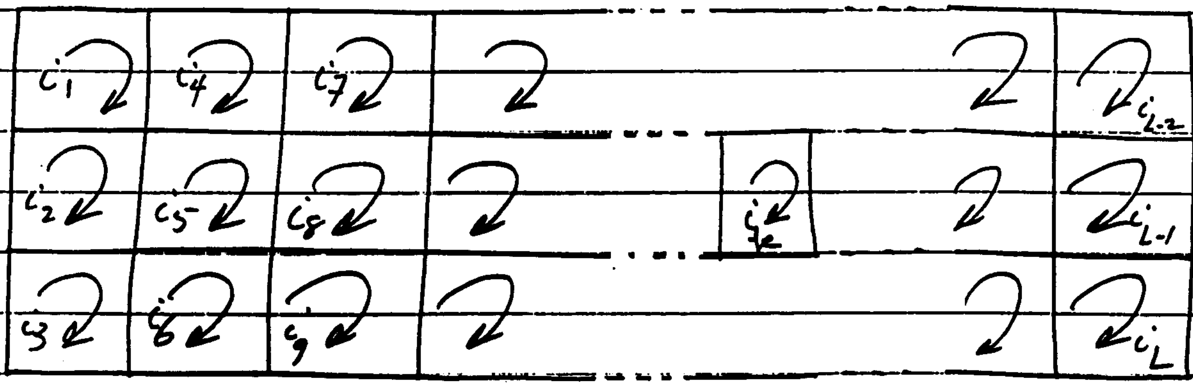
ENEL 341 Circuits I

Loop Variable Analysis

Consider a complex Network with L independent Loops, as shown in the graph below:



Each of the above L Loops may contain R, L and C in any one or all the branches that make up the loop.

Let us consider the kth Loop and define:

R_{kk} to be the total resistance in loop k ;
L_{kk} " " " " inductance " " " ;
D_{kk} " " " " elastance " " " ;

R_{kj} " " " " resistance common between loops k and j ;
L_{kj} " " " " inductance " " " " ;
D_{kj} " " " " elastance " " " " .

Note that $D_{kk} = D_1 + D_2 + \cdots = \frac{1}{C_1} + \frac{1}{C_2} \cdots = \frac{1}{C_{kk}}$

Writing KVL at loop k yields,

$$\underbrace{\left[R_{kk} + L_{kk}\frac{d}{dt} + D_{kk}\int dt\right] i_k(t)}_{\text{Voltage drop in loop } k \text{ produced by current in loop } k} - \underbrace{\sum_{\substack{j=1 \\ j\neq k}}^{L} \left[R_{kj} + L_{kj}\frac{d}{dt} + D_{kj}\int dt\right] i_j(t)}_{\text{Voltage drops in loop } k \text{ produced by currents flowing in all other neighbouring loops}} = \underbrace{\pm\, v_k}_{\text{voltage sources in loop } k}$$

(+) if v_k force current in the same direction as i_k
(−) if otherwise.

$k = 1, 2, \ldots, L$

$$\sum_{j=1}^{L} a_{kj}\, i_j(t) = v_k \qquad , k = 1, 2, \ldots, L$$

where $a_{kj} = -\left[R_{kj} + L_{kj}\frac{d}{dt} + D_{kj}\int dt\right]$

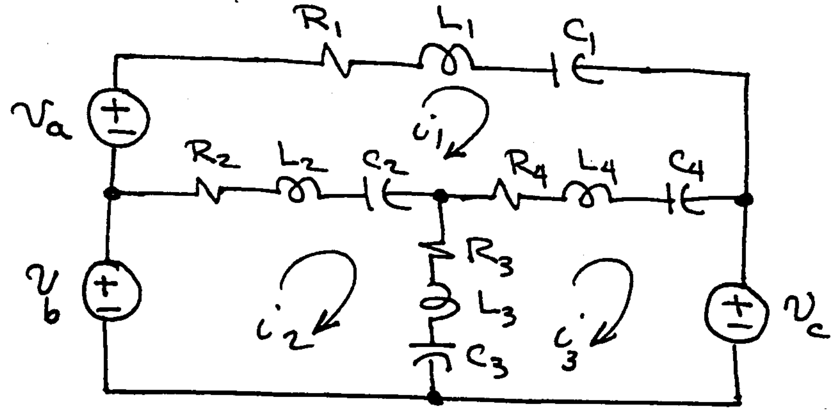
$a_{kk} = R_{kk} + L_{kk}\frac{d}{dt} + D_{kk}\int dt$

