MATH 311

Assignment (I)

Due: May 31, 2009 AT 4:30 pm

(1) Let $V = \{(x,y) \mid x,y \in \mathbb{R}\}$. Define addition and scalar multiplication on V by

$$(x_1,y_1) + (x_2,y_2) = (x_1+x_2, y_1+y_2+1)$$
$$K(x,y) = (Kx, Ky+K-1), \quad K \in \mathbb{R}$$

Show that V is a vector space under the stated operations.

(2) Let V be the set of all 2×2 matrices with real entries. Define addition on V to be the standard matrix addition and define scalar multiplication by $KA = KA^t$, $K \in \mathbb{R}$, $A \in V$. Show that V is not a vector space under the stated operations. List all axioms that fail to hold.

(3) Let V be a vector space, $\bar{u}, \bar{v}$ be vectors in V and $K, \ell \in \mathbb{R}$ be scalars. prove that

(a) If $K\bar{u} = \ell\bar{u}$, $\vec{u} \neq \vec{0}$, then $K = \ell$

(b) If $K\bar{u} = K\vec{v}$, $K \neq 0$, then $\vec{u} = \vec{v}$.

Hint: Use the following result which we have proved in class:

If $a\vec{w} = \vec{0}$, then either $a = 0$ or $\vec{w} = 0$.

(4) Let U be the subset of P_2 defined by $U = \{p(x) \mid p(2) = 0\}$. show that U is a subspace of P_2.

(5) Let W be the subset of M_{22} defined by

$$W = \{A \mid \det A \neq 0\}$$

Is W a subspace of M_{22}? Justify your answer.

(6) In each case determine whether the set S spans the vector space V

(a) $S = \{(2,1,-3), (4,1,2), (8,-1,8)\}$, $V = \mathbb{R}^3$

(b) $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right\}$, $V = M_{22}$

(7) For what values of K is the solution space of

$$\begin{aligned} x_1 + x_2 + Kx_3 &= 0 \\ x_1 + Kx_2 + x_3 &= 0 \\ Kx_1 + x_2 + x_3 &= 0 \end{aligned}$$

a line through the origin, a plane through the origin, the origin only or all of $\mathbb{R}^3$?

(8) Find the coordinate vector of $p(x) = 2 - x + x^2$ relative to the basis $B = \{p_1 = 1 + x,\ p_2 = 1 + x^2,\ p_3 = x + x^2\}$.

(9) Find a basis and the dimension of the subspace

$$W = \{(a, b, c, d) \mid d = a + b,\ c = a - b\} \quad \text{of } \mathbb{R}^4$$

(10) Let $S = \{\bar{u}, \bar{v}\}$ be a linearly independent set of vectors in a vectorspace V. Show that the set $S' = \{\bar{u} - \bar{v},\ \bar{u} + \bar{v}\}$ is also linearly independent.

<u>Note</u>: Not all Questions will be marked!

MATH 311

Assignment (II)

Due : June 21, 2002

(1) Let $A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$

(a) Find a basis for Row(A), Col(A), and null(A).

(b) Find the rank, and nullity of A

(2) Let $S = \{(1,-2,0,3), (2,-5,-3,6), (0,1,3,0), (2,-1,4,-7), (5,-8,1,2)\}$ and let W be the subspace of $\mathbb{R}^4$ spanned by S

(a) Find a basis for W consisting entirely of vectors in S

(b) Find dim W.

(c) Express the vectors in S not in the basis as a linear combination of the basis vectors

(3) If A is 7×10 matrix and if rank(A) = 4. Determine the dimension of the row space of A, the column space of A, the null space of A, and the null space of A^t.

(4) Find invertible matrices U, and V such that

$$UAV = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad r = \text{rank}(A) \text{ if } A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 3 & 2 & 1 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix}$$

(5) Show $\langle \bar{u}, \bar{v} \rangle = u_1 v_1 + u_3 v_3$ where $\bar{u} = (u_1, u_2, u_3)$, $\bar{v} = (v_1, v_2, v_3)$ is not an inner product on $\mathbb{R}^3$. List the axioms that do not hold.

(6) Let M_{22} have the i.p $\langle A, B \rangle = tr(AB^t)$. Find $d(A, B)$ if $A = \begin{bmatrix} 2 & 6 \\ 9 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 7 \\ 1 & 6 \end{bmatrix}$.

(7) Sketch the unit circle in $\mathbb{R}^2$ using the i.p

$$\langle \bar{u}, \bar{v} \rangle = \frac{1}{4} u_1 v_1 + \frac{1}{16} u_2 v_2, \quad \bar{u} = (u_1, u_2), \bar{v} = (v_1, v_2) \in \mathbb{R}^2$$

(8) Let P_2 have the i.p,

$$\langle p, q \rangle = \int_{-1}^{1} p(x) q(x) \, dx$$

Find $\|p\|$ if $p = 1$, $p = x^2$.

P. T. O ⇒

(9) Let W be the subspace of $\mathbb{R}^4$ spanned by $\{(1,0,-2,3),(1,-1,2,0)\}$. Find a basis and the dimension of $W^{\perp}$.

(10) Let $\mathbb{R}^3$ have the i.p $\langle \bar{u},\bar{v}\rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$. Use Gram-schmidt process to transform the basis $\{\bar{u}_1=(1,1,1),\ \bar{u}_2=(1,1,0),\ \bar{u}_3=(1,0,0)\}$ into an orthonormal basis.
