Respostas – Questões da lista de Matrizes e Determinantes

Exercícios de Fixação

1. $A_{2x2} = a_{ij} = \begin{cases} 2^{i+j}, \ i < j \\ i^2 + 1, \ se \ i \geq j \end{cases} \rightarrow A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1^2+1 & 2^{1+2} \\ 2^2+1 & 2^2+2 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 5 & 6 \end{pmatrix}$ resp: b)

2. $A_{3x3} = a_{ij} = \begin{cases} (-1)^{i+j}, \ i \neq j \\ 0, \ se \ i = j \end{cases} \rightarrow A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ resp: a)

3. $A_{mxn}.Bmxp \rightarrow se \ m = p$ → c) F

4. $A = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \ e \ B = \begin{pmatrix} 2 & -\frac{1}{2} \\ -1 & 3 \end{pmatrix} \rightarrow A.B = \begin{pmatrix} 4 & -1 \\ -2-3 & \frac{1}{2}+9 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -5 & \frac{19}{2} \end{pmatrix}$ Resp: ??
$-2.A.B = \begin{pmatrix} -8 & 1 \\ 10 & -19 \end{pmatrix}$

5. $A = a_{mxn} \ e \ B_{pxn}$ →

   a) F; $A = a_{mxn} \ e \ B_{pxn}$ → A+B se m=p ;

   b) F; A.B existe se n=p

   c) F; B.A existe se m=n

   d) F; $(A^t)_{nxm}.B_{pxn}$ existe se m=p

   e) V; $A_{mxn}.(B^t)_{nxp}$, temos n=n. e o produto tem ordem mxp.

6. $A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \ e \ B = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}$
$A.B = \begin{pmatrix} 0 & 1-1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ → resp. b)

7. $A_{2x2} \ e \ B_{2x2} \ com \ A.B = 0$, veja questão 6 (anterior). Resp. e)

8. $P=M^{-1}$ → $M = \begin{pmatrix} \frac{1}{3} & 0 \\ \frac{1}{7} & 1 \end{pmatrix} \rightarrow \det(M) = \frac{1}{3} - 0 = \frac{1}{3}; \ M^{-1} = \begin{pmatrix} \frac{3}{\ } & \frac{0}{\ } \\ -\frac{\frac{1}{7}}{\frac{1}{3}} & \frac{\frac{1}{3}}{\frac{1}{3}} \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -\frac{3}{7} & 1 \end{pmatrix}$;

   Soma dos elementos da diagonal principal da matriz P= 4

9. $A=\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix}$ e $B=\begin{pmatrix}0 & 1\\ -2 & 3\end{pmatrix}$; $\det A = 1 \to A^{-1} == \begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}$;

$C = A^{-1} + B == \begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix} + \begin{pmatrix}0 & 1\\ -2 & 3\end{pmatrix} = \begin{pmatrix}1 & 1\\ 0 & 4\end{pmatrix}$

10. $X = ?$

$A.X^{-1}.B^{-1} = I_n \to A^{-1}A.X^{-1}.B^{-1} = A^{-1}I_n \to I_n X^{-1}.B^{-1} = A^{-1} \to$

$I_n.X.X^{-1}.B^{-1} = X.A^{-1} \to I_n..B^{-1} = X.A^{-1} \to B^{-1} = X.A^{-1} \to$

$B^{-1}.A = X.A^{-1}.A \to B^{-1}.A = X.I_n \qquad \to X = B^{-1}.A$

11. $M.N = ? --> M.N = \begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}1 & 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$ resp: d)

12. $A_{2x2} = a_{ij} = \left[(-1)^{i+j}\right].i.j$

$A=\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} = \begin{pmatrix}1 & -2\\ -2 & 4\end{pmatrix}$, resp: b)

13. Temos: $A^t = -A$; $M = \begin{pmatrix}4+a & a_{12} & a_{13}\\ a & b+2 & a_{23}\\ b & c & 2c-8\end{pmatrix}$, numa matriz anti-simétrica os elementos da diagonal principal são todos nulos: 4+a=0 → a= -4 ; b+2=0 → b=-2; 2c-8=0 → c=4

$$M = -M^t = \begin{pmatrix}4+a & a_{12} & a_{13}\\ a & b+2 & a_{23}\\ b & c & 2c-8\end{pmatrix} = \begin{pmatrix}0 & -a & -b\\ -a_{12} & 0 & -c\\ -a_{13} & -a_{23} & 0\end{pmatrix} \to \begin{matrix}a_{12} = -a = 4\\ a_{13} = -b = 2\\ a_{23} = -c = -4\end{matrix}$$

14. $\begin{pmatrix}2x & 2x^2-2\\ -2 & -2x\end{pmatrix} = \begin{pmatrix}x^2+6x & 30\\ -2 & -2x\end{pmatrix} --> \begin{matrix}x^2+6x-2x=0 \to x^2+4x=0 \to x.(x+4)=0\\ 2x^2-2-30=0 \to 2x^2=32 \to x^2=16 \to x=\pm 4\end{matrix}$

