

Individual Events

SI	a	$\frac{2}{3}$	I1	a	*1 see the remark	I2	a	38	I3	a	10	I4	p	15	I5	a	4
	b	0		b	2		b	104		b	27		q	4		b	5
	c	3		c	4		c	100		c	23		r	57		c	24
	d	-6		d	24		d	-50		d	26		s	3		d	57

Group Events

SG	a	10	G1	p	4	G2	a	1110	G3	a	90	G4	a	0.13717421	G5	a	4290	GS	s	6
	b	73		q	3		b	1		b	1		b	90		b	18		b	10
	c	55		r	2		c	0		c	0		c	$\frac{665}{729}$		c	67		c	81
	d	16		a	9		d	6		d	1		d	50		d	30		d	50

Sample Individual Event

SI.1 Given that $\frac{3}{a} + \frac{1}{u} = \frac{7}{2}$ and $\frac{2}{a} - \frac{3}{u} = 6$ are simultaneous equations in a and u . Solve for a .

$$3(1) + (2): \frac{11}{a} = \frac{33}{2}; a = \frac{2}{3}$$

SI.2 Three solutions of the equation $px + qy + bz = 1$ are $(0, 3a, 1)$, $(9a, -1, 2)$ and $(0, 3a, 0)$. Find the value of the coefficient b .

$$\begin{cases} 3aq + b = 1 \\ 9ap - q + 2b = 1, \text{ sub. (3) into (1): } 1 + b = 1 \Rightarrow b = 0 \\ 3aq = 1 \end{cases}$$

SI.3 Find c so that the graph of $y = mx + c$ passes through the two points $(b + 4, 5)$ and $(-2, 2)$.

The 2 points are: $(4, 5)$ and $(-2, 2)$. The slope is $\frac{5-2}{4-(-2)} = \frac{1}{2}$.

The line $y = \frac{1}{2}x + c$ passes through $(-2, 2)$: $2 = -1 + c \Rightarrow c = 3$

SI.4 The solution of the inequality $x^2 + 5x - 2c \leq 0$ is $d \leq x \leq 1$. Find d .

$$x^2 + 5x - 6 \leq 0 \Rightarrow (x + 6)(x - 1) \leq 0$$

$$-6 \leq x \leq 1; d = -6$$

Individual Event 1

11.1 If a is the maximum value of $\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta$, find the value of a .

$$-1 \leq \sin 3\theta \leq 1 \text{ and } -1 \leq \cos 2\theta \leq 1$$

$$\frac{1}{2}\sin^2 3\theta \leq \frac{1}{2} \text{ and } -\frac{1}{2}\cos 2\theta \leq \frac{1}{2}$$

$$\frac{1}{2}\sin^2 3\theta - \frac{1}{2}\cos 2\theta \leq \frac{1}{2} + \frac{1}{2} = 1 = a,$$

Maximum occur when $\sin^2 3\theta = 1$ and $-\cos 2\theta = 1$

i.e. $3\theta = 90^\circ + 180^\circ n$ and $2\theta = 360^\circ m + 180^\circ$, where m, n are integers.

$$\theta = 30^\circ + 60^\circ n = 180^\circ m + 90^\circ \Rightarrow 60^\circ n = 180^\circ m + 60^\circ \Rightarrow n = 3m + 1; \text{ let } m = 1, n = 4, \theta = 270^\circ$$

*Remark: the original question is

If a is the maximum value of $\frac{1}{2}\sin^2 \theta + \frac{1}{2}\cos 3\theta$, find the value of a .

Maximum occur when $\sin^2 \theta = 1$ and $\cos 3\theta = 1$

i.e. $\theta = 90^\circ + 180^\circ n$ and $3\theta = 360^\circ m$, where m, n are integers.

$$\theta = 90^\circ + 180^\circ n = 120^\circ m \Rightarrow 3 + 6n = 4m, \text{ LHS is odd and RHS is even, contradiction.}$$

The question was wrong because we cannot find any θ to make the expression a maximum.