"OR"

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \\ \vdots \\ v_L \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1L} \\ a_{21} & a_{22} & \cdots & a_{2L} \\ \vdots & & \ddots & \vdots \\ a_{L1} & a_{L2} & \cdots & a_{LL} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_L \end{bmatrix}$$

"OR" $\underline{V} = \underline{A}\, \underline{i}$

Examples : (1) Passive R, L, C circuits.



$$a_{11} = (R_1+R_2+R_4) + (L_1+L_2+L_4)\frac{d}{dt} + (D_1+D_2+D_4)\int dt$$

$$a_{22} = (R_2+R_3) + (L_2+L_3)\frac{d}{dt} + (D_2+D_3)\int dt$$

$$a_{33} = (R_3+R_4) + (L_3+L_4)\frac{d}{dt} + (D_3+D_4)\int dt.$$

$$a_{12} = -R_2 - L_2\frac{d}{dt} - D_2\int dt = a_{21}$$

$$a_{13} = -R_4 - L_4\frac{d}{dt} - D_4\int dt = a_{31}$$

$$a_{23} = -R_3 - L_3\frac{d}{dt} - D_3\int dt = a_{32}$$

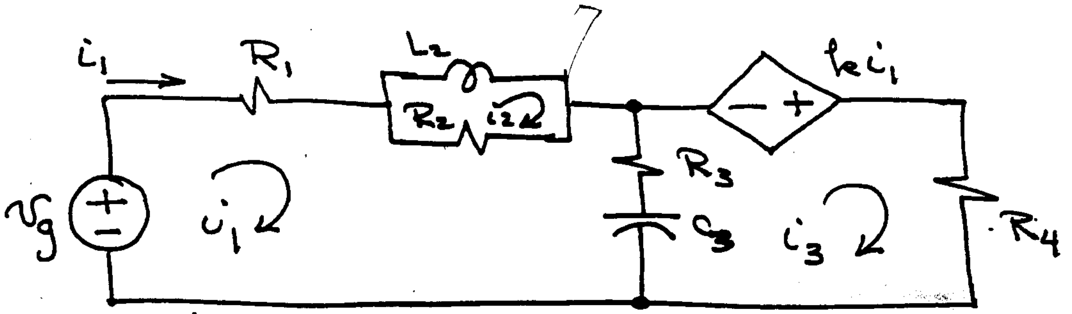
$V_1 = V_a$, $V_2 = V_b$ and $V_3 = -V_c$

Note that if the voltage source (drives) derives the current in a loop in the same direction then the plus sign used, otherwise, the (−) sign is used.

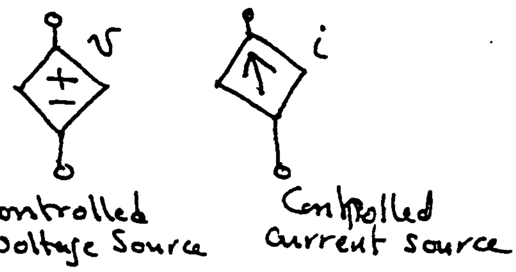
Note that in the above example A is symmetric; i.e, $a_{ij} = a_{ji}$.

Example (2): Active circuits:

Write a set of equilibrium equations on the Loop basis to describe the network in the fig. below. Note that the Net. containes one controlled source. Collect terms in your formulation so that your equations have the general form $A\underline{i} = \underline{V}$.