Para x= -4 temos $x^3=(-4)^3=(-4).(-4).(-4)=-64$

15. Inversa da matriz $A=\begin{pmatrix}4 & 3\\ 1 & 1\end{pmatrix} \to \det A = 4-3=1 \to A^{-1} = \begin{pmatrix}\frac{1}{1} & \frac{-3}{1}\\ \frac{-1}{1} & \frac{4}{1}\end{pmatrix} = \begin{pmatrix}1 & -3\\ -1 & 4\end{pmatrix}$

16. $\begin{vmatrix}1 & 2 & 3\\ x & -1 & 5\\ \frac{2}{3} & -\frac{1}{2} & 0\end{vmatrix} = \frac{20}{3} - \frac{3x}{2} + 2 + \frac{5}{2} = 0 \to -\frac{3x}{2} = -\frac{20}{3} - \frac{9}{2} \to -\frac{3x}{2} = -\frac{40+27}{6} \to$

$-3x = -\frac{67}{3} \to x = \frac{67}{9}$

17. $\begin{vmatrix} 1 & \frac{12}{5} & 9 \\ -\frac{1}{2} & 0 & k \\ -2 & \frac{2}{5} & -1 \end{vmatrix} = 0 - \frac{9}{5} - \frac{24k}{5} - \left(0 + \frac{2k}{5} + \frac{6}{5}\right) = 10 \rightarrow -\frac{9}{5} - \frac{24k}{5} - \frac{2k}{5} - \frac{6}{5} = 10 \rightarrow$

$-\frac{26k}{5} - \frac{15}{5} = 10 \rightarrow -\frac{26}{5}k - 3 = 10 \rightarrow -\frac{26}{5}k = 13 \rightarrow k = -\frac{13}{\frac{26}{5}} = -13.\frac{5}{26} = -\frac{5}{2}$

$A = ?$

18. $A_{2x2} = a_{ij} = j - i^2$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix} \rightarrow \det A = 0 - (-3) = 3$

19. Calcular x=? e y=?

Dados: $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & x & z \\ 0 & 0 & y \end{pmatrix}$, traço=9 e detB=15

Resolvendo:

$1 + x + y = 9 \rightarrow y = 9 - x - 1$

$1.x.y = 15 \rightarrow x.(9 - x - 1) = 15 \rightarrow x.(-x + 8) = 15 \rightarrow -x^2 + 8x - 15 = 0$

, resp: d)

$x^2 - 8x + 15 = 0 \rightarrow x = 4 \pm \sqrt{16 - 15} = 4 \pm 1 = \begin{cases} x = 5 \\ x = 3 \end{cases}$

20. $\begin{vmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 1 & 1 & 1 \end{vmatrix} < 0 \quad \rightarrow x^2 + 1 + 0 - (0 + x + 1) < 0 \rightarrow x^2 + 1 - x - 1 < 0 \rightarrow x^2 - x < 0$

$x.(x - 1) < 0 \rightarrow (0,1)$

21. $\frac{y}{x} = ? \quad \rightarrow \quad \frac{y}{x} = \frac{\begin{vmatrix} -2a & 2c \\ -3b & 3d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{-6ad + 6bc}{ad - bc} = \frac{-6(ad - bc)}{ad - bc} = -6$

22. detA=detA^t , que é um determinante de Vandermonde.

$\begin{vmatrix} 1 & x & x^2 \\ 1 & 2 & 4 \\ 1 & -3 & 9 \end{vmatrix} \xrightarrow{transposta} \begin{vmatrix} 1 & 1 & 1 \\ x & 2 & -3 \\ x^2 & 4 & 9 \end{vmatrix} = (2 - x).(-3 - 2).(-3 - x) = (2 - x).(-5).(x + 3) = -5.(x - 2).(x + 3)$

resp: a)

23. $\begin{vmatrix} x & 0 & 0 & 3 \\ -1 & x & 0 & 0 \\ 0 & -1 & x & 1 \\ 0 & 0 & -1 & -2 \end{vmatrix}$ =? → aplicando-se o teorema de Laplace a primeira coluna:

$$x.(-1)^{1+1}.\begin{vmatrix} x & 0 & 0 \\ -1 & x & 1 \\ 0 & -1 & -2 \end{vmatrix} -1.(-1)^{2+1}.\begin{vmatrix} 0 & 0 & 3 \\ -1 & x & 1 \\ 0 & -1 & -2 \end{vmatrix} = x.\begin{vmatrix} x & 1 \\ -1 & -2 \end{vmatrix} + 3.(-1)^{1+3}.\begin{vmatrix} -1 & x \\ 0 & -1 \end{vmatrix} = x.(-2x+1)+3$$

$$= -2x^2 + x + 3$$

24. $A = \begin{pmatrix} -2 & a^2 \\ 1 & 2 \end{pmatrix} \rightarrow \det A = -4 - a^2 \neq 0 \rightarrow a^2 + 4 \neq 0$, esse binômio é positivo para qualquer valor de a. resp. e)

25. A é uma matriz inversível logo detA é diferente de zero.

detA=? ; Temos: $\begin{cases} \det(2.A) = \det(A^2) \\ 2^2.\det A = \det A.\det A \rightarrow \ 4 = \det A \end{cases}$