11.2 If $\begin{cases} x + y = 2 \\ xy - z^2 = a \\ b = x + y + z \end{cases}$, find the value of b .

$$(2), xy = 1 + z^2 > 0; \text{ together with (1) we have } x > 0 \text{ and } y > 0$$

$$\text{by AM} \geq \text{GM in (1) } x + y \geq 2\sqrt{xy} \Rightarrow 2 \geq 2\sqrt{1 + z^2}$$

$$\text{after simplification, } 0 \geq z^2 \Rightarrow z = 0$$

$$(3): b = x + y + z = 2 + 0 = 2$$

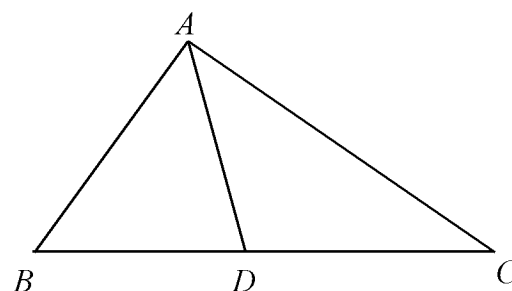
11.3 In the figure, $BD = b$ cm, $DC = c$ cm and area of

$$\Delta ABD = \frac{1}{3} \times \text{area of } \Delta ABC, \text{ find the value of } c.$$

Let the common height be h cm

$$\frac{1}{2}BD \times h = \frac{1}{3}BC \times h$$

$$2 = \frac{1}{3}(2 + c) \Rightarrow c = 4$$



11.4 Suppose d is the number of positive factors of $500 + c$, find the value of d .

$$500 + c = 504 = 2^3 \times 3^2 \times 7$$

A positive factor is in the form $2^i \times 3^j \times 7^k$, where $0 \leq i \leq 3, 0 \leq j \leq 2, 0 \leq k \leq 1$

$$\text{The total number of positive factors are } (1 + 3)(1 + 2)(1 + 1) = 24$$

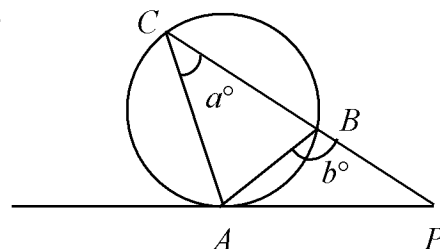
Individual Event 2

12.1 If $A(1, 3)$, $B(5, 8)$ and $C(29, a)$ are collinear, find the value of a .

The slopes are equal: $\frac{8-3}{5-1} = \frac{a-8}{29-5}$

$$\frac{a-8}{24} = \frac{5}{4} \Rightarrow a-8 = 30, a = 38$$

12.2 In the figure, PA is a tangent to the circle ABC , PBC is the straight line and $AB = PB$, if $\angle ABP = b^\circ$, $\angle ACB = a^\circ$, find the value of b .



$$\angle BAP = a^\circ = 38^\circ \text{ (\angle in alt. seg.)}$$

$$\angle BPA = 38^\circ \text{ (base \angle s isos. } \Delta)$$

$$38 + 38 + b = 180 \text{ (\angle s sum of } \Delta)$$

$$b = 104$$

12.3 If c is the minimum value of the quadratic function $y = x^2 + 4x + b$, find the value of c .

$$y = x^2 + 4x + 104 = (x + 2)^2 + 100 \geq 100 = c$$

12.4 If $d = 1 - 2 + 3 - 4 + \dots - c$, find the value of d .

$$d = (1 - 2) + (3 - 4) + \dots + (99 - 100) = -1 - 1 - \dots - 1 \text{ (50 times)} = -50$$

Individual Event 3

13.1 If $\{p, q\} = q \times a + p$ and $\{2, 5\} = 52$, find the value of a .

$$\{2, 5\} = 5 \times a + 2 = 52; a = 10$$

13.2 If $a, \frac{37}{2}, b$ is an arithmetic progression, find the value of b .

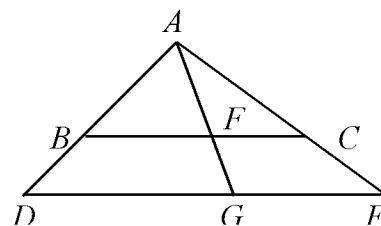
$$\frac{a+b}{2} = \frac{37}{2}; b = 27$$

