a}{x}$, find b .

$$\left(1 - \frac{2}{x}\right)^2 = 0; x = 2, b = \frac{2}{2} = 1$$

I5.3 If the number of real roots of the equation $x^2 - bx + 1 = 0$ is c , find the value of c .

$$x^2 - bx + 1 = 0; \Delta = 1^2 - 4 < 0, c = \text{number of real roots} = 0$$

I5.4 Let $f(1) = c + 1$ and $f(n) = (n - 1)f(n - 1)$, where $n > 1$. If $d = f(4)$, find d .

$$f(1) = 1, f(2) = f(1) = 1, f(3) = 2f(2) = 2, f(4) = 3f(3) = 3 \times 2 = 6$$

Sample Group

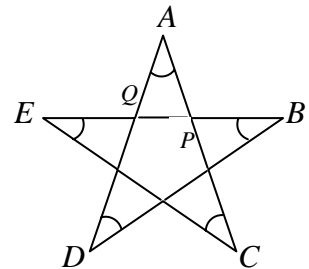
SG.1 In the given diagram, $\angle A + \angle B + \angle C + \angle D + \angle E = a^\circ$, find a .

In $\triangle APQ$, $\angle B + \angle D = \angle AQP \dots\dots(1)$ (ext. \angle of \triangle)

$\angle C + \angle E = \angle APQ \dots\dots(2)$ (ext. \angle of \triangle)

$$\begin{aligned} \angle A + \angle B + \angle C + \angle D + \angle E &= \angle A + \angle AQP + \angle APQ \text{ (by (1) and (2))} \\ &= 180^\circ \text{ (}\angle\text{s sum of } \triangle\text{)} \end{aligned}$$

$$\therefore a = 180$$



SG.2 If the sum of x terms $x^6 + x^6 + \dots + x^6 = x^b$, find b .

$$x \cdot x^6 = x^b \Rightarrow b = 7$$

SG.3 If $1 + 3 + 3^2 + 3^3 + \dots + 3^8 = \frac{3^c - 1}{2}$, find the value of c .

$$\text{By using the formula for the sum of } n \text{ terms of a geometric series: } \frac{3^9 - 1}{2} = \frac{3^c - 1}{2}; c = 9$$

SG.4 16 cards are marked from 1 to 16 and one is drawn at random. If the chance of it being a perfect square number is $\frac{1}{d}$, find d .

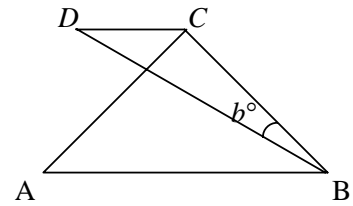
$$\text{Perfect square numbers are 1, 4, 9, 16, so the probability} = \frac{4}{16} = \frac{1}{4} = \frac{1}{d} \Rightarrow d = 4$$

Group Event 6

G6.1 Let $x * y = x + y - xy$, where x, y are real numbers. If $a = 1 * (0 * 1)$, find the value of a .

$$0 * 1 = 0 + 1 - 0 = 1; a = 1 * (0 * 1) = 1 * 1 = 1 + 1 - 1 = 1$$

G6.2 In figure 1, AB is parallel to DC , $\angle ACB$ is a right angle, $AC = CB$ and $AB = BD$. If $\angle CBD = b^\circ$, find the value of b .



$\triangle ABC$ is a right angled isosceles triangle.

$$\angle BAC = 45^\circ \text{ (}\angle\text{s sum of } \triangle, \text{ base } \angle\text{s isos. } \triangle\text{)}$$

$$\angle ACD = 45^\circ \text{ (alt. } \angle\text{s, } AB \parallel DC\text{)}$$

$$\angle BCD = 135^\circ$$

Apply sine law on $\triangle BCD$,

$$\frac{BD}{\sin 135^\circ} = \frac{BC}{\sin D}$$

$$AB\sqrt{2} = \frac{AB \sin 45^\circ}{\sin D}, \text{ given that } AB = BD$$

$$\sin D = \frac{1}{2}; D = 30^\circ$$

$$\angle CBD = 180^\circ - 135^\circ - 30^\circ = 15^\circ \text{ (}\angle\text{s sum of } \triangle BCD\text{), } b = 15$$