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$$A = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \rightarrow \det A = a^2 - b^2; \begin{cases} a = \dfrac{e^x + e^{-x}}{2} \\ b = \dfrac{e^x - e^{-x}}{2} \end{cases}$$

26. $\det A = \left(\dfrac{e^x + e^{-x}}{2}\right)^2 - \left(\dfrac{e^x - e^{-x}}{2}\right)^2 = \left(\dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2}\right).\left(\dfrac{e^x + e^{-x}}{2} - \dfrac{e^x - e^{-x}}{2}\right)$

$$\det A = e^x.e^{-x} = e^0 = 1$$

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27.

$$A_{2x2} = a_{ij} = i + j; \quad A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$$

$$A - k.I = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} -k & 0 \\ 0 & -k \end{pmatrix} = \begin{pmatrix} 2-k & 3 \\ 3 & 4-k \end{pmatrix}$$

$$\det(A - k.I) = (k-2).(k-4) - 9$$

$$\det A = 8 - 9 = -1$$

$$\det(A - kI) = \det A - k \rightarrow (k-2).(k-4) - 9 = -1 - k \rightarrow k^2 - 4k - 2k + 8 - 9 = -1 - k \rightarrow$$

$$k^2 - 6k - 1 = -1 - k \rightarrow k^2 - 5k = 0 \rightarrow k.(k-5) = 0 \rightarrow k = 0 \ \ ou \ \ k = 5$$

28. $A_{3x3} = \begin{cases} 0, \ se \ \ i < j \\ i + j, \ se \ \ i = j \\ i - j, \ se \ \ i > j \end{cases} \rightarrow \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 2 & 1 & 6 \end{pmatrix} \rightarrow \det A = 2x4x6 = 48$

29. $\det A = 12;\ 6.\frac{1}{4}.\det A --> \frac{3}{2}.\det A = 18$

30. Afirmações:
$I = F --> A = k.B \rightarrow \det A = k^n.\det B$
$II = F --> C = A + B \rightarrow \det C = \det(A + B)$
$III = V --> C = A.B \rightarrow \det C = \det A.\det B$

31. $\begin{vmatrix} x & 0 & 0 & 0 \\ 1 & x & 1 & 2 \\ 2 & 0 & x & 3 \\ 0 & 0 & 0 & 2 \end{vmatrix} = 16 \xrightarrow{Laplace\ 1a.linha} x.(-1)^{1+1}.\begin{vmatrix} x & 1 & 2 \\ 0 & x & 2 \\ 0 & 0 & 2 \end{vmatrix} = 16 \xrightarrow{Laplace\ 1a.coluna}$

resp: b)

$x.x.\begin{vmatrix} x & 2 \\ 0 & 2 \end{vmatrix} = 16 \rightarrow x.x.2x = 16 \rightarrow 2x^3 = 16 \rightarrow x^3 = 8 \rightarrow x = 2 \rightarrow a^2 = 4$

32. Determinante de

Vandermonde: $\begin{vmatrix} 1 & 1 & 1 \\ \log 7 & \log 70 & \log 700 \\ (\log 7)^2 & (\log 70)^2 & (\log 700)^2 \end{vmatrix} = (\log 70 - \log 7).(\log 700 - \log 70).(\log 700 - \log 7)$

$= \log\frac{70}{7}.\log\frac{700}{70}.\log\frac{700}{7} = \log 10.\log 10.\log 100 = 1.1.2 = 2$

33. $a = ?;$
$\begin{cases} x + 2y = 18 \\ 3x - ay = 54 \end{cases} \rightarrow \Delta = \begin{vmatrix} 1 & 2 \\ 3 & -a \end{vmatrix} = -a - 6 = 0 \rightarrow a = -6$

Para a=-6 temos: $\begin{matrix} 1 & 2 & 18 \\ 3 & 6 & 54 \end{matrix} \rightarrow \begin{matrix} 1 & 2 & 18 \\ 0 & 0 & 0 \end{matrix} \rightarrow x + 2y = 18 \rightarrow S = \{(18 - 2y; y)\}$

34. $\begin{matrix} 1 & -1 & 2 & 2 \\ 2 & 3 & 4 & 9 \\ 1 & 4 & 2 & 7 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 2 & 2 \\ 0 & 5 & 0 & 5 \\ 0 & 5 & 0 & 5 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 2 & 2 \\ 0 & 5 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 2 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix} \rightarrow \begin{matrix} x - 1 + 2z = 2 \rightarrow x = 3 - 2z \\ y = 1 \end{matrix}$

$\text{Resp}: S = \{(3 - 2z; 1; z)\}$

35. Cálculo do determinante da matriz incompleta: $\begin{vmatrix} 1 & 1 \\ a^2 & 1 \end{vmatrix} = 1 - a^2 = 0 \rightarrow a^2 = 1 \rightarrow a = \pm 1$

Verificando : Para a= -1 → $\begin{matrix} 1 & 1 & -1 \\ 1 & 1 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & -1 \\ 0 & 0 & 2 \end{matrix} \rightarrow 0.y = 2 \rightarrow y = \frac{2}{0}$

Para a=1 → $\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{matrix} \rightarrow x + y = 1 \rightarrow x = 1 - y$ → Resp: $S = \{(1 - y; y)\}$

36. Cálculo do determinante da matriz incompleta: $\begin{matrix} a & -b \\ 2 & 5 \end{matrix} \rightarrow 5a + 2b \neq 0 \rightarrow a \neq -\frac{2b}{5}$

37. Um sistema linear homogêneo é sempre possível (determinante da matriz incompleta diferente de zero) ou indeterminado (determinante da matriz incompleta igual a zero).

