

# Verification of global optimality of the OFC active power filters by means of genetic algorithms

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*Abstract-* In this paper, global optimality of the recently developed Optimal and Flexible Control (OFC) based Active Power Filters (APFs) is investigated and verified by means of Genetic Algorithms (GA). In particular, one of the well examined and suitable compensation strategies, that is the strategy maximizing the load power factor subject to some constraints on harmonics and unbalance currents, is taken into consideration. The GA optimization method acknowledges the global optimality of the previously published results.

*Key-Words:* - Active Power Filters, Optimal Compensation, Distorted Voltages, Global Optimization, Genetic Algorithms

## Nomenclature

$\eta$ :	Power factor
N:	Highest order considered for voltage harmonics
$\psi^*$ :	A scalar for balancing active power of compensated load
$\mathbf{i}^* = [i_a^*, i_b^*, i_c^*]^T$ :	Desired load current vector
$\mathbf{e} = [e_a, e_b, e_c]^T$ :	Voltage vector of the load
$\mathbf{e}^* = [e_a^*, e_b^*, e_c^*]^T$ :	Virtual voltage vector of the load
$\mathbf{e}(s)$ :	Laplace transform of $\mathbf{e}$
$\mathbf{e}^*(s)$ :	Laplace transform of $\mathbf{e}^*$
$\mathbf{G}(s)$ :	Transfer matrix of filter bank in abc coordinates
$P_{dc}$ :	Load active power
$H_a^i, H_b^i, H_c^i$ :	THDs of phase a, b, and c currents
$H_{a,n}^i, H_{b,n}^i, H_{c,n}^i$ :	Harmonic factors of the $n^{\text{th}}$ harmonic of phase a, b, and c currents of load
$H_a^v, H_b^v, H_c^v$ :	THDs of phase a, b, and c voltages of load
$H_{a,n}^v, H_{b,n}^v, H_{c,n}^v$ :	Harmonic factors of the $n^{\text{th}}$ harmonic of phase a, b, and c voltages of load

$i_n^0, i_n^-, i_n^+$ :	Zero, negative, and positive sequences for the $n^{\text{th}}$ harmonic of load current.
$i_n^0/i_1^+$ and $i_n^-/i_1^+$ :	Measure of current unbalance for the $n^{\text{th}}$ harmonic
$u_n^0$ and $u_n^-$ :	Adjustable bounds on $i_n^0/i_1^+$ and $i_n^-/i_1^+$
$G_a(i), G_b(i), G_c(i)$ :	Scalar gains of filter banks for $i^{\text{th}}$ harmonic
$\gamma_a, \gamma_b, \gamma_c$ :	Upper bounds on THDs of phase a, b, and c currents
$\lambda_{a,n}, \lambda_{b,n}, \lambda_{c,n}$ :	Upper bounds on Harmonic Factors of the $n^{\text{th}}$ harmonic of phase a, b, and c currents

## 1 Introduction

So far, there have been many efforts for developing compensating systems to mitigate the harmonics produced by nonlinear or non-stationary loads in the power network. One of the well-known equipments used to compensate harmonics is Active Power Filter (APF), which has found much attention in the literature in the past three decades [1]-[3]. Recently, some optimization based APF

systems have introduced to the field which are capable to operate under distorted voltage conditions. Ref. [1] introduced a new control system with a possibility to optimize any objective functions can be selected from a wide range of interesting functions subject to some suitable constraints. The objective and constraints can be formed based on any power quality indices such as the load power factor and current THD. The OFC based APF runs an iterative nonlinear optimization program to reach the optimal variables, that is the filter bank gains. Therefore, for any given set of objective functions and constraints, the following two important questions may arise:

- a. Is the optimization program convergent for any given initial values for the filter bank gains?
- b. Is there any guarantee for the optimization program to reach the globally optimal results? Or it may converge toward one of the local optimal points. It off course may occur whenever there is more than one optimal point in the search space.

The above questions are generally regarded as important issues in most of nonlinear optimization processes. Providing a comprehensive response for the general cases with any desired objective function/constraints is quite a difficult task. Here, we consider only one of the most suitable compensation strategies minimizing power factor subject to some constraints on the current harmonics and unbalance currents [1]-[3]. Investigations confirm that for the aforementioned strategy, the optimization program is almost always convergent for any given set of load voltage harmonics. Ref. [2] presents the results of a successful control system based on Neural Networks (NN). The optimization program should run for many different values of the load voltage harmonics to produce the required data for training the NN. The program was convergent for every given set of voltage harmonics. It also resulted in a unique optimal response for any initial values. However, there is an important issue needs more investigations about the global optimality of the results obtained. In this

system for APFs called Optimal and Flexible Control (OFC) system. OFC [1] and its variants [2,3] provide the compensating paper, the Genetic Algorithms (GAs) are used to prove the global optimality of the results. The optimization problem looks to be convex, however an analytical approach for evaluating the similar cases can be considered in the subsequent works.

