use sinh(at) &
cosh(at)
→ PTO...

# TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\, t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\,\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

**First Differentiation Formula**

Let $\mathcal{L}\{x(t)\} = X(s)$ or $X$

$$\mathcal{L}[D^n x] = s^n\mathcal{L}[x] - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - x^{(n-1)}(0)$$

To Solve IVP:
1. Take Laplace of both sides
2. Solve for $X = \mathcal{L}\{x(t)\}$
3. $\mathcal{L}^{-1}\{X\} = x(t)$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}\,F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$, so $f(t) = \mathcal{L}^{-1}[F(s)]$.

**First Shift Formula**

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\mathcal{L}^{-1}[F(s+a)]$$

**Second Differentiation Formula**

$$\mathcal{L}[t^n f(t)] = (-1)^n\,\frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

**Second Shift Formula**

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}\left[e^{-as}F(s)\right] = u_a(t)f(t-a)$$

To find Laplace of unit step function,
put $g=1$

The following are Some formulae that will not be supplied

$$\mathcal{L}[\sinh at] = \frac{a}{s^2-a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2-a^2}\right] = \frac{1}{a}\sinh at$$

$$\mathcal{L}[\cosh at] = \frac{s}{s^2-a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2-a^2}\right] = \cosh at$$

The formulae above are very Similar to the ones for trig. Sine and Cosine functions

Linearity of Laplace / Inverse Laplace Transforms

$$\mathcal{L}[c_1 f_1 \pm c_2 f_2 \pm \cdots] = c_1 \mathcal{L}[f_1] \pm c_2 \mathcal{L}[f_2] \pm \cdots$$

$$\mathcal{L}^{-1}[\alpha F(s) \pm \beta G(s) \pm \cdots] = \alpha \mathcal{L}^{-1}[F(s)] \pm \beta \mathcal{L}^{-1}[G(s)] \pm \cdots$$

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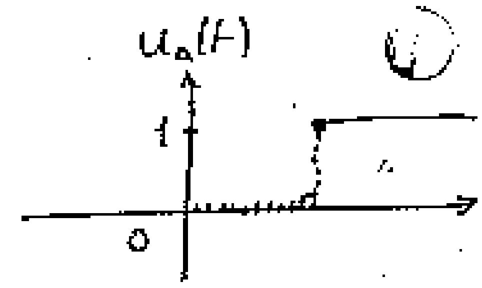
The unit step function : $u_a(t) = \begin{cases} 0 & t < a \\ 1 & a \le t \end{cases}$




$$\mathcal{L}[u_a(t)] = \frac{e^{-as}}{s}$$ "Know very well" !

oR : May find using the definition : $\mathcal{L}[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$ provided integral in R.H.S Converges.

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Completing the squares of the quadratic expression $s^2+\alpha s+\beta$

Just add / subtract $\left(\frac{\alpha}{2}\right)^2$ we get :

$$\left[s^2+\alpha s+\left(\tfrac{\alpha}{2}\right)^2\right] + \beta - \left(\tfrac{\alpha}{2}\right)^2 = \left(s+\tfrac{\alpha}{2}\right)^2 + \left(\beta - \left(\tfrac{\alpha}{2}\right)^2\right)$$

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An alternative to P.F.D : Very Powerful!

$$\frac{1}{(s+\alpha)(s+\beta)} = \frac{1}{\beta-\alpha}\left[\frac{1}{s+\alpha} - \frac{1}{s+\beta}\right], \quad \frac{1}{(s^2+\alpha)(s^2+\beta)} = \frac{1}{\beta-\alpha}\left[\frac{1}{s^2+\alpha} - \frac{1}{s^2+\beta}\right]$$