38. Cálculo do determinante da matriz incompleta: $\begin{vmatrix} m & -1 \\ 1 & m \end{vmatrix} = m^2 + 1 > 0$ para qualquer que seja m. logo este sistema linear será sempre determinado.

39. Cálculo do determinante da matriz incompleta: $\begin{vmatrix} 1 & 1 \\ 1 & m^2 \end{vmatrix} = m^2 - 1 \neq 0 \to m^2 \neq 1 \to m \neq \pm 1 \to$

Verificando (Testando): para m=0 → $\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{matrix} \to \begin{matrix} 1 & 1 & 1 \\ 0 & -1 & -1 \end{matrix} \to \begin{matrix} x+1=1 \to x=0 \\ -1.y=-1 \to y=1 \end{matrix}$

40. Este sistema linear será possível e indeterminado:

Por Cramer: determinante da matriz incompleta → $\Delta = \begin{vmatrix} k & -3 \\ 1 & 1 \end{vmatrix} = k+3=0 \to k=-3$

$x = \frac{\Delta x}{\Delta} = \frac{0}{0} = \frac{\begin{vmatrix} 6 & -3 \\ t & 1 \end{vmatrix}}{} = \frac{6+3t}{0} \to 6+3t=0 \to t=-2$; a soma pedida: k+t= -3-2= - 5.

41. Cálculo do determinante da matriz incompleta:

$\begin{vmatrix} 1 & 1 & 0 \\ a & 1 & -1 \\ 1 & -a & 1 \end{vmatrix} = 1-1+0-(0-a-a)=0 \to 2a=0 \to a=0$

$x = \frac{\Delta x}{\Delta} = \frac{0}{0} = \frac{\begin{vmatrix} b & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}}{0} = \frac{b-1}{0} \to b-1=0 \to b=1$

42. Resolvendo o sistema linear formado pelas duas primeiras equações

$\begin{matrix} 1 & 2 & 4 \\ 1 & -1 & 1 \end{matrix} \to \begin{matrix} 1 & 2 & 4 \\ 0 & -3 & -3 \end{matrix} \to \begin{matrix} x+2.1=4 \to x=2 \\ -3.y=-3 \to y=1 \end{matrix}$; agora substituindo x=2 e y=1 na terceira equação: 3.2-2.1=m → m=4. Resp: $m \neq 4$

43. Cálculo do determinante da matriz incompleta:

$\begin{vmatrix} k-4 & 8 & -4 \\ -1 & k & 0 \\ 0 & -1 & 1 \end{vmatrix} = k.(k-4)+0-4-(0+0-8)=0 \to k^2-4k-4+8=0 \to k^2-4k+4=0$

$k = 2 \pm \sqrt{4-4} = 2, \ \ raiz \ \ dupla$

44. Cálculo do determinante da matriz incompleta:

$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ 1 & 0 & -14 \end{vmatrix} \to \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & -9 \\ 0 & -2 & -18 \end{matrix} \to \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & -9 \\ 0 & 0 & 0 \end{matrix} \to \Delta = 0$, isso quer nos dizer que este sistema linear será sempre possível e indeterminado.

Resolvendo o sistema linear pelo método do escalonamento:

$$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ 1 & 0 & -14 \end{vmatrix} \to \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & -9 \\ 0 & -2 & -18 \end{matrix} \to \begin{matrix} 1 & 2 & 4 \\ 0 & -1 & -9 \\ 0 & 0 & 0 \end{matrix} \to \begin{matrix} x - 18z + 4z = 0 \to x = 14z \\ -y - 9z = 0 \to y = -9z \end{matrix} \to S = \{(14z; -9z;\ z)\}$$

45. Cálculo do determinante da matriz incompleta: $\begin{vmatrix} 2 & 2 \\ 2 & a \end{vmatrix} = 2a - 4 = 0 \to a = 2$

Calculando o valor de x: $y = \dfrac{\Delta x}{\Delta} = \dfrac{0}{0} = \dfrac{\begin{vmatrix} 2 & b \\ 2 & 6 \end{vmatrix}}{0} \to 12 - 2b = 0 \to b = 6$

Testes Propostos

Matrizes e Determinantes

1. $a_{ij} = (-1)^{i+j}.i.j; \quad A_{2x2} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} (-1)^{1+1}.1.1 & (-1)^{1+2}.1.2 \\ (-1)^{2+1}.2.1 & (-1)^{2+2}.2.2 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ -2 & 4 \end{pmatrix}$

2. $A_{3x2} = a_{ij} = \begin{cases} 1, \; se \; i = j \\ i^2, \; se \; i \neq j \end{cases} \rightarrow A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \\ 9 & 9 \end{pmatrix}$