G6.3 Let x, y be non-zero real numbers. If x is 250% of y and $2y$ is $c\%$ of x , find the value of c .

$$x = 2.5y \dots\dots\dots(1)$$

$$2y = \frac{c}{100} \cdot x \dots\dots\dots(2)$$

$$\text{sub. (1) into (2): } 2y = \frac{c}{100} \cdot 2.5y$$

$$c = 80$$

G6.4 If $\log_p x = 2, \log_q x = 3, \log_r x = 6$ and $\log_{pqr} x = d$, find the value of d .

$$\frac{\log x}{\log p} = 2; \frac{\log x}{\log q} = 3; \frac{\log x}{\log r} = 6$$

$$\frac{\log p}{\log x} = \frac{1}{2}; \frac{\log q}{\log x} = \frac{1}{3}; \frac{\log r}{\log x} = \frac{1}{6}$$

$$\frac{\log p}{\log x} + \frac{\log q}{\log x} + \frac{\log r}{\log x} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$$

$$\frac{\log pqr}{\log x} = 1$$

$$\frac{\log x}{\log pqr} = 1 \Rightarrow d = \log_{pqr} x = 1$$

Group Event 7

G7.1 If $a = x^4 + x^{-4}$ and $x^2 + x + 1 = 0$, find the value of a .

$$\frac{x^2 + x + 1}{x} = 0 \Rightarrow x + \frac{1}{x} = -1 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 1 \Rightarrow x^2 + \frac{1}{x^2} = -1 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 1; a = x^4 + \frac{1}{x^4} = -1$$

G7.2 If $6^b + 6^{b+1} = 2^b + 2^{b+1} + 2^{b+2}$, find the value of b .

$$6^b \cdot (1 + 6) = 2^b \cdot (1 + 2 + 4) \Rightarrow b = 0$$

G7.3 Let c be a prime number. If $11c + 1$ is the square of a positive integer, find the value of c .

$$11c + 1 = m^2 \Rightarrow m^2 - 1 = 11c \Rightarrow (m + 1)(m - 1) = 11c \Rightarrow m - 1 = 11 \text{ and } m + 1 = c; m = 13$$

G7.4 Let d be an odd prime number. If $89 - (d + 3)^2$ is the square of an integer, find the value of d .

$\therefore d$ is odd, $d + 3$ must be even. $89 - (d + 3)^2$ must be odd.

$$\text{By trial and error } m = 5, 89 = 8^2 + 5^2 \Rightarrow d + 3 = 8 \Rightarrow d = 5$$

Group Event 8

G8.1 Let a be the number of positive integers less than 100 such that they are both square and cubic numbers, find the value of a .

The positive integers less than 100 such that they are both square and cubic numbers are: 1 and $2^6 = 64$ only, so there are only 2 numbers satisfying the condition.

G8.2 The sequence $\{a_k\}$ is defined as: $a_1 = 1, a_2 = 1$ and $a_k = a_{k-1} + a_{k-2} (k > 2)$

If $a_1 + a_2 + \dots + a_{10} = 11 a_b$, find the value of b .

$$a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 3, a_5 = 5, a_6 = 8, a_7 = 13, a_8 = 21, a_9 = 34, a_{10} = 55$$

$$a_1 + a_2 + \dots + a_{10} = 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 + 55 = 143 = 11 \times 13 = 11a_7; b = 7$$

G8.3 If c is the maximum value of $\log(\sin x)$, where $0 < x < \pi$, find the value of c .

$$0 < \sin x \leq 1; \log(\sin x) \leq \log 1 = 0 \Rightarrow c = 0$$

G8.4 Given that $x + y = 18$. If the maximum value of $\sqrt{x} + \sqrt{y}$ is d , find the value of d .

