Pure Math Test
Differential Calculus

1. Let $f(x) = (x+1)(x-3)^3$.
   (a) Find the domain of f.
   (b) Find any symmetry of f.
   (c) Find the x and y-intecepts of f.
   (d) Find any asymptote of f.
   (e) Find $f'(x)$ and $f''(x)$.
   (f) Find the region such that
      (i) $f'(x) > 0$,
      (ii) $f'(x) < 0$,
      (iii) $f''(x) > 0$,
      (iv) $f''(x) < 0$.
   (g) Find all the extremum of f.
   (h) Find all the inflexion points of f.
   (i) Sketch the graph of f.

2. Let $f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!}$ and $g(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}$.
   (a) Show that $f(x)$ is strictly decreasing. Hence deduce that the equation $f(x) = 0$ has only one real root.
   (b) Let α be the real root of the equation $f(x) = 0$. Show that $g(x)$ attains its absolute minimum at $x = \alpha$. Hence deduce that the equation $g(x) = 0$ has no real root.

3. Evaluate the following limits:
   (a) $\lim_{x\to 0} \frac{a^x - 1}{x}$, where $a > 0$;
   (b) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$;
   (c) $\lim_{x\to 1} x^{\frac{1}{1-x}}$.