13.3 If $b^2 - c^2 = 200$, find the value of c .

$$27^2 - c^2 = 200$$

$$c^2 = 729 - 200 = 529; c = 23$$

13.4 Given that in the figure, $BC \parallel DE$, $BC : DE = 10 : c$ and $AF : FG = 20 : d$, find the value of d .



By similar triangles, $AF : AG = AC : AE = BC : DE$

$$20 : (20 + d) = 10 : 23$$

$$d = 26$$

Individual Event 4

14.1 Given that $\frac{10x-3y}{x+2y} = 2$ and $p = \frac{x+y}{y-x}$, find the value of p .

$$10x - 3y = 2(x + 2y)$$

$$8x = 7y$$

$$p = \frac{x+y}{y-x} = \frac{8x+8y}{8y-8x} = \frac{7y+8y}{8y-7y} = 15$$

14.2 Given that $a \neq b$ and $ax = bx$. If $p + q = 19(a - b)^x$, find the value of q .

$$a \neq b \text{ and } ax = bx \Rightarrow x = 0$$

$$p + q = 19(a - b)^x \Rightarrow 15 + q = 19; q = 4$$

14.3 Given that the sum of q consecutive integers is 222, the largest is r , find the value of r .

The smallest integer is $r - q + 1$

$$\frac{q}{2}(r - q + 1 + r) = 222 \Rightarrow 2(2r - 3) = 222; r = 57$$

14.4 If $\tan^2(r + s)^\circ = 3$ and $0^\circ \leq r + s \leq 90^\circ$, find the value of s .

$$\tan^2(57 + s)^\circ = 3$$

$$57 + s = 60; s = 3$$

Individual Event 5

15.1 If the sum of roots of $5x^2 + ax - 2 = 0$ is twice the product of roots, find the value of a .

$$\alpha + \beta = 2\alpha\beta$$

$$-\frac{a}{5} = 2\left(-\frac{2}{5}\right); a = 4$$

15.2 Given that $y = ax^2 - bx - 13$ passes through (3, 8), find the value of b .

$$8 = 4(3)^2 - b(3) - 13; b = 5$$

15.3 If there are c ways of arranging b girls in a circle, find the value of c .

First arrange the 5 girls in a line, the number of ways = $5 \times 4 \times 3 \times 2 \times 1 = 120$

Next, join the first girl and the last girl to form a circle. There are 5 repetitions.

The number of ways = $c = 120 \div 5 = 24$

15.4 If $\frac{c}{4}$ straight lines and 3 circles are drawn on a paper, and d is the largest numbers of points of intersection, find the value of d .

For the 3 circles, there are 6 intersections.

If each straight line is drawn not passing through these intersections, it intersects the 3 circles at 6 other points. The 6 straight lines intersect each other at $1+2+3+4+5$ points.

$$\therefore d = \text{the largest numbers of points of intersection} = 6 + 6 \times 6 + 15 = 57$$

Sample Group Event

SG.1 If a is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find the value of a .

$$a = 5k = 3m + 1$$

The smallest possible $a = 10$.

SG.2 In the following diagram, $FA \parallel DC$ and $FE \parallel BC$. Find the value of b .

Join AD and CF .

Let $\angle CFE = x$, $\angle AFC = y$

$\angle BCF = x$ (alt. \angle s, $FE \parallel BC$)

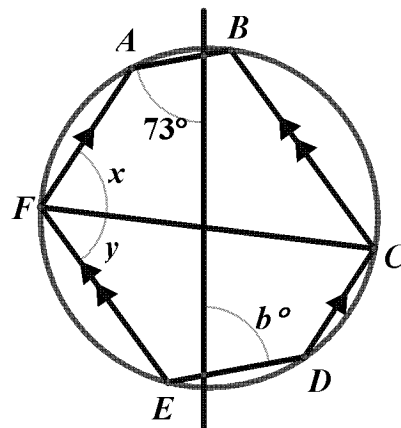
$\angle DCF = y$ (alt. \angle s, $FA \parallel DC$)

$\angle BCD = x + y$

$\angle BAD = 180^\circ - x - y = \angle ADE$ (opp. \angle cyclic quad.)

$\therefore AB \parallel ED$ (alt. \angle s eq.)

$b = 73$ (alt. \angle s $AB \parallel ED$)



SG.3 If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digit is 25, find the value of c .