Controlled-sources:



$v = k i_x$ CCVS

$v = k v_x$ VCVS (Op-amp)

$i = k i_x$ CCCS Bipolar

$i = k v_x$ VCCS Fet

(a) diamond shape

(b) don't physically exist.

$$a_{11} = (R_1 + R_2 + R_3) + D_3 \int dt$$

$$a_{12} = -R_2 = a_{21}$$

$$a_{13} = -R_3 - D_3 \int dt = a_{31}$$

$$a_{22} = R_2 + L_2 \frac{d}{dt}$$

$$a_{23} = 0 = a_{32}$$

$$a_{33} = (R_3 + R_4) + D_3 \int dt$$

$$v_1 = v_g$$

$$v_2 = 0$$

$$v_3 = k i_1$$

$$a_{11} i_1 + a_{12} i_2 + a_{13} i_3 = v_1$$
$$a_{21} i_1 + a_{22} i_2 + a_{23} i_3 = v_2$$
$$a_{31} i_1 + a_{32} i_2 + a_{33} i_3 = v_3$$

In matrix form:

$$\begin{bmatrix} R_1+R_2+R_3+\frac{1}{C_3}\int dt & -R_2 & -R_3-\frac{1}{C_3}\int dt \\ -R_2 & R_2+L_1\frac{d}{dt} & 0 \\ -R_3-\frac{1}{C_3}\int dt & 0 & R_3+R_4+\frac{1}{C_3}\int dt \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_a \\ 0 \\ k i_1 \end{bmatrix}$$

"OR"

$$\begin{bmatrix} R_1+R_2+R_3+\frac{1}{C_3}\int dt & -R_2 & -R_3-\frac{1}{C_3}\int dt \\ -R_2 & R_2+L_1\frac{d}{dt} & 0 \\ -k-R_3-\frac{1}{C_3}\int dt & 0 & R_3+R_4+\frac{1}{C_3}\int dt \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_a \\ 0 \\ 0 \end{bmatrix}$$

$$A i = v$$

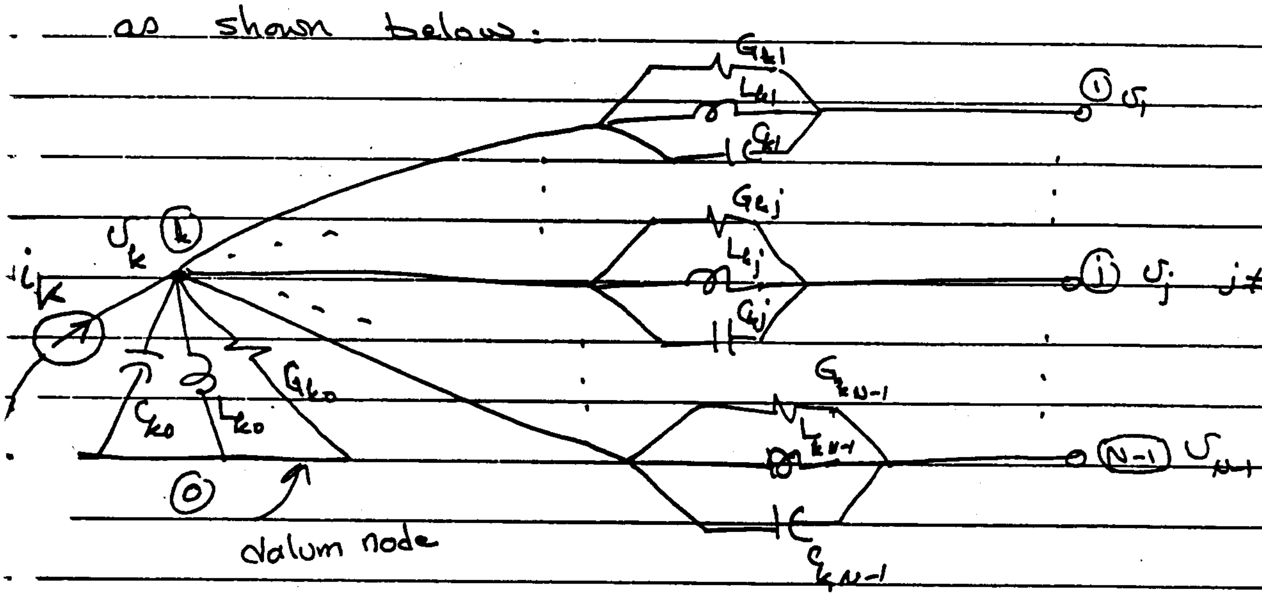
Note that A is not symmetric.

