An Optimal and Programmable Control Strategy for Flexible and Standard Active Filtering under Non-Sinusoidal Line Voltages

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Abstract:

This paper describes the concept of load compensation under distorted voltages conditions. At these conditions unity power factor achievement requires the compensated loads to have a current set like the voltages and so it will be non sinusoidal if the voltages are distorted. Conversely, the perfect compensation of current harmonics will result in a power factor lower than unity. Both the harmonics and power factor compensation are of the well known and very important concepts. So, this paper introduces several new compensation strategies, which can compromise between power factor and current harmonics. Following these strategies a generalized, optimal and flexible control strategy (OFC) for harmonic compensation of utility lines using Active Power Filter (APF) systems is proposed that can realize a wide range of suitable compensation strategies. One of the major contributions of this paper is developing the required structure and control algorithm of the needed control system. The control strategy is based on the new compensation concept for power quality improvement under non-sinusoidal line voltage situations. It provides a unified compensation framework and has the ability of programming for perfect current harmonics compensation, or Unity Power Factor (UPF) accomplishment, or other newly defined strategies. One of the defined suitable strategies has the ability of maximizing the power-factor subject to some adjustable constraints on the level of current harmonics and unbalancing via an on-line optimization algorithm to fulfill the IEEE-519 or other desired standards. The strategy guarantees the best achievable power factor and minimum required rating for the compensator. Flexibility of the control strategy has been proved mathematically and verified using extensive simulation results.

2. INTRODUCTION

Widespread use of power electronic systems has increased the harmonics in the utility power networks over the past three decades. Active power filters were developed for harmonic compensation and power factor correction. Sasaki and Machida introduced the fundamental concept of active filters in 1971 [1], while Gyugyi and Strycula performed the practical implementation using PWM inverters in 1976 [2]. Fig. 1 shows the block diagram of a parallel active filter. In the detector block, current harmonics and other undesirable components such as sub-harmonics, reactive power, and unbalanced currents are detected, which are to be compensated by the inverter. In the active filter, the compensation strategy is quite important, and various strategies have been proposed to improve the performance of active filters [3-12]. Generally, compensation strategies can be classified into two main categories [8]:

i. Techniques based on definitions of power quantities such as the p-q theory developed by Akagi et. al. [3] and definitions of Czarnecki [6].
ii. Techniques based on processing the load current by signal processing tools and filters to recognize the harmonics [9,10].

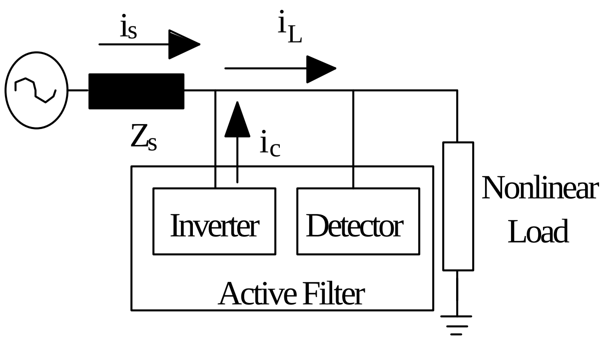


Fig. 1. Block Diagram of a parallel active filter.

The compensation strategy must fulfill the theoretical requirements and to be practically feasible. It is quite important to detect all the undesired components of the load currents. The decision about which components are undesirable is made based on the compensation strategy, which has been of interest for several decades, especially where the voltages are non-sinusoidal [13-15]. At present, the classical p-q theory developed by Akagi et al [3] is the state of art and is widely used [11,12]. In a modified version of the classical approach, the current in the supply line is controlled such that it provides UPF [4]. In fact, in the UPF approach, the load is compensated such that it is seen as a constant and symmetrical resistive load from the source. If the source voltages are sinusoidal and symmetrical, both previous methods are equivalent. In other words, it is possible to compensate the fundamental reactive power, harmonics, sub-harmonics, and improve the power factor simultaneously.

A couple of difficulties will arise when the voltages are non-sinusoidal. In the classical approach, by adjusting the instantaneous real power to a constant value, neither complete harmonic compensation nor UPF [4] can be provided. Also, in the UPF approach, total harmonic compensation is not achievable. In other words, regardless of the technique used, all the present compensation strategies seem to be not suitable if the voltages are non-sinusoidal. In these situations, perfect compensation of the harmonics and reactive power does not mean UPF accomplishment. Therefore, in this case there are two choices to make:

i. Unity power factor operation regardless of the amount of current harmonics.

So, this strategy does not guarantee perfect compensation of current harmonics.

ii. Perfec harmonics and sub-harmonics compensation.

So, this strategy does not guarantee the unity power factor.

These dificulties show that the above strategies are not suitable and in the case of distorted voltages it is necessary to define an alternative compensation strategy with the ability and flexibility of considering all power quality requirements such as the load voltage, current harmonics, unbalancing, and power factor. Such a flexible compensation strategy will be suitable for compensating the loads with or without considering the configuration of the power network and other loads [15]. The main goal of this paper is to develop such a compensation strategy with its required control system.

The structure of this paper is as follows: A brief review of the classical p-q based and UPF strategies is presented in section II. In section III, the concept of the indirect method and Perfect Harmonic Compensation (PHC) strategy and their features are presented. New compensation aim and strategies with considering the power quality requirements are discussed in section IV. Based on the idea of PHC, a unified compensation framework that can unify the compensation strategies is introduced in section V. A new optimal and flexible control strategy (OFC) for active filtering which can optimize the power factor subject to some constraints such as total harmonic distortion (THD) of currents and/or individual harmonics is proposed in this section. Also section V investigates the Flexibility characteristics of OFC system by a suitable mathematical theorem. The proposed strategy can be used in both the well-known parallel active filters and the recently developed Universal Power Line Conditioners (UPLC) [7]. Simulation results and discussions are presented in section VI. The conclusion of the paper appears in section VII.

II. P-Q BASED AND UPF STRATEGIES

developing a new structure for compensation.

I. Classical p-q Compensation Strategy

Any set of currents i_a , i_b , i_c , and voltages e_a , e_b , e_c in a three-phase four-wire system can be transformed to three-phase orthogonal (α-β-0) coordinates as follows:

$$\begin{pmatrix} f_0 \\ f_a \\ f_b \end{pmatrix} = C.\begin{pmatrix} f_a \\ f_b \\ f_c \end{pmatrix} \quad ; \qquad C = \sqrt{\frac{2}{3}}.\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{1}$$

where f can be voltage, e, or current, i. The instantaneous real power p, the instantaneous imaginary power q, and the instantaneous zero sequence power p_o as defined in [3], are given in the α-β-0 frame of reference as

$$\begin{pmatrix} p_0 \\ p \\ q \end{pmatrix} = \begin{pmatrix} e_0 & 0 & 0 \\ 0 & e_a & e_b \\ 0 & -e_b & e_a \end{pmatrix}\begin{pmatrix} i_0 \\ i_a \\ i_b \end{pmatrix} \tag{2}$$

The instantaneous power in a three-phase four-wire system is given by:

$$P = i_a e_a + i_b e_b + i_c e_c \tag{3}$$

Substituting the currents and voltages in terms of their equivalent α-β-0 variables results in:

$$P = i_a e_a + i_b e_b + i_0 e_0 = p + p_0 \tag{4}$$

Equation (4) shows that the instantaneous power is always equal to the sum of the real and zero sequence powers [3,16].

