

Let  $y = \frac{1}{x-1}$ . Then  $dy = -\frac{1}{(x-1)^2}dx$ . Also,  $x = \frac{1}{y} + 1$ .

$$\begin{aligned}\int \frac{dx}{(x-1)\sqrt{x^2-1}} &= \int \frac{-(x-1)^2 dy}{(x-1)\sqrt{x+1}\sqrt{x-1}} \\&= \int -\sqrt{\frac{x-1}{x+1}} dy \\&= -\int \sqrt{\frac{\frac{1}{y}}{\frac{1}{y}+2}} dy \\&= -\int \sqrt{\frac{1}{1+2y}} dy \\&= -\int \frac{1}{\sqrt{1+2y}} dy \\&= \frac{1}{2} \cdot 2\sqrt{1+2y} + C \\&= \sqrt{1+\frac{2}{x-1}} + C\end{aligned}$$