$c = 10x + y$, where $0 < x < 10$, $0 \leq y < 10$.

$$x + y = 10$$

$$xy = 25$$

Solving these two equations gives $x = y = 5$; $c = 55$

SG.4 Let S_1, S_2, \dots, S_{10} be the first ten terms of an A.P., which consists of positive integers.

If $S_1 + S_2 + \dots + S_{10} = 55$ and $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$, find d .

Let the general term be $S_n = a + (n - 1)t$

$$\frac{10}{2}[2a + (10 - 1)t] = 55 \Rightarrow 2a + 9t = 11$$

$\therefore a, t$ are positive integers, $a = 1, t = 1$

$$d = (S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = [a + 9t - (a + 7t)] + [a + 8t - (a + 6t)] + \dots + (a + 2t - a)$$

$$d = 2t + 2t + 2t + 2t + 2t + 2t + 2t + 2t = 16t = 16$$

Group Event 1

G1.1 If the area of a given sector $s = 4 \text{ cm}^2$, the radius of this sector $r = 2 \text{ cm}$ and the arc length of this sector $A = p \text{ cm}$, find the value of p .

By the formula $A = \frac{1}{2}rs$, where A is the sector area, r is the radius and s is the arc length

$$4 = \frac{1}{2}(2)p; p = 4$$

G1.2 Given that $\frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b}$ and $a+b+c \neq 0$. If $q = \frac{2b+c}{a}$, find the value of q .

$$\text{Let } \frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b} = k$$

$$a = (2b+c)k; b = (2c+a)k; c = (2a+b)k$$

$$a+b+c = (2b+c+2c+a+2a+b)k$$

$$a+b+c = (3a+3b+3c)k$$

$$k = \frac{1}{3}$$

$$q = \frac{2b+c}{a} = \frac{1}{k} = 3$$

G1.3 Let ABC be a right-angled triangle, CD is the altitude on AB , $AC = 3$,

$DB = \frac{5}{2}$, $AD = r$, find the value of r .

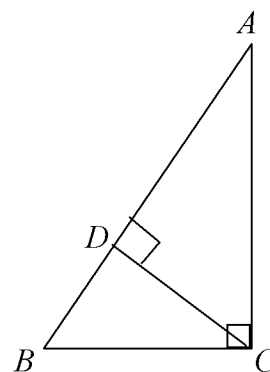
$$AD = AC \cos A = \frac{3AC}{AB} = \frac{9}{\frac{5}{2} + AD}$$

$$\frac{5}{2}AD + AD^2 = 9$$

$$2AD^2 + 5AD - 18 = 0$$

$$(2AD+9)(AD-2) = 0$$

$$AD = r = 2$$



G1.4 If $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$, find the value of a .

Compare the constant term: $17 = 8 + a; a = 9$

Group Event 2

G2.1 If $\frac{137}{a} = 0.1\dot{2}3\dot{4}$, find the value of a .

$$\frac{137}{a} = 0.1\dot{2}3\dot{4} = 0.1 + \frac{234}{9990} = \frac{999+234}{9990} = \frac{1233}{9990} = \frac{137}{1110}; a = 1110$$

G2.2 If $b = 1999 \times 19981998 - 1998 \times 19991999 + 1$, find the value of b .

$$b = 1999 \times 1998 \times 1001 - 1998 \times 1999 \times 1001 + 1 = 1$$

G2.3 If the parametric equation $\begin{cases} x = \sqrt{3-t^2} \\ y = t-3 \end{cases}$ can be transformed into $x^2 + y^2 + cx + dy + 6 = 0$, find

the values of c and d .

$$(1)^2 + (2)^2 : x^2 + y^2 = -6t + 12 = -6(y+3) + 12; c = 0, d = 6$$

Group Event 3

G3.1 In $\triangle ABC$, $\angle ABC = 2\angle ACB$, $BC = 2AB$. If $\angle BAC = a^\circ$, find the value of a .