In order to define a compensation strategy using (1)-(4), each power quantity is decomposed into a dc component and an ac component as follows:

$$\begin{aligned} p_0\,(t) &= p_{0dc} + p_{0ac} \\ p\,(t) &= p_{dc} + p_{ac} \\ q\,(t) &= q_{dc} + q_{ac} \end{aligned} \tag{5}$$

where p_{dc} is the desired active power of the load. It is obvious that if the voltages are symmetrical and sinusoidal, p(t) is equal to p_{dc}. Therefore, p_{ac} is that part of p(t) which is produced by the load harmonics. Similarly, q_{dc} is equal to the value of the fundamental reactive power of the load, and q_{ac} is that part of q(t) which is caused by harmonics of the load. Compensation of the load current is achieved by reducing p_0, p_{ac} and q_{ac} to zero.The compensating currents are obtained as:

$$\begin{pmatrix} i_{0c} \\ i_{ac} \\ i_{bc} \end{pmatrix} = \begin{pmatrix} e_0 & 0 & 0 \\ 0 & e_a & e_b \\ 0 & -e_b & e_a \end{pmatrix}^{-1} \begin{pmatrix} -p_0 \\ -p_{ac} \\ -q \end{pmatrix}; \; e_0 \neq 0 \qquad (6)$$

II. UPF Compensation Strategy

In the UPF approach, the three-phase load is compensated in such a way that it behaves like a symmetrical, constant resistive load as follows [4]:

$$[i_\alpha \quad i_\beta \quad i_0] = \psi(t) \, . \, [e_\alpha \quad e_\beta \quad e_0] \qquad (7)$$

where $\psi(t)$ is the conductance of the compensated load which must be a constant for the full compensation . i_α, i_β, i_0 and e_α, e_β, e_0 are the source currents and the load voltages in the $\alpha - \beta - 0$ frame. The instantaneous power P(t) can be defined as follows:

$$P(t) = \Psi(t).e(t) \quad ; \quad e(t) = e^2{}_a + e^2{}_b + e^2{}_0 \qquad (8)$$

As mentioned before, for perfect compensation, $\psi(t)$ should be a constant value, i.e., $\psi(t) = \psi_0$, so that the average power of the load can be written as follows and then the constant ψ_0 will result where ε_{dc} is calculated by (10) :

$$P_{dc} = \frac{1}{T}\int_0^T y_0 e(t)dt = y_0 e_{dc} \quad ; \quad y_0 = \frac{P_{dc}}{e_{dc}} \qquad (9)$$

$$e_{dc} = \frac{1}{T}\int_0^T e(t)dt \qquad (10)$$

Such a ψ_0 guarantees that the compensated load will be a constant and resistive load with UPF. If the voltages are symmetrical then so are the compensated currents. But in the cases where the voltages are unsymmetrical, this approach leads to an unsymmetrical current set.

In the following example, it is shown that the p-q and UPF approaches will fail in compensating the current harmonics when the voltages are non-sinusoidal. A three-phase symmetrical and non-sinusoidal voltage set is applied to a symmetrical and resistive three-phase load. Fig. 2 shows one phase current of the load. Using the classical p-q compensation strategy for compensating the p_{ac} and q leads to three-phase compensated currents as shown in Fig. 3. It is seen that the active filter is not only unable to compensate the current harmonics but also introduces some additional harmonics. On the other hand,

from the UPF strategy point of view, there is no undesirable component in the source current, since the load is pure resistive, constant and balance and so the load power factor is unity.
Hence the UPF strategy will not take part in harmonic cancellation. Therefore, both the classical and UPF strategies are unable to recognize and compensate the harmonics.

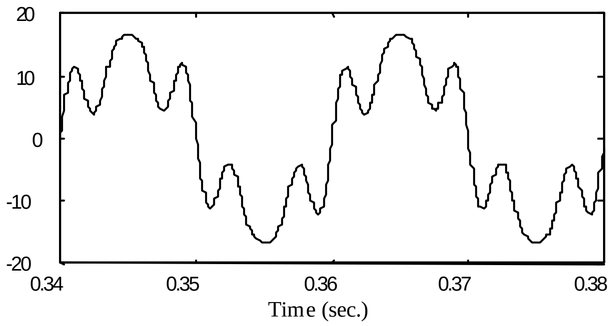


Fig. 2. Phase a load current (Amperes).

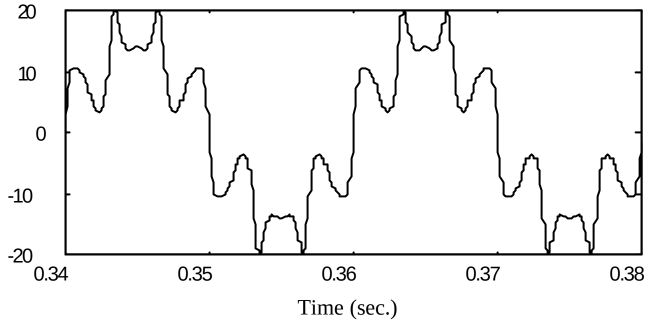


Fig. 3. Phase a current of P-Q compensated load (Amperes).

III. INDIRECT COMPENSATION STRATEGIES

I. Indirect p-q Compensation Strategy

In the case of sinusoidal voltage condition, the classical p-q method is based on determining the compensating current by calculating the undesirable components of the power quantities (p, q, p_0). Alternatively, it is possible to determine the desired current of the load by calculating the desired power quantities. For complete compensation, it is necessary to set the p to a constant and q and p_0 to zero, and then calculate the corresponding desired currents:

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \begin{pmatrix} e_0 & 0 & 0 \\ 0 & e_a & e_b \\ 0 & -e_b & e_a \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ P_{dc} = p_{dc} + p_{odc} \\ 0 \end{pmatrix} \quad (11)$$

where $_{dc}$ is the active power of the load. Equation (11) can be simplified as follows:

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \frac{P_{dc}}{e_a^2 + e_b^2} \begin{pmatrix} 0 \\ e_a \\ e_b \end{pmatrix} \qquad (12)$$

Next, the compensating currents are calculated by subtracting the desired currents from the original currents of the load as follows:

$$\begin{pmatrix} i_{ad} \\ i_{bd} \\ i_{cd} \end{pmatrix} = C^T . \begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} \quad ; \quad \begin{pmatrix} i_{ac} \\ i_{bc} \\ i_{cc} \end{pmatrix} = \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} - \begin{pmatrix} i_{ad} \\ i_{bd} \\ i_{cd} \end{pmatrix} \qquad (13)$$

The implementation of indirect method has the advantage that it is not necessary to compute q and p_0, and only the average load power is required. Therefore, this method can be implemented using simpler and lower cost micro-controllers. Regardless of practical features, the indirect method has no theoretical advantages over the classical one. But it will provide a suitable framework which can help in developing new and more powerful strategies as shown in next sections.

II. Perfect Harmonic Compensation Strategy

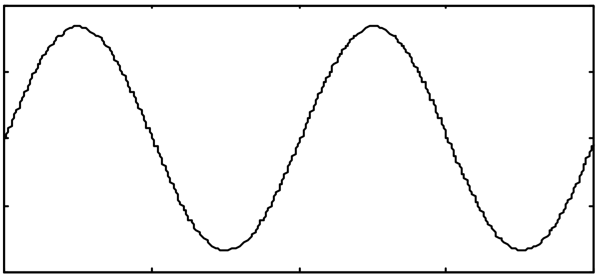
The indirect method is not able to compensate all current harmonics. But a simple modification enables it to recognize all the harmonics and sub-harmonics. In this proposed strategy (PHC) the load active power is calculated as follows:

$$P = i_a e_a + i_b e_b + i_c e_c = P_{dc} + P_{ac} \qquad (14)$$

Then, only the fundamental components of the load voltages are considered for determining the desired currents. Therefore,

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \frac{P_{dc}}{(\overline{e}_a)^2 + (\overline{e}_b)^2} \begin{pmatrix} 0 \\ \overline{e}_a \\ \overline{e}_b \end{pmatrix} \qquad (15)$$

where $\overline{e}_a$ and $\overline{e}_b$ are the fundamental components of the load voltages. These voltage components can be obtained from the original voltages by means of two simple digital band-pass filters (BPF). Also, P_{dc} is filtered from p(t) using a simple low-pass filter (LPF). The desired calculated currents obtained from this method are symmetrical and sinusoidal. In addition, the average power of the compensated load does not differ from the uncompensated condition and the average power of the active filter is equal to zero. Fig. 4 shows the resulting phase current in the compensated load while the uncompensated current has been shown in Fig. 2. It can be seen that the resulting current is a pure sinusoid.