<u>Comments</u>:

In Loop analysis all current sources in loops should be converted to voltage sources. In this case the variables will be the loop's currents.

Node Variables Analysis

Let us consider a net. with N nodes and only one part. Assign a node to be a reference node (datum) node. In this case the variables are the (N−1) node-datum voltages $(v_1, v_2, \dots, v_{N-1})$ as shown below:



Define:

G_{kj} = sum of all conductance bet. nodes k and j

$\frac{1}{L_{kj}}$ = sum of the reciprocal of inductance bet. nodes k and j

C_{kj} = sum of all cap. bet. nodes k and j

Writing KCL @ k yields

$$i_k = \left(G_{k0} + C_{k0}\frac{d}{dt} + \frac{1}{L_{k0}}\int dt\right) v_k + \sum_{\substack{j=1 \\ j \neq k}}^{N-1} \left[G_{kj} + C_{kj}\frac{d}{dt} + \frac{1}{L_{kj}}\int dt\right](v_k - v_j)$$

note $i_k > 0$ in (entering)
$i_k < 0$ out (leaving)

$k = 1, 2, \dots, N-1$

"OR"

$$i_k = \sum_{j=1}^{N-1} b_{kj} v_j \quad , \quad k = 1, 2, \dots, N-1$$

where $b_{kj} = \begin{cases} -G_{kj} - C_{kj}\dfrac{d}{dt} - \dfrac{1}{L_{kj}}\int dt & , \quad k \neq j \\ G_{kk} + C_{kk}\dfrac{d}{dt} + \dfrac{1}{L_{kk}}\int dt & , \quad k = j \end{cases}$

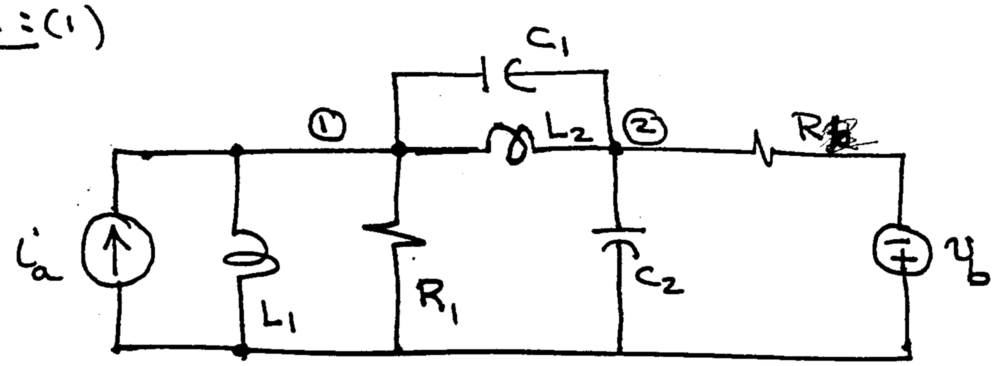
"OR"

$$\underline{i} = B\,\underline{v}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_k \\ \vdots \\ i_{N-1} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N-1} \\ b_{21} & b_{22} & \cdots & b_{2N-1} \\ \vdots & & & \\ b_{1N-1} & b_{2N-1} & \cdots & b_{N-1,N-1} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{N-1} \end{bmatrix}$$

$$\cancel{q_{kk} = q_{k0} + \sum_{\substack{j=1\\ j\neq k}}^{N} q_{kj}} \quad \text{~~and~~} \quad \cancel{L_{kk} = L_{k0} + \sum_{\substack{j=1\\ j\neq k}}^{N} L_{kj}}$$

Example: (1)