3. $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -4 & 0 \end{bmatrix} \; e \; B = \begin{bmatrix} 1 & 2 \\ 3 & -3 \\ 2 & 1 \end{bmatrix}; \; 2.A - B = \begin{bmatrix} 4 & 2 \\ 6 & 4 \\ -8 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ -3 & 3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 3 & 7 \\ -10 & -1 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix} \rightarrow A^2 = \begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix} . \begin{bmatrix} 1 & 3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 1-9 & 3+12 \\ -3-12 & -9+16 \end{bmatrix} = \begin{bmatrix} -8 & 15 \\ -15 & 7 \end{bmatrix}$ →Resp: e)

5. Resolvendo cada item:

a) falso; → $B.A = I = ? \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2+0 & 4+8 \\ 0+0 & 0+16 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 0 & 16 \end{bmatrix}$

b) falso →

$$B.A \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2+0 & 4+8 \\ 0+0 & 0+16 \end{bmatrix} = \begin{bmatrix} 2 & 12 \\ 0 & 16 \end{bmatrix}$$

$$A.B = \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4+16 \\ 0 & 16 \end{bmatrix}$$

c) Falso:

$$A = \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} \rightarrow 2.A = \begin{bmatrix} 4 & 8 \\ 0 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 12 \\ 0 & 16 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 24 \\ 0 & 32 \end{bmatrix} \rightarrow A \neq 2.B$$

d) Falso:

$$A.I = \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & 4 \end{bmatrix}$$

$$B.Z = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} . \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

6.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ .0 & 0 & 3 \end{bmatrix} . \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ x & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -16 & 0 \\ 3x & 0 & 6 \end{bmatrix}$$

$$.\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ x & 0 & 2 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ .0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -16 & 0 \\ x & 0 & 6 \end{bmatrix}$$

→ 3x=x→ x=0

7.
$$2.A.B+B^2 = 2.\begin{bmatrix}0&1&1\\1&1&0\\1&1&1\end{bmatrix}.\begin{bmatrix}0&-2&1\\2&-1&0\\0&1&0\end{bmatrix}+\begin{bmatrix}0&-2&1\\2&-1&0\\0&1&0\end{bmatrix}.\begin{bmatrix}0&-2&1\\2&-1&0\\0&1&0\end{bmatrix}$$
$$=2.\begin{bmatrix}2&0&0\\2&-3&1\\2&-2&1\end{bmatrix}+\begin{bmatrix}-4&&\\&&\\&&\end{bmatrix}=\begin{bmatrix}4&0&0\\4&-6&2\\4&-6&2\end{bmatrix}+\begin{bmatrix}-4&&\\&&\\&&\end{bmatrix}=$$

8. $M.N=?\ -->\ M.N=\begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}1&1&1\end{pmatrix}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ resp: d)

9. Se $\begin{bmatrix}-2&1\\1&-2\end{bmatrix}.\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}9\\3\end{bmatrix}$, resolvendo pelo método do escalonamento

$$\begin{matrix}-2&1&9\\1&-2&3\end{matrix}\rightarrow\begin{matrix}1&-2&3\\-2&1&9\end{matrix}\rightarrow\begin{matrix}1&-2&3\\0&-3&15\end{matrix}\rightarrow\begin{matrix}x+10=3\rightarrow x=7\\-3y=15\rightarrow y=-5\end{matrix}$$

10.
$$A=\begin{bmatrix}1&2\\1&4\end{bmatrix}\rightarrow\det A=4-2=2\rightarrow A^{-1}=\begin{bmatrix}\frac{4}{2}&\frac{-2}{2}\\\frac{-1}{2}&\frac{1}{2}\end{bmatrix}=\begin{bmatrix}2&-1\\\frac{-1}{2}&\frac{1}{2}\end{bmatrix}$$
$$B=\begin{bmatrix}2&-1\\x&y\end{bmatrix};\ Temos\quad B=A^{-1}\rightarrow\begin{bmatrix}2&-1\\x&y\end{bmatrix}=\begin{bmatrix}2&-1\\\frac{-1}{2}&\frac{1}{2}\end{bmatrix}\rightarrow x=-\frac{1}{2}\ e\ y=\frac{1}{2}$$

11. $\begin{bmatrix}3y-x&3\\2&1\end{bmatrix}=\begin{bmatrix}5&3\\2&3y+x\end{bmatrix}\rightarrow\begin{cases}-x+3y=5\\x+3y=1\end{cases}\rightarrow\begin{matrix}1&3&1\\-1&3&5\end{matrix}\rightarrow\begin{matrix}1&3&1\\0&6&6\end{matrix}\rightarrow\begin{matrix}x=-2\\y=1\end{matrix}$

12.
$$A_{3x2}=a_{ij}=2i-3j$$
$$A=\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\\a_{31}&a_{32}\end{bmatrix}=\begin{bmatrix}-1&-4\\1&-2\\3&0\end{bmatrix}$$
resp: b)

13. $\begin{bmatrix}0&1\\1&0\end{bmatrix}.\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&1\\1&0\end{bmatrix}$

14. $\begin{bmatrix}3a-2b\\a+4b\end{bmatrix}=\begin{bmatrix}5\\7\end{bmatrix}\rightarrow\begin{cases}\begin{matrix}1&4&7\\3&-2&5\end{matrix}\end{cases}\rightarrow\begin{matrix}1&4&7\\0&-14&-16\end{matrix}\rightarrow\begin{matrix}a=7-\frac{32}{7}=\frac{17}{7}\\-14.b=-16\rightarrow b=\frac{16}{14}=\frac{8}{7}\end{matrix}$

15. $M=\begin{bmatrix}1&0\\0&1\end{bmatrix};\ N=\begin{bmatrix}0&0\\0&0\end{bmatrix};\ P=\begin{bmatrix}6&8\\10&7\end{bmatrix};$ MxP=P, M é a matriz identidade, elemento neutro da multiplicação de matrizes.