Time (sec.)

Fig. 4. Phase a current of PHC compensated load (Amperes).

IV. NEW COMPENSATION STRATEGIES

Generally, in load compensation we deal with the following two essential topics:

1. Aims of compensation: That is, which components of the load current are undesirable and need to be compensated.
2. Technique of compensation: That is how the undesirable components should be recognized and compensated.

Obviously, the compensation technique should be developed after defining the aims of compensation. The major question, which

Research efforts have generally focused on compensation techniques with less attention being paid to the aims of compensation. Historically, active filters were developed to compensate the harmonics of loads while the voltages were considered to be sinusoidal. Where the voltages are sinusoidal, complete compensation of current harmonics will result in UPF. But with the utility line voltages becoming non-sinusoidal, one of the responsibilities of the new active filtering systems to compromise between at least power factor and current harmonics. The amount of desired current harmonics and power factor after compensation can be determined only after considering many theoretical and practical factors which depends on the load characteristics and waveforms of voltages and are not discussed in this paper. The following shows one of the several compensation strategies that can be defined that compromise between power factor and current harmonics:

Compensation Strategy:

Maximize power factor

Subject to: Total Harmonics Distortion (THD) < x

where x is the adjustable upper bound on current harmonics after compensation. In some cases, rejecting the harmonics because of their impacts on the system may be so important that the power factor can be ignored. In other conditions, a near unity power factor load which has lower line current and provides a better performance in the network [15] may be preferred. Although, the voltage compensation is very important but it should be done using other active filter topologies [7].

V. OPTIMAL AND FLEXIBLE CONTROL (OFC) STRATEGY

Many different compensation strategies can be defined and used. This paper does not want to introduce and discuss all the suitable strategies but we want to provide an active filter with very flexible control system that can realize any new compensation strategy like the strategy introduced in the previous section. In this section an adaptive control system with its control algorithm is proposed .

A. Essential

Using the indirect method, it is possible to represent the classical p-q, UPF and PHC by the following equations:

1. Indirect p-q:

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \frac{P_{dc}}{e_a^2 + e_b^2} \begin{pmatrix} 0 \\ e_a \\ e_b \end{pmatrix} \qquad (16)$$

2. PHC:

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \frac{P_{dc}}{(\overline{e}_a)^2 + (\overline{e}_b)^2} \begin{pmatrix} 0 \\ \overline{e}_a \\ \overline{e}_b \end{pmatrix} \qquad (17)$$

3. UPF:

$$\begin{pmatrix} i_{0d} \\ i_{ad} \\ i_{bd} \end{pmatrix} = \frac{P_{dc}}{(e_a^2 + e_b^2 + e_0^2)_{dc}} \begin{pmatrix} e_0 \\ e_a \\ e_b \end{pmatrix} \qquad (18)$$

Equations (16-18) show that the resulting compensated currents from all strategies with different performances have a similar structure. They can be represented by the product of a scalar, and a vector, which is a function of voltages. The above representation facilitates the development of a unified compensation framework that can cover and extend all the above-mentioned techniques. The following relation shows the new compensation structure:

$$i^* = \Psi_0^* . e^* \qquad (19)$$

ψ^*_0 is a constant and $i^* = [i^*_\alpha, i^*_\beta, i^*_0]^T$ is the desired load current vector in the α, β, and 0 coordinates means the transpose of the vector. Virtual voltage $e^* = [e^*_\alpha, e^*_\beta, e^*_0]^T$ is generally a filtered version of the load voltage $e = [e_\alpha, e_\beta, e_0]^T$ by the filtering matrix G(s) :

$$e^*(s) = G(s).e(s) \qquad (20)$$

$e^*(s)$ and e(s) are the Laplace transforms of e^* and e vectors respectively . In the new structure, the desired current vector i^* is considered to have the same shape as the filtered load voltage vector e^* instead of the exact load voltage vector e. G(s) is designed by an optimization algorithm such that the resulting e^* and i^* provide some desired power quality features like maximizing the power factor or minimizing the total harmonic distortion (THD) of the source currents. These can be taken as objective functions and can be used with an appropriate set of the following constraints.

i. Upper bound on THD of the source currents.
ii. Upper bound on harmonic factors of individual harmonics of the source currents.
iii. Upper bound on unbalancing factor of the source currents.
iv. Lower bound on power factor.

For example, the following are two suitable compensation strategies based on the above objective functions and constraints. h is the power-factor and H_a^i, H_b^i, H_c^i are the THDs of phase a, b and c currents respectively . Also, $H_{a,n}^i, H_{b,n}^i, H_{c,n}^i$ are the harmonic factors of the nth harmonic of the phase a,b and c currents respectively.

Strategy 1

Maximize h

Subject to: $H_a^i \le g_a$, $H_b^i \le g_b$, $H_c^i \le g_c$,

$$H_{a,n}^i \le l_{a,n} ,\ H_{b,n}^i \le l_{b,n} ,\ H_{c,n}^i \le l_{c,n}$$

$$\frac{i_n^0}{i_1^+} \le u_n^0 \quad \text{and} \quad \frac{i_n^-}{i_1^+} \le u_n^- \quad , \text{n=1,2,...,N}$$

where i_n^0 , i_n^- and i_n^+ are the source zero, negative and positive sequences for the n^{th} harmonic respectively and can be derived from harmonics in a-b-c phases as follows :

$$\begin{pmatrix} i_n^0 \\ i_n^+ \\ i_n^- \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & h_n & h_n^2 \\ 1 & h_n^2 & h_n \end{pmatrix} \begin{pmatrix} i_{a_n} \\ i_{b_n} \\ i_{c_n} \end{pmatrix} ; \qquad h_n = e^{n(j2p/3)} \qquad (21)$$

i_n^0/i_1^+ *and* i_n^-/i_1^+ are the measures of unbalance for the n^{th} harmonic while u_n^0 and u_n^- are adjustable bounds on them. Constants g_a, g_b, g_c and $l_{a,n}, l_{b,n}, l_{c,n}$ may be chosen according to the IEC or IEEE standards for harmonics [17]. N is the highest order considered for the voltage harmonics.

Strategy 2

Minimise $H^i = k_a H_a^i + k_b H_b^i + k_c H_c^i$

Subject to: $h \ge h_0$ (lower bound on h)

$$H_{a,n}^i \le l_{a,n} ,\ H_{b,n}^i \le l_{b,n} ,\ H_{c,n}^i \le l_{c,n}$$

$$\frac{i_n^0}{i_1^+} \le u_n^0 \quad \text{and} \quad \frac{i_n^-}{i_1^+} \le u_n^- \quad , n=1,2,...,N$$

where k_a , k_b and k_c are adjustable weights. To develop the compensation strategies , (20) can be rewritten as

$$e_a^*(s) = G_a(s) \, . \, e_a(s)$$
$$e_b^*(s) = G_b(s) \, . \, e_b(s)$$
$$e_0^*(s) = G_0(s) \, . \, e_0(s) \quad (22)$$

where G_α (s) , G_β (s) and G_0 (s) are α, β, and 0 coordinates filter banks that process the harmonic components of e_α , e_β and e_0 respectively. By the above definitions, the average power of the load is derived by (23

$$P_{dc} = (e^T.i^*)_{dc} = \Psi_0^*(e^T.e^*)_{dc} \quad (23)$$

Then the constant ψ^*_0 is calculated in such a way that it does not change the average power of the compensated load.

$$y_0^* = \frac{P_{dc}}{e_{dc}^*} \quad ; \quad e_{dc}^* = (e^T.e^*)_{dc} = (e_a e_a^* + e_b e_b^* + e_0 e_0^*)_{dc} \quad (24)$$

Fig. 5 shows the block diagram of the proposed OFC strategy while G_x (s) is a LPF which extracts the dc component of $(e^T.e^*)$ and provides $e_{dc}^* = (e^T.e^*)_{dc}$. Also, Table I shows the conditions on G_α (s) , G_β (s) , G_0(s) that reduce the OFC strategy to the UPF, PHC, and p-q strategies .

