

1994 Paper II Question 12(a)

Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a continuously differentiable function satisfying the following conditions for all $x \in \mathbf{R}$:

- A. $f(x) > 0$;
- B. $f(x+1) = f(x)$;
- C. $f(\frac{x}{4})f(\frac{x+1}{4}) = f(x)$.

Define $g(x) = \frac{d}{dx} \ln f(x)$ for $x \in \mathbf{R}$.

a. Show that for all $x \in \mathbf{R}$,

- (a) $f'(x+1) = f'(x)$;
- (b) $g(x+1) = g(x)$;
- (c) $\frac{1}{4} (g(\frac{x}{4}) + g(\frac{x+1}{4})) = g(x)$.

(8 marks)