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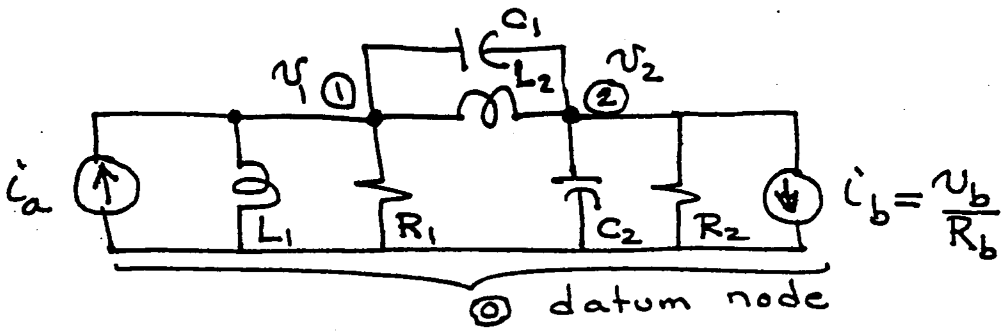


$N = 3 \quad (0, 1, 2)$

$$\left.\begin{aligned} i_1 &= \sum_{j=1}^{2} b_{1j} v_j \\ i_2 &= \sum_{j=1}^{2} b_{2j} v_j \end{aligned}\right\} \text{"OR"} \qquad \begin{aligned} i_1 &= b_{11} v_1 + b_{12} v_2 \\ i_2 &= b_{21} v_1 + b_{22} v_2 \end{aligned}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \text{OR} \quad i = B v$$

where, $\underbrace{i_1 = i_a, \quad i_2 = -i_b}$ and

Note that i is pos. if entering a node and neg. otherwise.

$$b_{11} = G_1 + C_1 \frac{d}{dt} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \int dt \ ,$$

$$b_{12} = b_{21} = -C_1 \frac{d}{dt} - \frac{1}{L_2} \int dt \ , \text{ and}$$

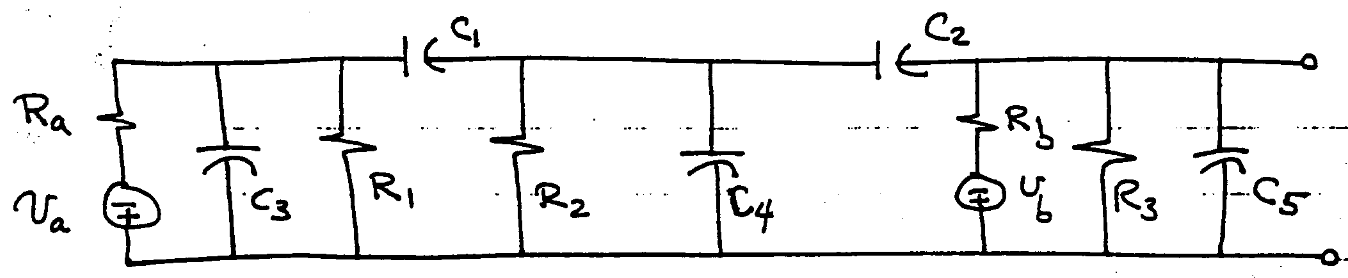
$$b_{22} = G_2 + (C_1 + C_2) \frac{d}{dt} + \frac{1}{L_2} \int dt$$