B. Power Factor Study

Generally, the power factor of a three-phase load can be represented as

$$h = P_{dc} \big/ E.I^* \quad (25)$$

where E and I^* are the effective values of the voltages and currents of the compensated load. E and I^* are defined as:

$$E = \sqrt{\frac{1}{T}\int_0^T e^T.e.dt} = \sqrt{(e^T.e)_{dc}} = \sqrt{(e_a^2 + e_b^2 + e_0^2)_{dc}} \quad (26)$$

$$I^* = \sqrt{(i^{*\,2}_{\,a} + i^{*\,2}_{\,b} + i^{*\,2}_{\,0})_{dc}} \quad (27)$$

Using (26),(27), the power factor of the UPF, PHC and OFC can be represented as follows:

$$h_{PHC} = \sqrt{\frac{(\bar{e}_a^{\,2} + \bar{e}_b^{\,2})_{dc}}{(e_a^2 + e_b^2 + e_0^2)_{dc}}} = \sqrt{\frac{(\bar{e}_a^{\,2} + \bar{e}_b^{\,2})}{(e_a^2 + e_b^2 + e_0^2)_{dc}}} \qquad (28)$$

$$h_{UPF} = \sqrt{\frac{(e_a^2 + e_b^2 + e_0^2)_{dc}}{(e_a^2 + e_b^2 + e_0^2)_{dc}}} \qquad (29)$$

$$h_{OFC} = \frac{P_{dc}}{E.I^*} = \frac{P_{dc}}{y_0^*E.E^*} = \frac{(e^T e^*)_{dc}}{E.E^*} \qquad (30)$$

Table I Special cases of OFC

Strategy	G_0 (s)	G_α (s)	G_β (s)	G_x (s)
UPF	1	1	1	LPF
PHC	0	BPF	BPF	1 or LPF
p-q	0	1	1	1

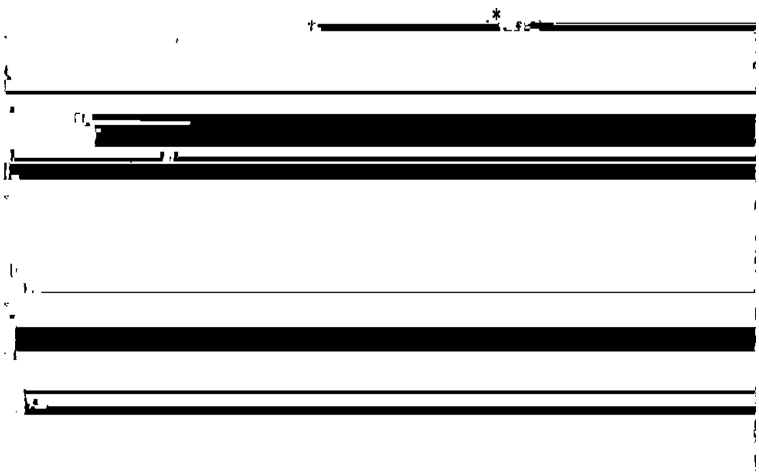


Fig. 5. Block Diagram of OFC Strategy

where E^* is the effective value of the vector e^*. It is easy to show that the power factor η will be equal to one if and only if $e=e^*$ as in the UPF approach. Comparing (28) and (29) provides the following important result: $h_{PHC} \le h_{UPF} = 1$

C. Control Algorithm

Considering the first N terms of the Fourier series for the load voltage vector e, we have

$$e_a = e_{a_0} + \sum_{i=1}^{N} e_{a_i} \cos(iwt + j_{a_i}) \quad (31)$$

$$e_b = e_{b_0} + \sum_{i=1}^{N} e_{b_i} \cos(iwt + j_{b_i}) \quad (32)$$

$$e_0 = e_{0_0} + \sum_{i=1}^{N} e_{0_i} \cos(iwt + j_{0_i}) \quad (33)$$

Hence, the Fourier series coefficients of e^* are given by:

$$e^*_{a_i} = e_{a_i}.G_a(i) \quad , \quad i = 1,2,3,\ldots,N \quad (34)$$

$$e^*_{b_i} = e_{b_i}.G_b(i) \quad , \quad i = 1,2,3,\ldots,N \quad (35)$$

$$e^*_{0_i} = e_{0_i}.G_0(i) \quad , \quad i = 1,2,3,\ldots,N \quad (36)$$

Where $G_a(i), G_b(i)$, and $G_0(i)$ are the gains of the $\alpha, \beta, 0$ coordinate filter banks for the i[th] harmonic frequency. The phase response of the filters for all harmonic frequencies is set to zero as this has no effect on the harmonic cancellation ability of the system and it may decrease the power factor. E, E^*, and ε^*_{dc} can be represented by using the Fourier series coefficients of e and filter gains $G_a(i), G_b(i)$, and $G_0(i)$ as follows:

$$E^2 = e^2_{a_0} + e^2_{b_0} + e^2_{0_0} + \frac{1}{2}\sum_{i=1}^{N}(e^2_{a_i} + e^2_{b_i} + e^2_{0_i}) \quad (37)$$

$$(E^*)^2 = e^2_{a_0} G^2_a(0) + e^2_{b_0} G^2_b(0) + e^2_{0_0} G^2_0(0) + \frac{1}{2}\sum_{i=1}^{N}(e^2_{a_i} G^2_a(i) + e^2_{b_i} G^2_b(i) + e^2_{0_i} G^2_0(i)) \quad (38)$$

$$e^*_{dc} = (e^T e^*)_{dc} = e^2_{a_0} G_a(0) + e^2_{b_0} G_b(0) + e^2_{0_0} G_0(0) + \frac{1}{2}\sum_{i=1}^{N}(e^2_{a_i} G_a(i) + e^2_{b_i} G_b(i) + e^2_{0_i} G_0(i)) \quad (39)$$

The THD of phase a current is given by:

$$H^i_a = \frac{1}{i^*_{a_1}}\sqrt{\sum_{i\neq 1}^{N}(i^*_{a_i})^2} = \frac{1}{e^*_{a_1}}\sqrt{\sum_{i\neq 1}^{N}(e^*_{a_i})^2} \quad (40)$$

and

$$\begin{pmatrix} e^*_{a_i} \\ e^*_{b_i} \\ e^*_{c_i} \end{pmatrix} = C^T . \begin{pmatrix} e^*_{0_i} \\ e^*_{a_i} \\ e^*_{b_i} \end{pmatrix} = C^T \begin{pmatrix} e_{0_i} .G_0 (i) \\ e_{a_i} .G_a (i) \\ e_{b_i} .G_b (i) \end{pmatrix} \tag{41}$$

While THDs of phases b and c are similar to (40) and are functions of $G_a(i), G_b(i),$ and $G_0(i)$. So, the desired current of the load i^* is calculated by solving the nonlinear programming problem introduced by strategy 1 or 2 for finding the optimal values of $G_a(i), G_b(i),$ and $G_0(i)$ where the power factor h is described as follows:

$$h_{OFC} = \frac{e^2_{a_0} G_a(0) + e^2_{b_0} G_b(0) + e^2_{0_0} G_0(0) + \frac{1}{2}\sum_{i=1}^{N} (e^2_{a_i} G_a(i) + e^2_{b_i} G_b(i) + e^2_{0_i} G_0(i))}{E.\sqrt{e^2_{a_0} G^2_a(0) + e^2_{b_0} G^2_b(0) + e^2_{0_0} G^2_0(0) + \frac{1}{2}\sum_{i=1}^{N} (e^2_{a_i} G^2_a(i) + e^2_{b_i} G^2_b(i) + e^2_{0_i} G^2_0(i))}} \tag{42}$$

Figure. 6 shows the flow chart of the proposed control algorithm.

