

1992 Paper 2 Question 7

Let f be a differentiable function such that

$$f(x + y) = f(x) + f(y) + 3xy(x + y)$$

for all $x, y \in \mathbf{R}$.

a. Show that $f'(0) = \lim_{h \rightarrow 0} \frac{f(h)}{h}$.

b. Hence, or otherwise, show that for all $x \in \mathbf{R}$,

$$f'(x) = f'(0) + 3x^2,$$

and deduce that

$$f(x) = f'(0)x + x^3.$$

(6 marks)