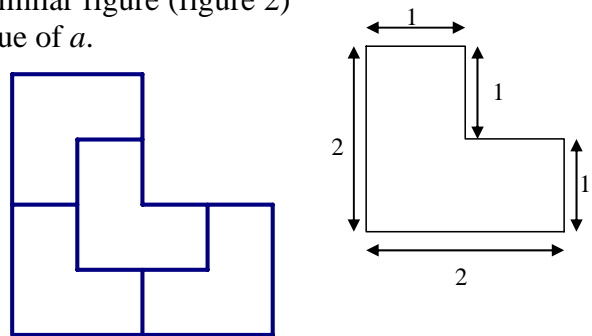
$$x + y = (\sqrt{x} + \sqrt{y})^2 - 2\sqrt{xy} \Rightarrow (\sqrt{x} + \sqrt{y})^2 = 18 + 2\sqrt{xy} \leq 18 + 2\left(\frac{x+y}{2}\right) = 36 \quad (\text{GM} \leq \text{AM})$$

$$\sqrt{x} + \sqrt{y} \leq 6 = d$$

Group Event 9

G9.1 If a tiles of L-shape are used to form a larger similar figure (figure 2) without overlapping, find the least possible value of a .

From the figure, $a = 4$.



G9.2 Let α, β be the roots of $x^2 + bx - 2 = 0$. If $\alpha > 1$ and $\beta < -1$, and b is an integer, find the value of b .

$$\alpha - 1 > 0 \text{ and } \beta + 1 < 0 \Rightarrow (\alpha - 1)(\beta + 1) < 0 \Rightarrow \alpha\beta + \alpha - \beta - 1 < 0 \Rightarrow \alpha - \beta < 3$$

$$\alpha + \beta = -b < 2\beta + 3 < 1 \Rightarrow b > -1,$$

$$\alpha + \beta = -b > 2\alpha - 3 > -1 \Rightarrow b < 1$$

Combined solution $-1 < b < 1; \because b$ is an integer, $\therefore b = 0$

G9.3 Given that m, c are positive integers less than 10. If $m = 2c$ and $0.\dot{m}\dot{c} = \frac{c+4}{m+5}$, find the value of c .

$$0.\dot{m}\dot{c} = \frac{10m+c}{99} = \frac{c+4}{m+5} \Rightarrow \frac{20c+c}{99} = \frac{c+4}{2c+5} \Rightarrow \frac{7c}{33} = \frac{c+4}{2c+5} \Rightarrow 14c^2 + 35c = 33c + 132$$

$$14c^2 + 2c - 132 = 0 \Rightarrow 7c^2 + c - 66 = 0 \Rightarrow (7c + 22)(c - 3) = 0 \Rightarrow c = 3$$

G9.4 A bag contains d balls of which x are black, $x + 1$ are red and $x + 2$ are white. If the probability of drawing a black ball randomly from the bag is less than $\frac{1}{6}$, find the value of d .

$$\frac{x}{3x+3} < \frac{1}{6} \Rightarrow \frac{x}{x+1} < \frac{1}{2} \Rightarrow 2x < x+1 \Rightarrow x < 1 \Rightarrow x = 0 \Rightarrow d = 3x+3 = 3$$

Group Event 10

G10.1 If the roots of $x^2 - 2x - p = 0$ differ by 12, find the value of p .

$$\alpha + \beta = 2, \alpha \beta = -p$$

$$\alpha - \beta = 12 \Rightarrow (\alpha - \beta)^2 = 144 \Rightarrow (\alpha + \beta)^2 - 4\alpha \beta = 144 \Rightarrow 4 + 4p = 144 \Rightarrow p = 35$$

G10.2 Given that the roots of $x^2 + ax + 2b = 0$ and $x^2 + 2bx + a = 0$ are both real and $a, b > 0$. If the minimum value of $a + b$ is Q , find the value of Q .

