Q1: (15)

$Z_1, Z_2 \sim N(0,1)$, $X_1, X_2 \sim N(1,1)$

$X_1 - X_2 \sim N(0,2)$, $Z_1 - Z_2 \sim N(0,2)$, $Z_1 + Z_2 \sim N(0,2)$

$\bar{X} \sim N(1, \frac{1}{2})$, $\bar{Z} \sim N(0, \frac{1}{2})$. all indep. RVs

a) $\bar{X} + \bar{Z} \sim N(1, 1)$ ←

b) $\frac{1}{2}\left[(X_1 - X_2)^2 + (Z_1 - Z_2)^2 + (Z_1 + Z_2)^2\right]$

$\overset{d}{=} \chi^2_1 + \chi^2_1 + \chi^2_1 \equiv \chi^2_3$ ←

c) $U = \frac{Z_1 + Z_2}{\sqrt{2}} \sim N(0,1)$, $W = \frac{(X_2 - X_1)^2}{2} + \frac{(Z_2 - Z_1)^2}{2} \sim \chi^2_2$

$\frac{U}{\sqrt{\frac{W}{2}}} \sim t_2 \Rightarrow \frac{Z_1 + Z_2}{\sqrt{2}\sqrt{\frac{(X_2 - X_1)^2 + (Z_2 - Z_1)^2}{2}}/2} \equiv \frac{2}{\sqrt{2}} \frac{Z_1 + Z_2}{\sqrt{(\,)^2 + (\,)^2}}$

d) $(X_1 + X_2) \sim N(2,2) \Rightarrow \frac{X_1 + X_2 - 2}{\sqrt{2}} \sim N(0,1)$

$\Rightarrow (X_1 + X_2 - 2)^2/2 \sim \chi^2_1$

$\frac{(X_2 - X_1)}{\sqrt{2}} \sim N(0,1) \Rightarrow \frac{(X_2 - X_1)^2}{2} \sim \chi^2_1$

$\therefore \frac{(Z_2 + Z_1 - 2)^2}{(X_2 - X_1)^2} \sim F_{1,1}$ ←

e) $\frac{(n_1 - 1)S_Z^2}{\sigma_Z^2} \sim \chi^2_{n_1 - 1}$, $\frac{(n_2 - 1)S_X^2}{\sigma_X^2} \sim \chi^2_{n_2 - 1}$

$S_Z^2 = \sum^2 (Z_i - \bar{Z})^2 \sim \chi^2_1$, $S_X^2 = \sum^2 (X_i - \bar{X})^2 \sim \chi^2_1$

$\therefore \sum^2 (Z_i - \bar{Z})^2 + \sum^2 (X_i - \bar{X})^2 \sim \chi^2_2$ ←