Let $\angle ACB = \theta$, $\angle ABC = 2\theta$

$AB = c$, $BC = 2c$

$\angle BAC = 180^\circ - \theta - 2\theta$ (\angle s sum of Δ)

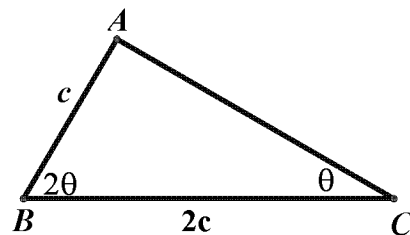
By sine formula, $\frac{c}{\sin \theta} = \frac{2c}{\sin(180^\circ - 3\theta)}$

$\sin 3\theta = 2\sin \theta$

$3 \sin \theta - 4 \sin^3 \theta = 2 \sin \theta$

$4 \sin^2 \theta - 1 = 0$

$\sin \theta = \frac{1}{2}$; $\theta = 30^\circ$, $\angle BAC = 180^\circ - 3\theta = 90^\circ$; $b = 90$



G3.2 Given that $x + \frac{1}{x} = \sqrt{2}$, $\frac{x^2}{x^4 + x^2 + 1} = b$, find the value of b .

$\left(x + \frac{1}{x}\right)^2 = 2 \Rightarrow x^2 + 2 + \frac{1}{x^2} = 2 \Rightarrow x^2 + \frac{1}{x^2} = 0$ (remark: x is a complex number)

$b = \frac{x^2}{x^4 + x^2 + 1} = \frac{1}{x^2 + 1 + \frac{1}{x^2}} = 1$

G3.3 If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is c , find the value of c .

$2x + 2y + 4xy = 282 \Rightarrow 2x + 2y + 4xy + 1 = 283$, which is a prime number

$(2x + 1)(2y + 1) = 1 \times 283$

$2x + 1 = 1$, $2y + 1 = 283$

Solving the above equations, there is no positive integral roots; $c = 0$

G3.4 Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2(x^4 + y^4 + z^4)$, find the value of d .

$(x + y + z)^2 = 0 \Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 0 \Rightarrow xy + yz + zx = -\frac{1}{2}$

$(xy + yz + zx)^2 = \frac{1}{4} \Rightarrow x^2y^2 + y^2z^2 + z^2x^2 + 2xyz(x + y + z) = \frac{1}{4} \Rightarrow x^2y^2 + y^2z^2 + z^2x^2 = \frac{1}{4}$

$(x^2 + y^2 + z^2)^2 = 1 \Rightarrow x^4 + y^4 + z^4 + 2(x^2y^2 + y^2z^2 + z^2x^2) = 1 \Rightarrow d = 2\left(1 - 2 \times \frac{1}{4}\right) = 1$

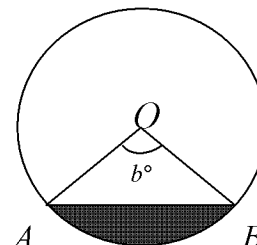
Group Event 4

G4.1 If $0.\dot{1} + 0.0\dot{2} + 0.00\dot{3} + \dots + 0.00000000\dot{9} = a$, find the value of a (Give your answer in decimal)

$$a = \frac{1}{9} + \frac{2}{90} + \frac{3}{900} + \dots + \frac{9}{900000000} = \frac{100000000 + 20000000 + 3000000 + \dots + 9}{900000000}$$

$$a = \frac{123456789}{900000000} = \frac{13717421}{100000000} = 0.13717421$$

G4.2 The circle in the figure has centre O and radius 1, A and B are points on the circle. Given that $\frac{\text{Area of shaded part}}{\text{Area of unshaded part}} = \frac{\pi - 2}{3\pi + 2}$

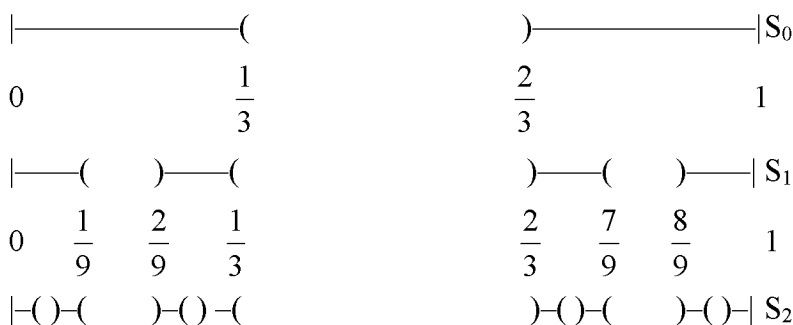


and $\angle AOB = b^\circ$, find the value of b .