16. $A_{2x2} = a_{ij} = \begin{cases} i+2j, \; se \;\; i \neq j \\ i-2j, \; se \;\; i = j \end{cases}$ resp e)

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 4 & -2 \end{bmatrix}$

17. $\begin{bmatrix} x & y \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} x+y & x-1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 2x+y & x-1+y \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 0 & 2 \end{bmatrix}$

$\begin{cases} 2x+y=7 \\ x+y=5 \end{cases} \rightarrow \begin{matrix} 1 & 1 & 5 \\ 2 & 1 & 7 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 5 \\ 0 & -1 & -3 \end{matrix} \rightarrow \begin{matrix} x=2 \\ y=3 \end{matrix}$

18. A alternativa falsa é c); Verdade: adição de matrizes de mesma ordem.

19. a_{51} está na linha 5; a_{15} está na primeira linha.

20. A) $M^{-1}.M=I$ e o detM diferente de zero.

21. $A.B = B$ é porque a matriz A é o elemento neutro da multiplicação de matriz, A=I.

22. $M + N = \begin{bmatrix} 5 & 0 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 1 \end{bmatrix}$

23. $N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow N^t = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

24. $M - N = \begin{bmatrix} 5 & 0 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -3 & 1 \end{bmatrix}$

25. $M = \begin{bmatrix} 5 & 0 \\ -2 & 1 \end{bmatrix} \rightarrow 3.M = \begin{bmatrix} 15 & 0 \\ -6 & 3 \end{bmatrix}$

26. $M.N = \begin{bmatrix} 5 & 0 \\ -2 & 1 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix}$

27. $A_{3x3} \;\; e \;\; \det A = \Delta; \;\; B = 2.A \rightarrow \det B = \det(2.A) = \rightarrow \det B = 2^3 \det A = 8.\Delta$

28. $\det \begin{bmatrix} a & b \\ b & a \end{bmatrix} = a^2 - b^2$

29. $\begin{vmatrix} 3x-1 & 2x+3 \\ -1 & 2 \end{vmatrix} = \begin{vmatrix} x-1 & -x+1 \\ -2 & 2 \end{vmatrix} \rightarrow 6x-2+2x+3 = 2x-2-2x+2$

$8x+1=0 \rightarrow \; x = -\dfrac{1}{8}$

30. $\begin{bmatrix} 2x & -3 \\ x+1 & x \end{bmatrix} = 5 \rightarrow \; 2x^2+3x+3=5 \rightarrow \; 2x^2+3x-2$

$\Delta = 9+16 = 25; \;\; x = \dfrac{-3 \pm 5}{4} = \begin{cases} x = \dfrac{-3+5}{4} \\ x = \dfrac{-3-5}{4} \end{cases}$

31. $\begin{bmatrix} x & y \\ -2 & 1 \end{bmatrix} = 8 \quad e \quad \begin{bmatrix} x & \frac{y}{2} \\ 4 & 3 \end{bmatrix} = 0 \rightarrow \begin{cases} x+2y=8 \\ 3x-2y=0 \end{cases} \rightarrow \begin{matrix} 1 & 2 & 8 \\ 3 & -2 & 0 \end{matrix} \rightarrow \begin{matrix} 1 & 2 & 8 \\ 0 & -8 & -24 \end{matrix} \rightarrow \begin{matrix} x=2 \\ y=3 \end{matrix}$

32. $\begin{vmatrix} \cos a & -sena \\ sena & \cos a \end{vmatrix} = \cos a.\cos a - (-sena).sena = \cos^2 a + sen^2 a = 1$

$\begin{vmatrix} x+1 & 1 & 3 \\ -3 & 2x & -x \\ 4 & -2 & 2 \end{vmatrix} = 0 \rightarrow 4x.(x+1) - 4x + 18 - 24 - 2x.(x+1) + 6 = 0$

33. / $(x+1).(4x-2x) - 4x = 0 \rightarrow 2x.(x+1) - 4x = 0 \rightarrow 2x^2 - 2x = 0 \rightarrow 2x.(x-1)$

$x = 0; \quad x = 1$

34. Podemos observar que uma matriz é a transposta da outra de detA=detA^t. resp c)

35. $\begin{vmatrix} 2 & -3 & 1 \\ x & 2 & 3 \\ 1 & -1 & 4 \end{vmatrix} = 16 - 9 - x - (2 - 6 - 12x) = 0 \rightarrow 7 - x + 4 + 12x = 0 \rightarrow -11x + 11 = 0$

36. Para o produto de Matrizes não vale a propriedade comutativa, resp: c)

$A = \begin{bmatrix} x & y \\ 3 & 3 \end{bmatrix} \quad e \quad B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$

37. $A.B = \begin{bmatrix} x & y \\ 3 & 3 \end{bmatrix} . \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} x+3y & 3x+y \\ 12 & 12 \end{bmatrix}$ → A.B=B.A x=? e y=?