D. OFC Characteristics

Equations(19-20) show the following features for the new structure:

1. The compensated load will be linear but not necessarily pure resistive and balanced.
2. The new structure provides a parameterized harmonic extraction system. It can be programmed based on any desirable current after compensation, which may be determined by any of the compensation strategies.
3. Only the harmonics, which are incorporated in the voltages, can appear in the currents after compensation.

E. OFC Applications and Features

Generally the following are the two wide areas of OFC applications:

1. OFC as a compensation tool: the designer chooses a compensation strategy like strategy 1 based on his/her own goal, and then it is realized using OFC system.
2. OFC as an analysis tool: in this application, one can examine several compensation strategies with different objective functions and constraints and then decide to choose the best strategy. However, this exactly depends on the load characteristics and the desired theoretical and practical parameters.

A highly important feature of OFC is its programming ability based on any reasonable compensation strategy. This feature is presented and proved using the following theorem.

F. OFC Flexibility Theorem

In the following it is shown that the OFC can be used for implementing any desired compensation strategy. For any desired current vector i_d after compensation which may be derived based on any compensation strategy, there exist a set of $G_\alpha(i)$, $G_\beta(i)$, $G_0(i)$ (i=1, 2 , N), and ψ^*_0 which can realize the strategy. $G_\alpha(i)$, $G_\beta(i)$, $G_0(i)$ are the $a-b-0$ axis filter bank gains for the i^{th} harmonics. The only constraint on i_d is that it can not contain any harmonic component, which does not appear in the voltages.

Moreover the harmonics of the compensated current should be in phase with their corresponding voltages harmonics. This is a very attractive and reasonable constraint since any other harmonics in the current will result in a negative impact on both power factor and THD and should be removed.

Proof :
Suppose i_d be any desired current vector after compensation with the above constraints. Based on the Fourier expansion of i_d the following relations can be derived. $i_{a_{d_i}}$ is the Fourier series coefficient of the i-th harmonics in the a-axis component of the i_c where i=1,2

$$i_{a_{d_i}} = y_0^* e_{a_i}^* = y_0^* G_a(i) e_{a_i} \qquad (43)$$

$$i_{b_{d_i}} = y_0^* e_{b_i}^* = y_0^* G_b(i) e_{b_i} \qquad (44)$$

$$i_{0_{d_i}} = y_0^* e_{0_i}^* = y_0^* G_0(i) e_{0_i} \qquad (45)$$

$$y_0^* = \frac{P_{dc}}{e_{dc}^*} \; ; \; e_{dc}^* = (e^T.e^*)_{dc} = (e_a e_a^* + e_b e_b^* + e_0 e_0^*)_{dc} \qquad (46)$$

Equations (43) to (46) show a set of 3N equations with 3N+1 unknown parameters which are the G_α(i), G_β(i), G_0(i), and ψ^* need to be found. Without loss of generality suppose $i_{a_{d_1}}$ (and so e_{a_1}) be a non-zero component then both sides of equations (43) to (45) can be divided by $i_{a_{d_1}}$. Since $i_{a_{d_i}}$, $i_{b_{d_i}}$, $i_{0_{d_i}}$ and e_{a_i}, e_{b_i}, e_{0_i} are the known parameters hence (43) to (45) reduce to the following 3N linear equations with 3N unknown parameters in terms of $G_a(i)/G_a(1)$, $G_b(i)/G_a(1)$, and $G_0(i)/G_a(1)$. Hence always there exist a set of filter banks that can be used for achieving the desired i_d .

$$\frac{i_{a_{d_i}}}{i_{a_{d_1}}} = \frac{G_a(i) e_{a_i}}{G_a(1) e_{a_1}} \; ; \qquad \frac{i_{b_{d_i}}}{i_{a_{d_1}}} = \frac{G_b(i) e_{b_i}}{G_a(1) e_{a_1}} \; ; \qquad \frac{i_{0_{d_i}}}{i_{a_{d_1}}} = \frac{G_0(i) e_{0_i}}{G_a(1) e_{a_1}} \qquad (47)$$

Then, e_{dc}^* can be calculated based on the voltages harmonics and filter bank gains using (46).

VI. SIMULATION RESULTS

The following simulation results have been provided not to show all the OFC applications. It requires a separate paper but they can show some different and interesting results which can be achieved by OFC. The results also verify the OFC flexibility. To demonstrate the ability of the proposed OFC strategy, two non-sinusoidal but symmetric voltage and current sets for a nonlinear load are considered and shown in Fig. 7. Table II shows the normalized values of the voltages and currents harmonics. The harmonics of the voltages up to the 7^{th} harmonic are considered and each filter bank is constructed by placing N simple Band Pass Filters with center frequencies f_0, $2f_0$, $3f_0$ in parallel, where f_0 is the line frequency. The bandwidths of the filters are selected as 4, 8, 12... Hz respectively. The control algorithm is simulated optimization toolbox. Optimization is started with all the filter bank gains set to one. That is, initially the algorithm is in UPF mode. Several cases of compensation are considered as follows and the results have been shown in Table III:

A. Strategy 1 :No Constraint on Individual Harmonics & Unbalancing.

Case 1: The THD upper bounds, γ are chosen so small that the control strategy results like the PHC approach.

Case 2: The γ are selected so large that the algorithm presents a behavior like the UPF approach.

Cases 3: Moderate values for γ (5%) have been selected. Fig. 8 shows the phase a current of compensated load of cases 1-3.

Case 4: The THD constraint is only for the current of phase a and so only the current harmonics of phase a are removed. Fig. 9 shows the currents of compensated load.

B. Strategy 1 :Constraints on Individual Harmonics.

Case 5: THD constraints on the currents are assigned to be 30% but the bounds on harmonic factors of harmonics 5 and 7 (λ_n)are set at 5%. The resulting current of the compensated load shown in Fig. 10 . Since the load voltage has no third harmonic component, it is not necessary to constrain it.