$$\frac{\text{Area of shaded part}}{\text{Area of the circle}} = \frac{\pi - 2}{\pi - 2 + 3\pi + 2} = \frac{\pi - 2}{4\pi}$$

$$\frac{\pi(1)^2 \cdot \frac{b}{360} - \frac{1}{2}(1)^2 \sin b^\circ}{\pi(1)^2} = \frac{\pi - 2}{4\pi} \Rightarrow \frac{\pi b}{90} - 2 \sin b^\circ = \pi - 2; b = 90$$

G4.3 A sequence of figures S_0, S_1, S_2, \dots are constructed as follows. S_0 is obtained by removing the middle third of $[0,1]$ interval; S_1 by removing the middle third of the two intervals in S_0 ; S_2 by removing the middle thirds of the four intervals in S_1 ; S_3, S_4, \dots are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 (Give your answer in fraction).



The total length in $S_0 = \frac{2}{3}$

The total length in $S_1 = 4 \times \frac{1}{9} = \frac{4}{9}$

The total length in $S_2 = 8 \times \frac{1}{27} = \frac{8}{27}$

Deductively, the total length in $S_5 = 2^6 \times \frac{1}{3^6} = \frac{64}{729}$

The total length removed in $S_5 = 1 - \frac{64}{729} = \frac{665}{729}$

G4.4 All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is d , find the value of d .

Integer	-3	-2	-1	0	1	2	3
Code	7	5	3	1	2	4	6

Sum of integers code as 102, 104, ..., 200 is $51 + 52 + \dots + 100$

Sum of integers code as 101, 103, ..., 199 is $-50 - 51 - \dots - 99$

Sum of all integers = $1 + 1 + \dots + 1$ (50 times) = 50

Group Event 5

G5.1 If $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots + 10 \times 11 \times 12 = a$, find the value of a .

$$a = \frac{1}{4}n(n+1)(n+2)(n+3) = \frac{1}{4}10(11)(12)(13) = 4290$$

G5.2 Given that $5^x + 5^{-x} = 3$. If $5^{3x} + 5^{-3x} = b$, find the value of b .

$$(5^x + 5^{-x})^2 = 9 \Rightarrow 5^{2x} + 2 + 5^{-2x} = 9 \Rightarrow 5^{2x} + 5^{-2x} = 7$$

$$b = 5^{3x} + 5^{-3x} = (5^x + 5^{-x})(5^{2x} - 1 + 5^{-2x}) = 3(7 - 1) = 18$$

G5.3 Given that the roots of equation $x^2 + mx + n = 0$ are 98 and 99 and $y = x^2 + mx + n$. If x takes on the values of 0, 1, 2, ..., 100, then there are c values of y that can be divisible by 6. Find the value of c .

$$m = -98 - 99 = -197; n = 98 \times 99 = 49 \times 33 \times 6, \text{ which is divisible by } 6$$

$$y = x^2 - 197x + 98 \times 99 = x^2 + x - 198x + 49 \times 33 \times 6 = x(x + 1) - 6(33x + 49 \times 33)$$

If y is divisible by 6, then $x(x + 1)$ is divisible by 6

One of $x, x + 1$ must be even. If it is divisible by 6, then one of $x, x + 1$ must be divisible by 3.

We count the number of possible x for which y cannot be divisible by 6

These x may be 1, 4, 7, 10, ..., 97, 100; totally 34 possible x .

$$c = 101 - 34 = 67$$

G5.4 In the figure, $ABCD$ is a square, $BF \parallel AC$, and $AEFC$ is a rhombus. If $\angle EAC = d^\circ$, find the value of d .