## 2 Brief Review Of OFC

### A. OFC Structure

Fig. 1 shows the block diagram of the OFC system [1]. The OFC structure in a-b-c frame is characterized by following equations:

$$i^* = \Psi_0^* \cdot e^* \quad (1)$$

$$e^*(s) = G(s) \cdot e(s) \quad (2)$$

Where  $\Psi_0^*$  is a constant scalar for balancing the active power of the compensated load and voltage vector  $e^*$  is a filtered version of the load actual voltage,  $e$ . Filter bank  $G(s)$  is designed based on a selected compensation strategy through an optimization algorithm [1].

### B. Compensation Strategy

The following indicates one of the most interesting load compensation strategies that can be realized by the OFC system used in [1]-[3].

$$\text{Max } \eta(G) \quad (3)$$

Subject to:

$$H_a^i \leq \gamma_a, \quad H_b^i \leq \gamma_b, \quad H_c^i \leq \gamma_c, \\ H_{a,n}^i \leq \lambda_{a,n}, \quad H_{b,n}^i \leq \lambda_{b,n}, \quad H_{c,n}^i \leq \lambda_{c,n}, \quad \frac{i_n^-}{i_1^+} \leq u_n^-,$$

$$\frac{i_n^0}{i_1^+} \leq u_n^0, \quad n=1,2,\dots,N$$

For example, the harmonic constraints can be selected according to the IEEE standards for harmonics [5]. Moreover, any other power quality indices, e.g. THD of the compensated load currents can be considered as the cost function as well [1].

### C. Control Algorithm

Considering the first N terms of the Fourier series for  $e$  and  $e^*$ , we have:

$$e_a = e_{a_0} + \sum_{i=1}^N e_{a_i} \cos(i\omega t + \varphi_{a_i}) \quad (4)$$

$$e_{a_i}^* = e_{a_i} G_a(i) \quad , \quad i = 1,2,3,\dots,N \quad (5)$$

Similar equations can also be developed for phases b and c. The phase responses of the filters for all harmonic frequencies are set to zero. Neglecting the dc components, the effective load voltage  $E$ , the effective filtered voltage  $E^*$ , and  $\varepsilon_{dc}^* = (e^T \cdot e^*)_{dc}$  can be represented by the Fourier series coefficients of  $e$  and the filter gains as [4]:

$$E = \sqrt{\frac{1}{2} \sum_{i=1}^N (e_{a_i}^2 + e_{b_i}^2 + e_{c_i}^2)} \quad (6)$$

$$E^* = \sqrt{\frac{1}{2} \sum_{i=1}^N (e_{a_i}^2 G_a^2(i) + e_{b_i}^2 G_b^2(i) + e_{c_i}^2 G_c^2(i))} \quad (7)$$

$$\varepsilon_{dc}^* = \frac{1}{2} \sum_{i=1}^N (e_{a_i}^2 G_a(i) + e_{b_i}^2 G_b(i) + e_{c_i}^2 G_c(i)) \quad (8)$$

The desired load current ( $i^* = \Psi_0^* \cdot e^*$ ) is calculated by solving the non-linear programming problem introduced by the compensation strategy e.g. (3), to determine the optimal values of  $G_a(i)$ ,  $G_b(i)$ , and  $G_c(i)$ . Power factor  $\eta$  is calculated by (9) and is a function of the filter bank gains and the load voltage harmonics. After calculating the optimal values of the filter bank gains, parameter  $\psi_0^*$  is calculated by (10) to balance the active power of the load.

$$\eta = \frac{P_{dc}}{E \cdot I^*} = \frac{P_{dc}}{\psi_0^* \cdot E \cdot E^*} = \frac{(e^T \cdot e^*)_{dc}}{E \cdot E^*} \quad (9)$$

$$\psi_0^* = \frac{P_{dc}}{\varepsilon_{dc}^*} \quad (10)$$

### 3 Genetic Optimization

This section, considers the application of classic Genetic Algorithms [5] to calculate the optimal results of OFC systems with the compensation strategy expressed in the previous section. Due to lack of space, a review of GA is omitted; interested readers can be referred to [6,7]. GAs are well-know approaches for calculating optimal results of any unconstrained or constrained optimization problems with no need in knowing the explicit form of the objective

function or constraints. In brief, in a GA program, there are some main operators called selection, crossover, and mutation, which incorporate in producing any new generation from the current generation. The first generation is formed by a set of randomly produced strings, which are candidates for the optimal design vector. In this problem, each string consists all gains of the filter banks placed in phases a,b, and c. The specifications of the problem under study are given as follow:

1. The number of filter bank gains in each phase has been selected to be equal to 11. Hence, the total number of the unknown parameters to be calculated is equal to 33.
2. Each string consists 33 elements, which are real numbers that is we do not consider a binary GA optimization.
3. Each generation has 40 elements, and the first generation is produced randomly with uniform distribution between 0 and 1.
4. The fitness function used to evaluate the strings is the same power factor produced by each string.
5. To improve the algorithm performance, the string with the highest fitness in each generation is copied into the next generation. By this, it is guaranteed that each generation contains the best string of all of the previous generations.
6. Each generation has 40 members. Every new generation is produced based on the previous one. 75 percents of its population are produced by means of the crossover and mutation operators, acting on the strings with highest fitness. For each generation, thirty strings are randomly selected from the current generation. Then, for each pair(  $A=[a_1 a_2 \dots a_{33}]$  and  $B=[b_1 b_2 \dots b_{33}]$ ) of strings selected randomly, the aforementioned operators act on and result in two new strings. One is the average of the selected strings ( $[A+B]/2$ ), and the other is the one its elements are a random selection of the corresponding

- elements of one of the initial string A or B.
7. Random selecting within the current generation produces the rest of elements. The selected strings' members are then changed randomly. The maximum change in each element is limited to  $\pm 10\%$ . This will extend the search space of the procedure and reduces the risk of terminating at local optima.
  8. To assure about the constraints must be fulfilled, the fitness value of any string violating any constraints is multiplied by 0.6. That significantly reduces the chance of such strings in contributing to the next generation.
  9. The algorithm continues for 370 successive generations.

#### 4 Simulation Results

In this section the results obtained for two different optimization approaches are presented and compared. The two approaches are a) the classic one obtained from the MATLAB 7 optimization toolbox, and b) the genetic algorithm described in the previous section. Considering the large number of variables/ constraints, the GA based optimization is not an easy task. The GA optimization program runs for more than 2 hours while a 1700 MHz Pentium IV personal computer is used. Many different cases with different level of constraints have been evaluated, but the simulation result of only one case is reported with details. Owing to high number of design variables and constraints, the optimization is not an easy task. In this case study, the constraints on the current harmonics are based on IEEE 519 standards [4].

Figure 2 shows the filter banks frequency response obtained from classic and GA approaches respectively. It clearly shows that the two results are very close to each other. Also, Fig. 3 shows the Phase "a" waveforms of the load voltage and current before compensation together with the compensated current when the GA optimization method has been used. The results obtained from the two different optimization methods are so close that a separate drawing of the results for the classic method is not necessary. Fig. 3 can be

considered as the result of the GA method as well. Fig. 3 also indicates the corresponding current spectra before and after compensation as well. The final power factors obtained are as follow, which are almost equal:

- a) Classic optimization: 0.9313
- b) GA optimization: 0.9318

To explore more details on the results, Table 1 explores the phase "a" filter bank gains resulted from the two different methods. Table 2 also indicates the last generation's power factor, while flag "\*" beside the some of the table's entities means that the associated entity complies with the optimization constraints. Fig. 4 depicts the power factors associated with the last 35 generations. It confirms that the total number of steps equal to 370 is quite sufficient to reach the global optima with high certainty, as there are no significant changes in the strings existing in the subsequent generations in the last steps of optimization.

#### 5 Conclusions

In this paper, the global optimality of the OFC filters was taken into consideration. A genetic algorithm optimization program was developed to verify the optimality of the previously reported OFC system. Through a case study as a one of the most suitable compensation strategies, it was shown that the results of the genetic algorithm are quite close to those obtained from the classic optimization method by means of MATLAB 7 optimization toolbox. The results validate the optimality of the classic approach when the compensation strategy with maximum power factor is selected. Analytically investigating the optimization problem would be an interesting topic for the future researches.