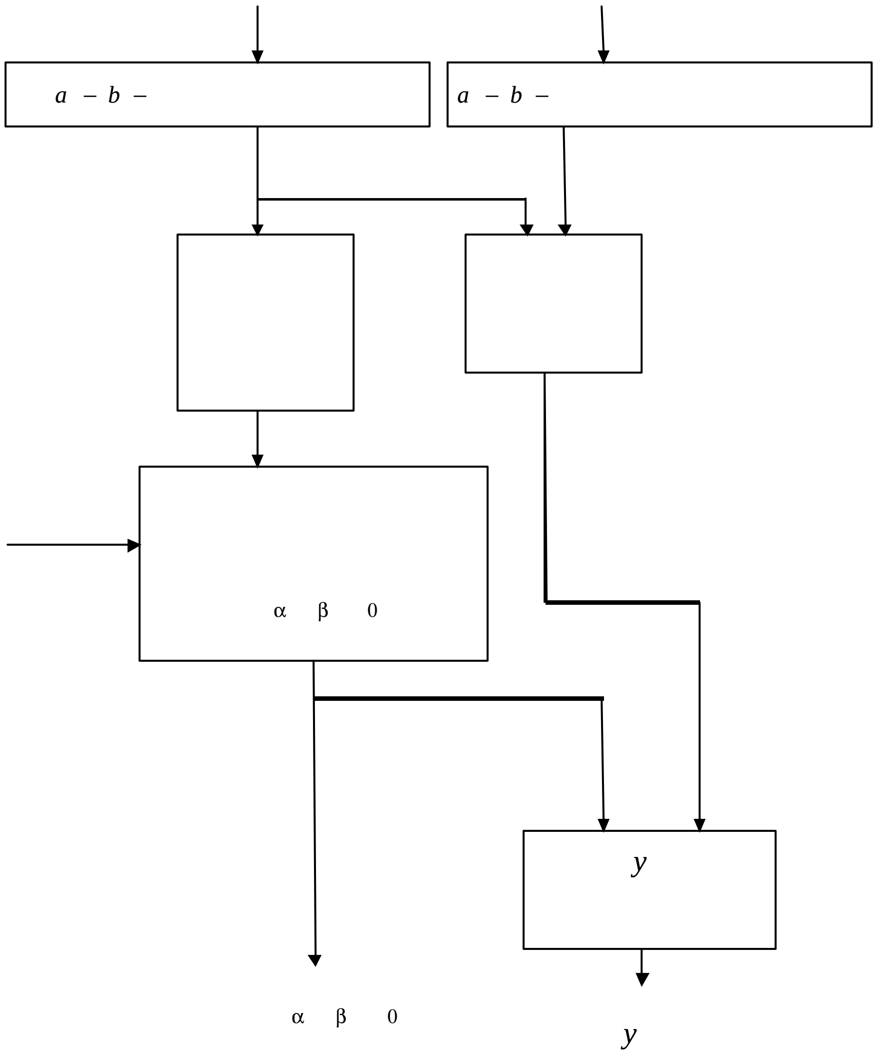


Fig. 6 :Flow chart of the proposed control algorithm.

C. Strategy 1: Asymmetric Voltages - no Constraint on Unbalancing.

Case 6: Fundamental component of phase a voltage increased by 20% and no constraints on unbalancing are considered. Hence the resulting currents of the compensated load are harmonic -free but unbalanced as shown in Fig. 11.

D. Strategy 1 :Asymmetric Voltages - Constraint on Unbalancing.

Case 7: Constraints on unbalancing are considered as shown in table III. The resulting currents are harmonic free and balanced. Fig. 12 shows the currents of compensated load. Balancing the currents leads to a lower power factor.

E. Strategy 2: Minimization of THDs

Case 8: Strategy 2 has been used to reduce the current harmonics while keeping the power factor over 0.95 and weights a,b and c in strategy 2 are chosen to be one . The original load voltages and currents in Fig. 7 have been considered. Fig. 13 shows the phase a current of compensated load. If there is no constraint on power factor, the strategy leads to PHC strategy and results in a lower power factor than 0.95.The figures do not show the transients introduced by the transient response of the filters. Fig. 14. shows the block diagramof $G_0(i)$ filter bank and other filter bancks are similar. Also, it is important to recognize that shape of the resulting currents in OFC strategy is independent of the load currents. To verify this, the following cases in which the currents are unbalance have been simulated and the results have been shown in table V.

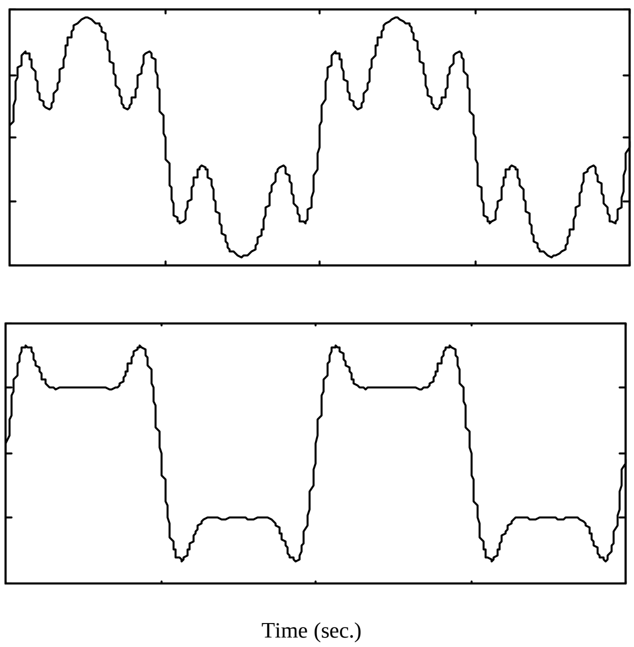


Fig. 7. Load Phase a Waveforms, Up: Voltage (Volts) Down: Current(Amperes)

Table II. Harmonics in the load voltages and currents (Normalized)

Harmonic Order	1	2	3	4	5	6	7
Load Voltage	1	0	0	0	.4	0	.2
Load Current	1	0	.5	0	.3	0	.1

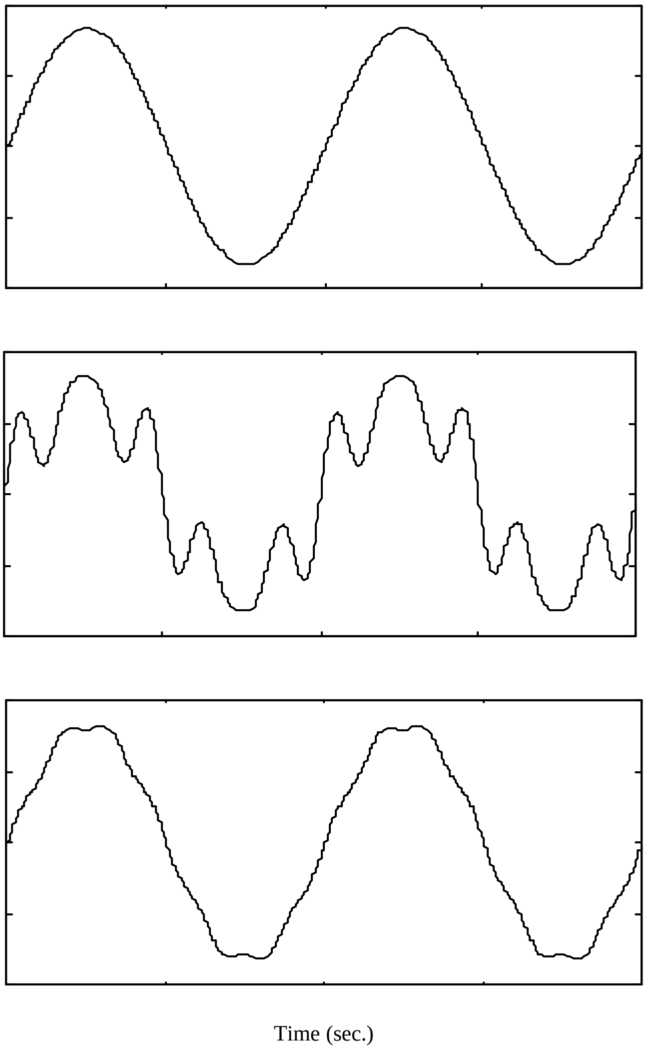
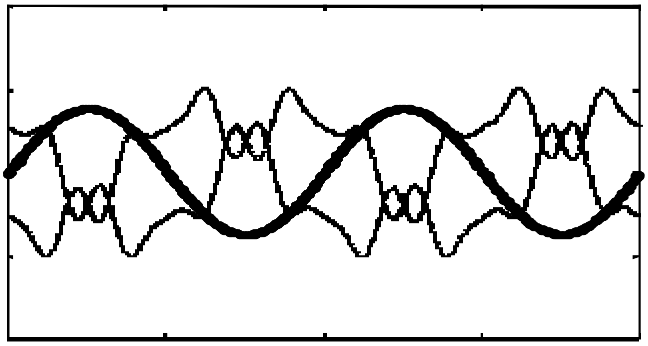
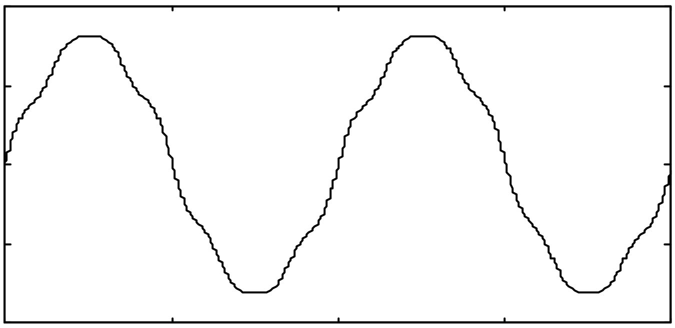


Fig. 8. Up to Down : Case 1 , Case 2 , and Case3 phase a current
Vertical Axes : Amperes



Time (sec.)

