Part (c) was a conceptually confusing question. When $g(x)$ is defined as

$$\frac{d}{dx}\ln f(x),$$

which is also equal to

$$\frac{1}{f(x)}\frac{d}{dx}f(x),$$

does $g(\frac{x}{4}) = \frac{1}{f(\frac{x}{4})}\frac{d}{dx}f(\frac{x}{4})$? The answer is NO.

Think about $g(\frac{x}{4})$ as the function $g(x)$, computed from $\frac{d}{dx}\ln f(x)$, finished, and then plugging in $\frac{x}{4}$ for $x$. So, $\frac{x}{4}$ has NOTHING to do during the differentiation.

Perhaps, we can for sake of clarity, write $g(\frac{x}{4}) = \frac{d}{dx}\ln f(x)|_{x\to\frac{x}{4}}$, or simply as $\frac{d}{dx}\ln f(x)|_{\frac{x}{4}}$.

So, a smart kid like you may figure out that $g(\frac{x}{4})$ should be equal to

$$\begin{aligned}\frac{d}{dx}\ln f(x)|_{\frac{x}{4}} &= \frac{1}{f(x)}\frac{d}{dx}f(x)|_{\frac{x}{4}}\\ &= \frac{1}{f(\frac{x}{4})}\frac{d}{d\frac{x}{4}}f(\frac{x}{4})\end{aligned}$$

Why?

Similarly for $g(\frac{x+1}{4})$.

Then, it's straightforward to verify the required equality.

$$\begin{aligned}\frac{1}{4}(g(\frac{x}{4})+g(\frac{x+1}{4})) &= \frac{1}{4}\left(\frac{1}{f(\frac{x}{4})}\frac{d}{d\frac{x}{4}}f(\frac{x}{4}) + \frac{1}{f(\frac{x+1}{4})}\frac{d}{d\frac{x+1}{4}}f(\frac{x+1}{4}))\right)\\ &= \frac{1}{f(\frac{x}{4})}\frac{d}{dx}f(\frac{x}{4}) + \frac{1}{f(\frac{x+1}{4})}\frac{d}{dx}f(\frac{x+1}{4})\\ &= \frac{1}{f(\frac{x}{4})f(\frac{x+1}{4})}\left(f(\frac{x+1}{4})\frac{d}{dx}f(\frac{x}{4}) + f(\frac{x}{4})\frac{d}{dx}f(\frac{x+1}{4})\right)\\ &= \frac{1}{f(x)}\left((\frac{d}{dx}\left(f(\frac{x}{4})f(\frac{x+1}{4})\right)\right)\\ &= \frac{1}{f(x)}\frac{d}{dx}f(x)\\ &= g(x)\end{aligned}$$