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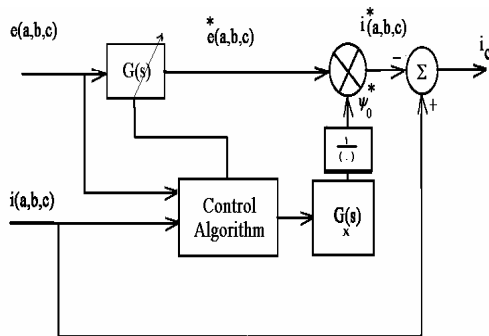


Fig. 1: Block diagram of the OFC active filter

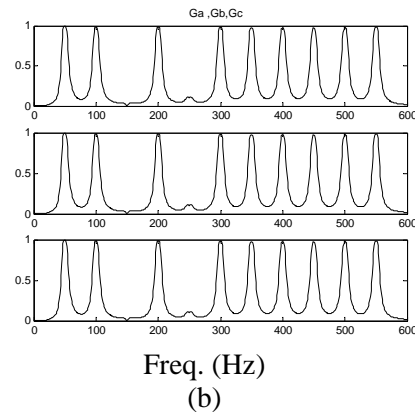
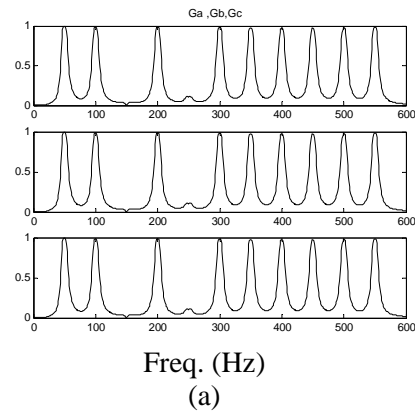


Fig.2: Frequency response of each filter bank.  
 a) results obtained from the classic optimization  
 b) results obtained from the GA

Table 1: Optimal filter gains (phase "a") obtained for the classic and the GA optimizations

GA Optimization	Classic Optimization
1.0135	1.0000
0.9760	1.0000
1.0241	1.0000
0.0356	0.0323
1.0051	1.0000
0.1329	0.1333
0.9290	1.0000
1.0237	1.0000
0.9796	1.0000
1.0373	1.0000
0.9530	1.0000

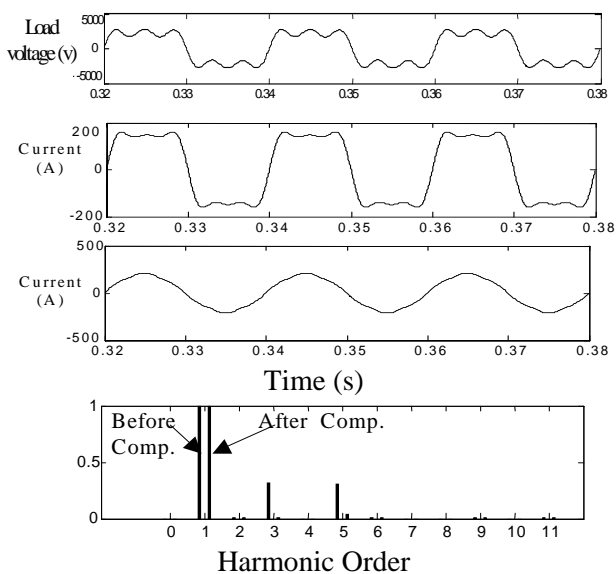


Fig.3: Phase “a” Results associated with GA optimization method.

Up to down: load voltage, current, and current after compensation, and current spectra.

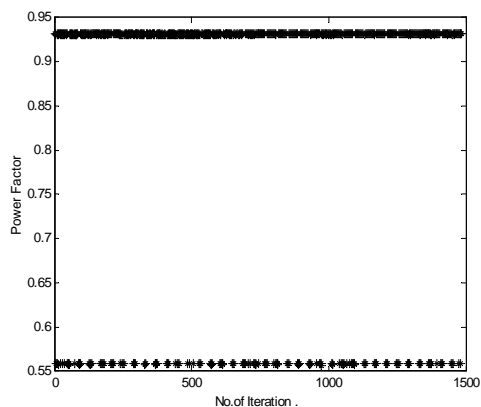


Fig. 4: Power factor associated with the strings of the last 35 generations

Table 2: Values of the fitness function for the last generation of GA.

0.9316*	0.9315*	0.9315*	0.9314*	*0.9312	
0.9315*	0.9317*	0.9317*	0.9316*	0.9316*	0.9316*
0.5590	0.9315*	0.9315*	0.9317*	0.9317*	0.9315*
0.9315*	0.9317*	0.9317*	0.9315*	0.9315*	0.9316*
0.9316*	0.9316*	0.9315*	0.9313*	0.5587*	0.9315*
0.9312*	0.9313*	0.5590	0.5588	0.5589	0.9315*
	0.9318*	0.5585	0.5590	0.5582	0.5589