Fig. 9. case 4 : Phase a,b, and c Currents (Amperes)

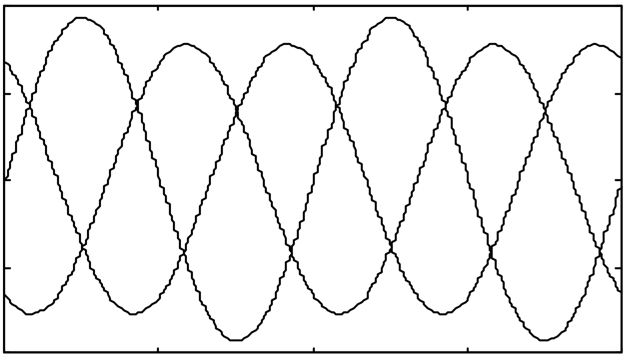


Time (sec.)

Fig. 10. case 5 : Phase a Current (Amperes)

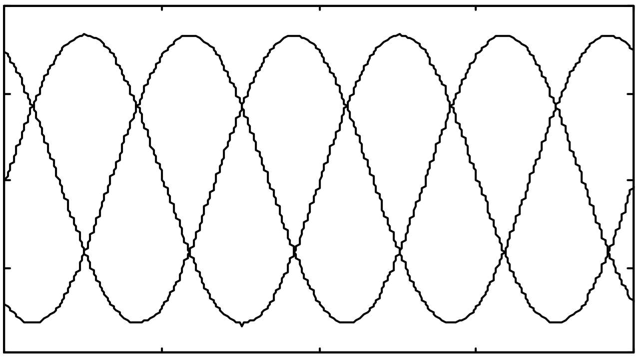
Table III. Results of OFC strategy 1 for active filtering.

	Harmonics Contraints (%)				Unbalance Contraints (%)		Power Factor
Case	γ_a	γ_b	γ_c	λ_n	u_n^0	u_n^-	h
1	0.1	0.1	0.1	---	---	---	0.9131
2	100	100	100	---	---	---	1.000
3	5	5	5	---	---	---	0.9208
4	.1	100	100	---	---	---	0.9575
5	30	30	30	5	---	---	0.9379
6	.1	.1	.1	---	---	---	0.9229
7	.1	.1	.1	---	.1	.1	0.9197



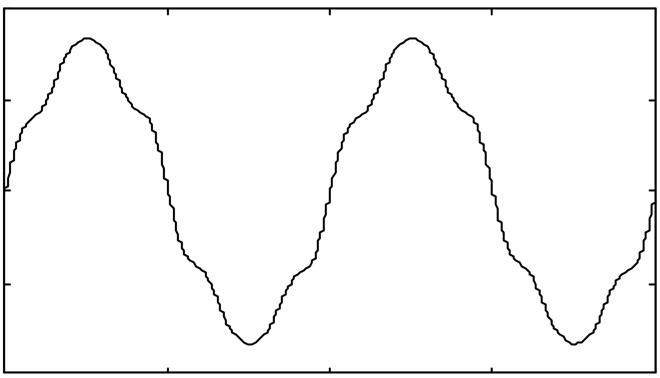
Time (sec.)

Fig.11. case 6 : Phase a,b, and c Currents(Amperes)



Time (sec.)

Fig.12. case 7 : Phase a,b, and c Currents (Amperes).



Time (sec.)

Fig. 13. case 8 : Phase a Current(Amperes)

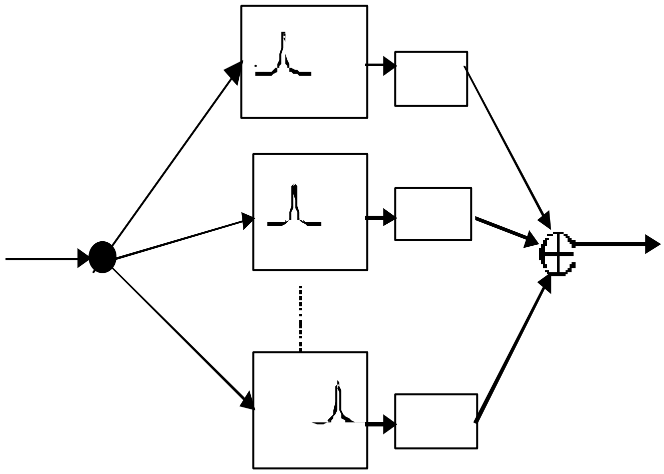


Fig. 14.Block Diagram of $G_0(i)$ Filter Bank .

F. Strategy 1: Unbalance Currents

Figures 15 and 16 show the three- phases voltages and currents of the uncompensated load while the currents are unbalanced. Table IV. shows the normalized values of the voltages and currents harmonics and as seen the voltages have both the even and odd harmonics.

Case 9: The THD upper bounds, γ are chosen so small that the control strategy results like the PHC approach. The resulting currents have been shown in Fig. 17 and as seen they are symmetric and harmonic free.

Case 10: The γ are selected so large that the algorithm presents a behavior like the UPF approach The resulting currents have been shown in Fig. 18 and as seen they are symmetric and have the shape like the voltages.

Case 11: The THD constraint is only for the current of phase a and so only the current harmonics of phase a are removed. Fig.1 9 shows the currents of compensated load.

G. Strategy 1 : Unbalance Currents: IEEE-519 Standards

Case 12: The IEEE-519 Standards for the harmonics [17] have been used . This standards on individual current harmonics and THD of Currents for 2.4 kv ~ 69 kv range of voltages have been shown in table VI . The table shows only the constraints on the odd harmonics and for the even harmonics the quantities in the table should be divided by 4. The resulting standard currents have been shown in Fig. 20. Also the resulting frequency responses of the filter banks have been shown in Fig. 21.

Table IV. Harmonics in the load voltages and currents (Normalized)

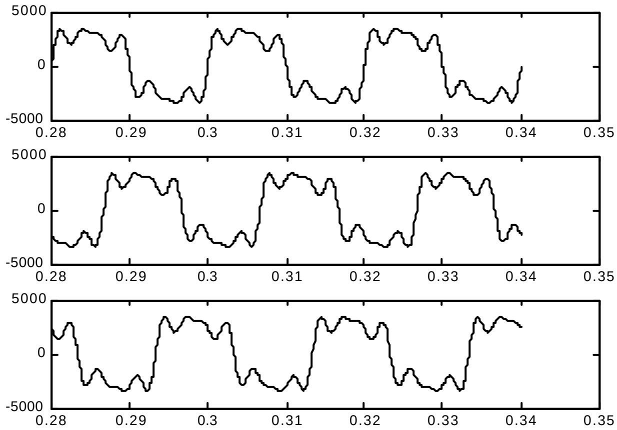
Harmonic Order	1	2	3	4	5	6	7
Voltage Phase a	1	.1	.2	0	.3	0	.2
Current Phase a	1	0	.7	0	.3	0	.1