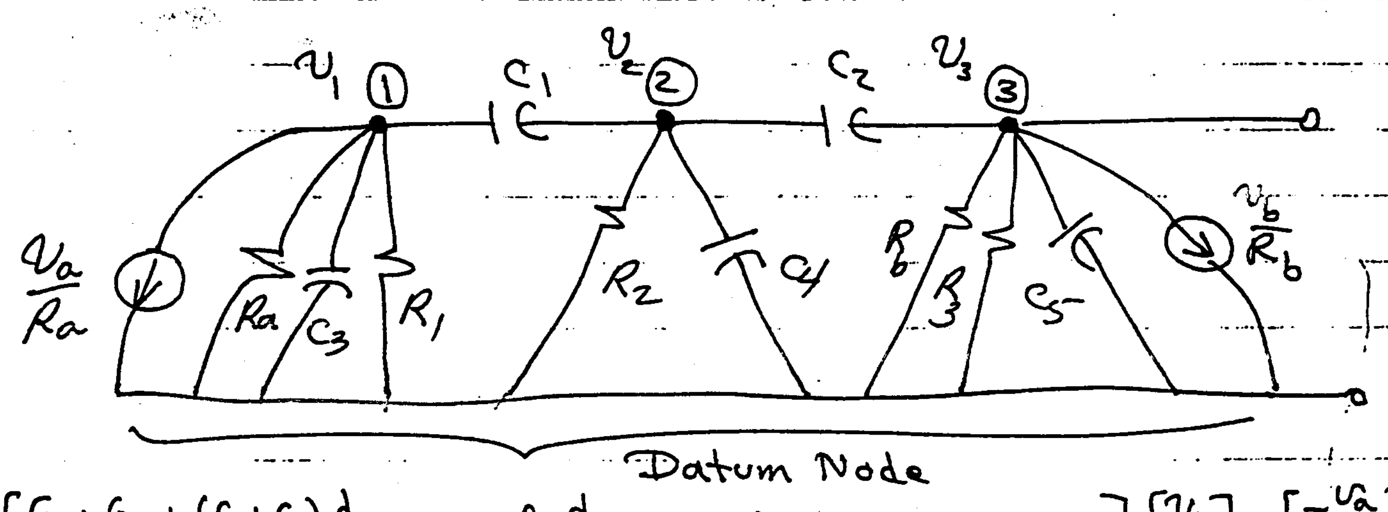
Thus,

$$\begin{bmatrix} i_a \\ -i_b \end{bmatrix} \begin{bmatrix} G_1 + C_1 \frac{d}{dt} + \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \int dt & -C_1 dt - \frac{1}{L_2} \int dt \\ -C_1 \frac{d}{dt} - \frac{1}{L_2} \int dt & G_2 + (C_1 + C_2) dt + \frac{1}{L_2} \int dt \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Note that B in this ex. is symmetric.

<u>Example (2)</u>:





symmetric

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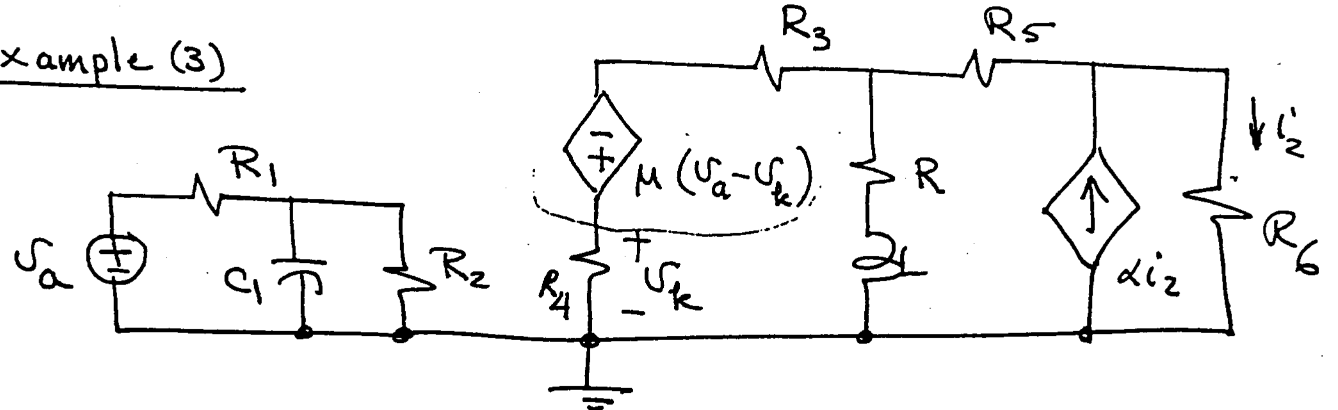
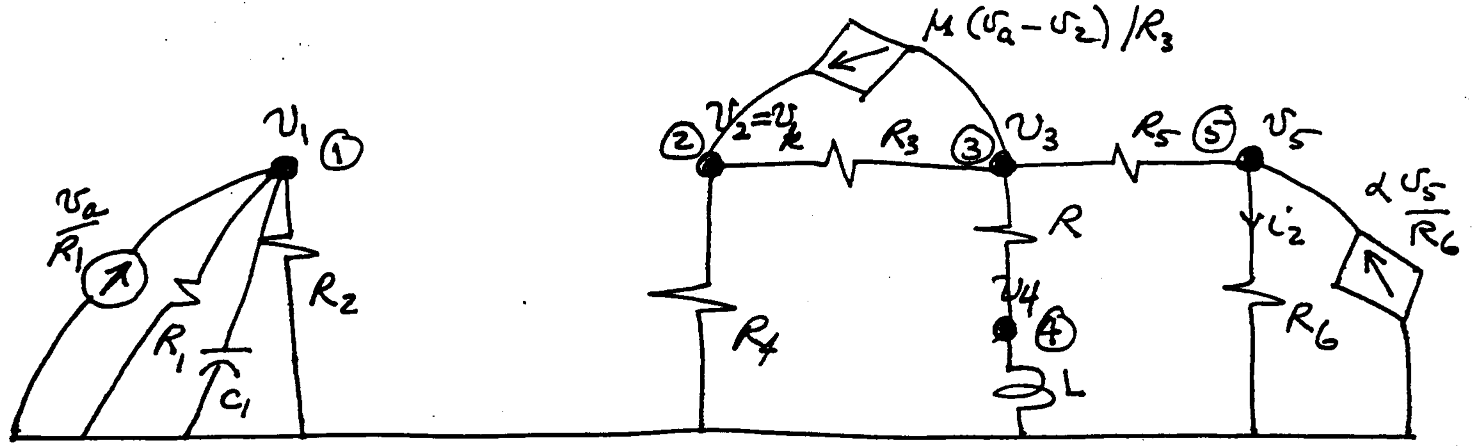
$$\begin{bmatrix} G_a + G_1 + (C_1 + C_3)\frac{d}{dt} & -C_1 \frac{d}{dt} & 0 \\ -C_1 \frac{d}{dt} & G_2 + (C_1 + C_2 + C_4)\frac{d}{dt} & -C_2 \frac{d}{dt} \\ 0 & -C_2 \frac{d}{dt} & (G_b + G_3) + (C_2 + C_5)\frac{d}{dt} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -\frac{v_a}{R_a} \\ 0 \\ -\frac{v_b}{R_b} \end{bmatrix}$$

Example (3)



Explain why R_3 and VCVS (instead of R_4 and VCVS) are converted to current source.



$$b_{11} = (G_1+G_2)+C_1\frac{d}{dt}, \quad b_{12}=0, \quad b_{13}=0, \quad b_{14}=0, \quad b_{15}=0$$

$$b_{21}=0, \quad b_{22}=G_3+G_4, \quad b_{23}=-G_3, \quad b_{24}=0, \quad b_{25}=0$$

$$b_{31}=0, \quad b_{32}=-G_3, \quad b_{33}=(G+G_3+G_5), \quad b_{34}=-G, \quad b_{35}=-G_5$$

$$b_{41}=0, \quad b_{42}=0, \quad b_{43}=-G, \quad b_{44}=G+\frac{1}{L}\int dt, \quad b_{45}=0$$

$$b_{51}=0, \quad b_{52}=0, \quad b_{53}=-G_5, \quad b_{54}=0, \quad b_{55}=G_5+G_6.$$

$$i_1=\frac{V_a}{R_1}, \quad i_2=\mu(V_a-V_2)/R_3, \quad i_3=-\mu(V_a-V_2)/R_3$$

$$i_4=0, \quad i_5=\alpha\frac{V_5}{R_6}$$