From B and E draw 2 lines $h, k \perp AC$

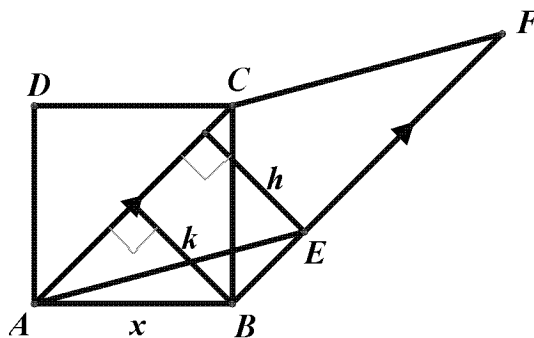
$$h = k (\because BF \parallel AC)$$

Let $AB = x, \angle CAB = 45^\circ$

$$k = x \sin 45^\circ = \frac{x}{\sqrt{2}} = h$$

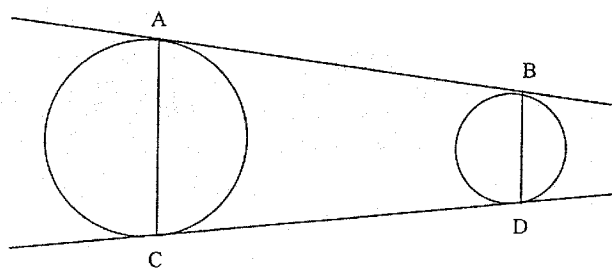
$AC = x \div \cos 45^\circ = \sqrt{2}x = AE$ ($\because AEFC$ is a rhombus)

$$\sin \angle EAC = \frac{h}{AE} = \frac{\frac{x}{\sqrt{2}}}{\sqrt{2}x} = \frac{1}{2}, d = 30$$



Group Spare Event

GS.1 In the figure, there are two common tangents. These common tangents meet the circles at points A, B, C and D . If $AC = 9$ cm, $BD = 3$ cm, $\angle BAC = 60^\circ$ and $AB = s$ cm, find the value of s .



Produce AB and CD to meet at E .

$AE = CE, BE = DE$ (tangent from ext. pt.)

$\triangle EAC$ and $\triangle EBD$ are isosceles triangles

$\angle ECA = \angle BAC = 60^\circ$ (base \angle s isos. \triangle)

$\angle AEC = 60^\circ$ (\angle s sum of \triangle)

$\angle EBD = \angle EDB = 60^\circ$ (\angle s sum of \triangle , base \angle s isos. \triangle)

$\therefore \triangle EAC$ and $\triangle EBD$ are equilateral triangles

$EB = BD = 3, EA = AC = 9$ (sides of equilateral triangles)

$AB = 9 - 3 = 6$

GS.2 In the figure, $ABCD$ is a quadrilateral, where the interior angles $\angle A, \angle B$ and $\angle D$ are all equal to 45° . When produced, BC is perpendicular to AD . If $AC = 10$ and $BD = b$, find the value of b .

reflex $\angle BCD = 360^\circ - 45^\circ - 45^\circ - 45^\circ = 225^\circ$ (\angle s sum of polygon)

$\angle BCD = 360^\circ - 225^\circ = 135^\circ$ (\angle s at a point)

Produce BC to meet AD at $E, \angle AEB = 90^\circ$ (given)

$\angle BAE = 45^\circ = \angle ABE$ (given)

$\triangle ABE$ and $\triangle CDE$ are right angled isosceles triangles

Let $AE = x, DE = y$, then $BE = x, CE = y, BC = x - y$

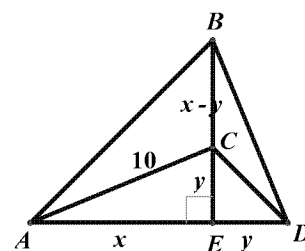
In $\triangle ACE, x^2 + y^2 = 10^2 \dots (1)$ (Pythagoras' Theorem)

$CD = \sqrt{y^2 + y^2} = \sqrt{2}y$ (Pythagoras' Theorem)

Apply cosine rule on $\triangle BCD$

$$BD^2 = (x - y)^2 + 2y^2 - 2(x - y)\sqrt{2}y \cos 135^\circ$$

$$BD^2 = x^2 - 2xy + y^2 + 2y^2 + 2(x - y)y = x^2 + y^2 = 10^2 \Rightarrow BD = b = 10$$



GS.3 If $\log_c 27 = 0.75$, find the value of c .

$$c^{0.75} = 27 \Rightarrow c = (3^3)^{\frac{4}{3}} = 81$$

GS.4 If the mean, mode and median of the data 30, 80, 50, 40, d are all equal, find the value of d .

$$\text{Mean} = \frac{30 + 80 + 50 + 40 + d}{5} = 40 + \frac{d}{5} = \text{mode}$$

By trial and error, $d = 50$