$B.A = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} . \begin{bmatrix} x & y \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} x+9 & y+9 \\ 3x+3 & 3y+3 \end{bmatrix}$

$3x + 3 = 12 \rightarrow x = 3$

38. $\begin{matrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{matrix} \rightarrow \begin{matrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{matrix} = 1.(3-1) = 2$

39. $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow a_{22} = 1 \quad e \quad i = j \leq 3$

40.

1. Verdadeiro; $A^{-1} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \rightarrow \det A = 5x2 - 3x3 = 1 \rightarrow A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$

2. Falso;

3. Falso; $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} \rightarrow A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} . \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 21 \\ 7 & 28 \end{bmatrix}$ resp: c)

41. Uma matriz é inversível se o determinante for diferente de zero:

$$\det\begin{bmatrix} 1 & m \\ m & 2 \end{bmatrix} = 2 - m^2 = 0 \rightarrow m^2 = 2 \rightarrow m = \pm\sqrt{2}$$

42. Uma matriz é inversível se o determinante for diferente de zero:

$$\det\begin{bmatrix} 1 & 0 & -1 \\ k & 1 & 3 \\ 1 & k & 3 \end{bmatrix} = 3 + 0 - k^2 - (-1 + 0 + 3k) = 0 \rightarrow 3 - k^2 + 1 - 3k = 0 \rightarrow k^2 + 3k - 4 = 0$$

$$\Delta = 9 + 16 = 25 \rightarrow k = \frac{-3 \pm 5}{2} = \begin{cases} k = \dfrac{-3+5}{2} \\ k = \dfrac{-3-5}{2} \end{cases}$$

43. $\det\begin{bmatrix} x & y-x \\ y & x-y \end{bmatrix} = x.(x-y) - y.(y-x) = x.(x-y) + y.(x-y) = (x-y).(x+y)$

44. $\dfrac{y}{x} = \dfrac{\det\begin{bmatrix} -2a & 2c \\ -3b & 3d \end{bmatrix}}{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}} == \dfrac{-6ad + 6bc}{ad - bc} = \dfrac{-6.(ad - bc)}{ad - bc} = -\dfrac{6}{1}$

45. $\begin{bmatrix} x & 1 \\ 0 & x \end{bmatrix} + \begin{bmatrix} 0 & y \\ y & 1 \end{bmatrix} = \begin{bmatrix} x & 1+y \\ y & x+1 \end{bmatrix}$

46.

$$\begin{vmatrix} x & -1 & 3 \\ -4 & x & 5 \\ 6 & -3 & 7 \end{vmatrix} = 7x^2 - 30 + 36 - 18x + 15x - 28 = 0 \rightarrow 7x^2 - 3x - 22 = 0$$

$$\Delta = 9 + 616 = 625 \rightarrow x = \frac{3 \pm 25}{14} = \begin{cases} x = \dfrac{3+25}{14} \\ x = \dfrac{3-25}{14} \end{cases}$$

47.

$$\begin{vmatrix} x & 0 & 0 \\ 0 & a & x \\ 0 & 1 & 1 \end{vmatrix} = 0 \rightarrow a.x - x^2 = 0 \rightarrow -x.(x-a) = 0 \rightarrow x^2 - ax = 0 \rightarrow \Delta = a^2 - 4 > 0$$

$$(a-2).(a+2)0 \rightarrow (-\infty, -2) \cup (2, +\infty)$$

48. $\begin{vmatrix} 1 & -3 & 1 \\ 2 & -5 & -3 \\ -1 & 1 & -2 \end{vmatrix} = 10 - 9 + 2 - (5 - 3 + 6) = 3 - 8 = -5 \neq 0 \rightarrow resp: c)$

49. $\begin{vmatrix} m & 4 \\ 1 & m \end{vmatrix} = m^2 - 4 = 0 \to m = \pm 2$

Para m=2 → $\begin{matrix} 2 & 4 & 1 \\ 1 & 2 & 7 \end{matrix} \to \begin{matrix} 1 & 2 & 7 \\ 2 & 4 & 1 \end{matrix} \to \begin{matrix} 1 & 2 & 7 \\ 0 & 0 & -13 \end{matrix} \to y = \frac{-13}{0}$

Para m= -2 → $\begin{vmatrix} -2 & 4 & 1 \\ 1 & -2 & 7 \end{vmatrix} \to \begin{vmatrix} 1 & -2 & 7 \\ -2 & 4 & 1 \end{vmatrix} \to \begin{vmatrix} 1 & -2 & 7 \\ 0 & 0 & 15 \end{vmatrix} \to y = \frac{15}{0}$ Resp: a)