Table V. Results of Strategy 1 for unbalance currents

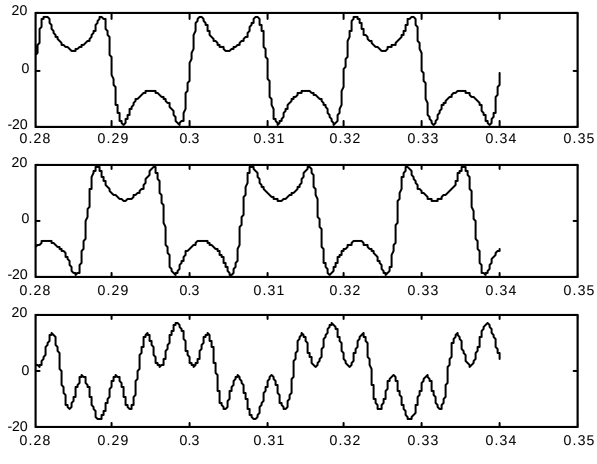
	Harmonics Constraints (%)				Power Factor
Case	γ_a	γ_b	γ_c	λ_n	h
9	0.1	0.1	0.1	---	0.9207
10	100	100	100	---	1.000
11	20	100	100	---	0.9881
12	IEEE-519	IEEE-519	IEEE-519	IEEE-519	0.9389

Table VI. IEEE-519 Standards

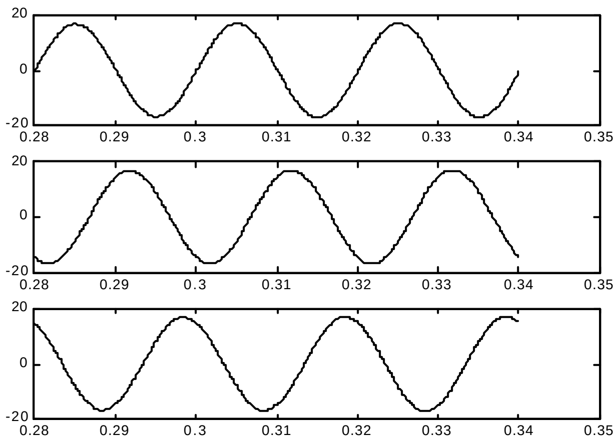
I_{SC}/I_L	$h <$	$h \geq 11$ $h < 17$	$h \geq 17$ $h < 23$	$h \geq 23$ $h < 35$	$h \geq 35$	THD (%)
<20	4	2	1.5	.6	.3	5
20-50	7	3.5	2.5	1	.5	8
50-100	10	4.5	4	1.5	.7	12
100-1000	12	5.5	5	2	1	15
>1000	15	7	6	2.5	1.4	20



Time (sec.)
Fig. 15. Phases a, b, and c, Voltages of The Load (Volts).

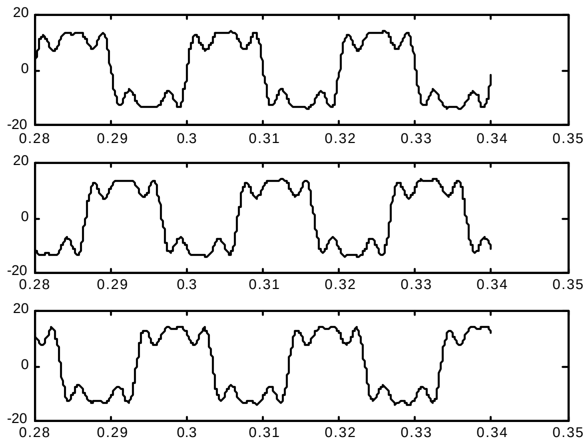


Time (sec.)
Fig. 16. Phases a, b, and c, Currents of The Load (Amperes)



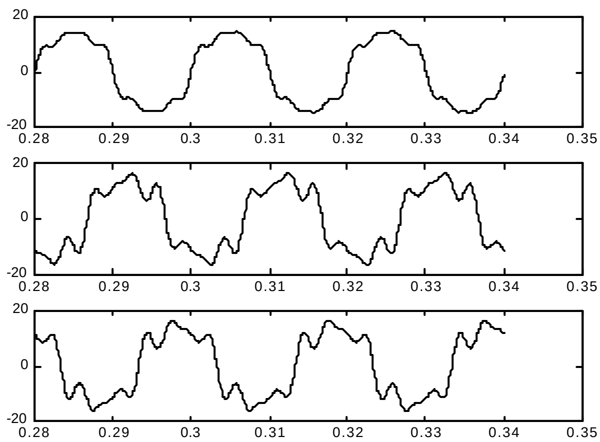
Time (sec.)

Fig. 17. Case 9 : Phases a, b, and c, Currents of The Compensated Load



Time (sec.)

Fig. 18. Case 10 : Phases a, b, and c, Currents of The Compensated Load (Amperes).



Time (sec.)

Fig. 19. Case 11 : Phases a, b, and c, Currents of The Compensated Load (Amperes).

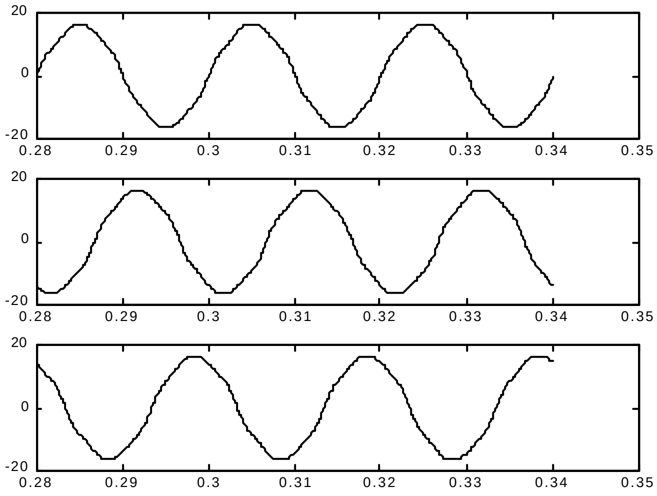


Fig. 20. Case 12 (1EEE-519): Phases a, b, and c, Currents of The Compensated Load (Amperes).

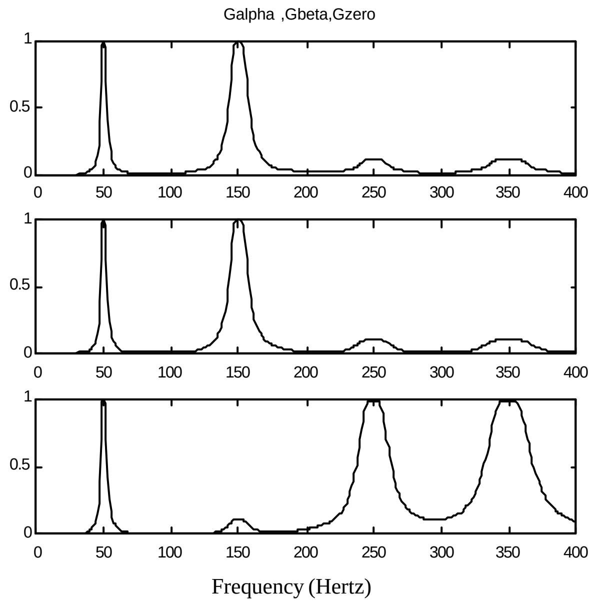


Fig. 21. Case 12 : Frequency Response of The Filter Banks .Up to down : á â 0 axes Filter Banks

VII. CONCLUSION

In this paper, an important concept regarding the current harmonics compensation under distorted voltage conditions was addressed. In these conditions the simultaneous perfect compensation of both power factor and current distortion is not possible. Hence, It was suggested to define and use a suitable strategy, which can compromise between the harmonics and power factor. A New optimal and flexible control strategy (OFC) for active filter in non-sinusoidal voltage conditions was proposed and its flexibility was proved analytically. The control algorithm was formulated by a nonlinear optimization approach. It can be programmed for different types of useful cost functions and desirable constraints, which might be selected based on the power quality requirements. As examples, two strategies were developed and validated by several simulations. The first strategy comprises the highest power factor subject to some adjustable constraints on THD, harmonic factors and current unbalances. The second strategy minimizes the THD with a constraint on the power factor. OFC is a suitable strategy for achieving the IEEE-519 [17] or other national or international standards, in which the required power factor and optimum rating of the compensator are obtained regarding the conditions of load or system under compensation. These conditions can be a wide range of theoritical, practical and economical problems. More clear applications of OFC will appear in the subsequent papers.

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