$$a^2 - 8b \geq 0 \text{ and } 4b^2 - 4a \geq 0$$

$$a^2 \geq 8b \text{ and } b^2 \geq a \Rightarrow a^4 \geq 64b^2 \geq 64a \Rightarrow a^4 - 64a \geq 0 \Rightarrow a(a^3 - 64) \geq 0 \Rightarrow a^3 \geq 64 \Rightarrow a \geq 4$$

$$\text{minimum } a = 4, b^2 \geq a \Rightarrow b^2 \geq 4 \Rightarrow \text{minimum } b = 2$$

$$Q = \text{minimum value of } a + b = 4 + 2 = 6$$

G10.3 If $R^{2000} < 5^{3000}$, where R is a positive integer, find the largest value of R .

$$(R^2)^{1000} < (5^3)^{1000} \Rightarrow R^2 < 5^3 = 125 \Rightarrow R < \sqrt{125} < 12, \text{ the largest integral value of } R = 11$$

G10.4 In figure 3, $\triangle ABC$ is a right-angled triangle and $BH \perp AC$.

If $AB = 15$, $HC = 16$ and the area of $\triangle ABC$ is s , find the value of s .

It is easy to show that $\triangle ABH \sim \triangle BCH \sim \triangle ACB$.

Let $\angle ABH = \theta = \angle BCH$

In $\triangle ABH$, $BH = 15 \cos \theta$

In $\triangle BCH$, $CH = BH \div \tan \theta \Rightarrow 16 \tan \theta = 15 \cos \theta$

$$16 \sin \theta = 15 \cos^2 \theta \Rightarrow 16 \sin \theta = 15 - 15 \sin^2 \theta$$

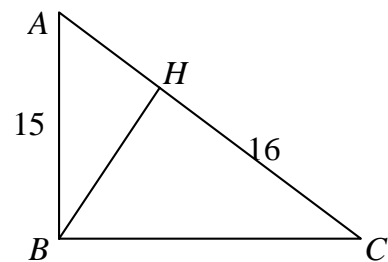
$$15 \sin^2 \theta + 16 \sin \theta - 15 = 0$$

$$(3 \sin \theta + 5)(5 \sin \theta - 3) = 0$$

$$\sin \theta = \frac{3}{5}; \tan \theta = \frac{3}{4}$$

$$BC = AB \div \tan \theta = 15 \times \frac{4}{3} = 20$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \cdot 15 \times 20 = 150 = s$$



Group Event (Spare)

GS.1 If a number N is chosen randomly from the set of positive integers, the probability of the unit digit of N^4 being unity is $\frac{P}{10}$, find the value of P .

If the unit digit of N^4 is 1, then the unit digit of N may be 1, 3, 7, 9. So the probability = $\frac{4}{10}$

$$P = 4$$

GS.2 Given that $x + y = 18$. If the maximum value of $\sqrt{x} + \sqrt{y}$ is d , find the value of d .

$$x + y = (\sqrt{x} + \sqrt{y})^2 - 2\sqrt{xy} \Rightarrow (\sqrt{x} + \sqrt{y})^2 = 18 + 2\sqrt{xy} \leq 18 + 2\left(\frac{x+y}{2}\right) = 36 \quad (\text{GM} \leq \text{AM})$$

$$\sqrt{x} + \sqrt{y} \leq 6 = d$$

GS.3 If the roots of $x^2 - 2x - R = 0$ differs by 12, find the value of R .

$$\alpha + \beta = 2, \alpha \beta = -R$$

$$\alpha - \beta = 12 \Rightarrow (\alpha - \beta)^2 = 144 \Rightarrow (\alpha + \beta)^2 - 4\alpha \beta = 144 \Rightarrow 4 + 4R = 144 \Rightarrow R = 35$$

GS.4 If the product of a 4-digit number $abSd$ and 9 is equal to another 4-digit number $dSba$, find the value of S .

$$a = 1, d = 9, \text{ Let the carry digit in the hundred digit be } x. \text{ Then } 9s + 8 = 10x + b \dots\dots(1)$$

$$9b + x = s \dots\dots(2); x = s - 9b \dots\dots(3)$$

$$\text{Sub. (3) into (1): } 9s + 8 = 10(s - 9b) + b \Rightarrow 8 = s - 89b \Rightarrow s = 8, b = 0$$