50. $\begin{cases} bx - y = b \\ x + ay = a \end{cases}$ →determinante da matriz incompleta: $\begin{vmatrix} b & -1 \\ 1 & a \end{vmatrix} = ab + 1$

$$x = \frac{0}{0} = \frac{\begin{vmatrix} b & -1 \\ a & a \end{vmatrix}}{ab+1} = \frac{ab-a}{ab+1} = \frac{a.(b-1)}{ab+1}$$
$$a.(a-1) = 0 \to a = 0 \ ou \ a = 1$$
$$ab + 1 = 0 \to ab = -1 \to b = -1 \ e \ a = 1$$

51. $\begin{cases} bx - y = b \\ x + ay = a \end{cases}$ e $x = \frac{\Delta x \neq 0}{0} = \frac{\begin{vmatrix} b & -1 \\ a & a \end{vmatrix}}{ab+1} = \frac{ab-a}{ab+1} = \frac{a.(b-1)}{ab+1}$ resp: a)

$a.(a-1) \neq 0 \to a \neq 0 \ ou \ a \neq 1$

$$\begin{vmatrix} 3 & 2 & 1 \\ 5 & a & 5 \\ 1 & -1 & 2 \end{vmatrix} = 6a + 10 - 5 - (a - 15 + 20) = 0 \to 6a + 5 - a - 5 = 0$$

52. $5a = 0 \to a = 0$

$$x = \frac{0}{0} = \frac{\Delta x}{\Delta} = \frac{\begin{vmatrix} 4 & 2 & 1 \\ 8 & a & 5 \\ 2 & -1 & 2 \end{vmatrix}}{0} = \frac{8a + 20 - 8 - 2a + 20 - 32}{0} = \frac{6a}{0} \to a = 0$$

53. $\begin{vmatrix} 2 & 3 \\ 4 & a \end{vmatrix} = 2a - 12$; para a=6 → $\begin{matrix} 2 & 3 & 1 \\ 4 & 6 & 5 \end{matrix} \to \begin{matrix} 2 & 3 & 1 \\ 0 & 0 & 3 \end{matrix} \to y = \frac{3}{0}$,

$Para \ 2a - 12 \neq 0 \to a \neq 6 \to sistema \ SPD$

sistema impossível.

54. $\begin{vmatrix} -3 & k \\ -k & 12 \end{vmatrix} = -36 + k^2 \neq 0 \to k^2 \neq 36 \to x \neq \pm 6$ -> d)

55. $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & -2 \\ 2 & 3 & 0 & -1 \end{vmatrix} = \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & -2 \\ 0 & 1 & 0 & -1 \end{matrix} \to \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & -1 \end{matrix} \to 3.y = 0 \to \begin{matrix} x = 1 \\ z = 0 \\ y = -1 \end{matrix}$ resp. a)

56. Cálculo do determinante da matriz incompleta:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -3 & -1 \\ 1 & -4 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -5 & -3 \\ 0 & -5 & -1 \end{vmatrix} = 5 - 15 = -10$$

Cálculo de x pela regra de Cramer: $x = \dfrac{\begin{vmatrix} 10 & 1 & 1 \\ -3 & -3 & -1 \\ 0 & -4 & 0 \end{vmatrix}}{-10} = \dfrac{}{-10}$ resp: 2,8

57. $\begin{vmatrix} 1 & k & k+1 \\ 1 & 1 & 2 \\ 2 & k & 1 \end{vmatrix} = 1 + 4k + k.(k+1) - (2.(k+1) + 2k - k) \neq 0$ resp: e)

58. $A = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix}$ e $B = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} . \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + 2y \\ 2x - 3y \end{pmatrix}$

(1) → falso; B é uma matriz coluna.

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(2) → falso; B é uma matriz de ordem 2x1

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(4) → Verdadeiro; A_{2x2} e B_{2x1}

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(8) → Verdadeiro; $A^t = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix}$ =A isso é a definição de matriz simétrica.

$A+A^t = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 4 & -6 \end{pmatrix}$

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(16) → Verdadeiro; $A^2 = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} . \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 13 & 0 \\ 0 & 13 \end{pmatrix} = 13. \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A.X = B \rightarrow A^{-1}.A.X = A^{-1}.B \rightarrow X = A^{-1}.B$$

(32) Verdadeiro→

$$A = \begin{pmatrix} 3 & 2 \\ 2 & -3 \end{pmatrix} \quad \det A = -9 - 4 = -13 \quad \rightarrow \quad A^{-1} = \begin{pmatrix} \frac{3}{13} & \frac{2}{13} \\ \frac{2}{13} & -\frac{3}{13} \end{pmatrix}$$

$$\frac{3}{13}x + \frac{2}{13}y = 10, \quad \frac{2}{13}x - \frac{3}{13}y = 24$$

$$X = A^{-1}.B = \begin{pmatrix} \frac{3}{13} & \frac{2}{13} \\ \frac{2}{13} & -\frac{3}{13} \end{pmatrix} . \begin{pmatrix} 10 \\ 24 \end{pmatrix} = \begin{pmatrix} \frac{30}{13} + \frac{48}{13} \\ \frac{20}{13} - \frac{72}{13} \end{pmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix} \rightarrow$$