Crystallization in Non-Cooperative Games - Research Proposal

Instructor: Prof. Rann Smorodinsky
MA Student: Sergey Kuniavsky

September 20, 2005

1 Introduction

In Game Theory we differentiate between two basic approaches - non-cooperative and cooperative games. Most models fall into one of the two disciplines. In our work we plan to investigate a model on the border line of these two disciplines.

We consider a model of a non-cooperative game and ask whether it is worthwhile for agents to create coalitions within the non-cooperative framework. In each such coalition agents will coordinate their action, in order to receive a better outcome for each of them. The joint profit will be divided among the agents according to some pre-specified rule.

In a non-cooperative game every member from the set of agents N, chooses a strategy, without any knowledge of what other agents choose. Based on the agents' choices, each agent has a utility (possibly negative).

In this environment joining coalitions and coordinating among the agents may lead to better outcome, for the coalition as a whole, but possibly for each of the coalition members.

We will look into "Crystallization" of the non-cooperative game, which is a non-cooperative game, where agents group together, and each group of agents acts as one agent (i.e., agents within the group coordinate actions). One can think of the agents as choosing some benevolent dictator to choose their action tuple. Thus, the agents in the crystallization is a partition of the original agents set. The set of actions for such agent is the coalition members strategy space Cartesian Product, which is the strategy space for the coalition. The utility vector associated with a vector of actions (one for each coalition) is the sum of utility gained by the members of the coalition.

In the suggested research we will look into how agents will crystalize? This is dome by comparing individual payoffs before and after a crystallization. If there exists a crystallization which improves agents' individual utilities, is there a notion of an "optimal" crystallization? If so, is it unique and does it necessarily involve the grouping of all agents into the grand coalition.

In our study we consider various families of games, such as potential games and more specifically congestion games. We also study how results change as the mechanism which distributes a coalition's gain changes.

2 Model

A non-cooperative game $G = (N, A, U)$ is composed of a finite set of agents, denoted N, with N agents. For each agent $i \in N$, a strategy set A_i and a utility function $U_i : \prod A_i \to R$. We denote by $A = (A_1, \dots, A_n)$ and $U = (U_1, \dots, U_N)$.

A partition $\tau = (T_1, \dots, T_k)$ of N is a collection of subsets of N which are mutually exclusive and such that $\cup T_i = N$ and $T_i \neq \phi$.

A game G and a partition τ of N induces a new G^τ which we shall refer to as a *crystallization* of G. G^τ is defined as follows:
(1) The set of agents is $N^\tau = \{T_1, \dots, T_k\}$.
(2) The set of action for agent T_j is $\prod_{i \in T_j} A_i$, and will be denoted as A_j^τ.
(3) The utility functions are: $U_j^\tau(y_1, \dots, y_k) = \sum_{i \in T_j} U_i(x_1, \dots, x_n)$,
where $y_j = \prod_{i \in T_j} x_i$ (note that y_j denotes a generic element in A_j^τ and x_i is a generic element in A_i).

A function $\sigma : \prod A_j^\tau \to R^N$, that satisfies:

$$\sum_{i \in T_j} \sigma_i(y_1, \dots, y_k) = U_j^\tau(y_1, \dots, y_k) \tag{1}$$

will be called a *distribution rule*. Depending on the outcome of G^τ, the distribution rule tells us how the coalition utility is distributed among members, when (1) ensures that exactly all coalition utility is distributed, not lost or created.

A *simple distribution rule* gives each agent the utility he gained, without any side payments. Meaning: $\sigma_i(y_1, \dots, y_k) = U_i(x_1, \dots, x_n)$, where $y_j = \prod_{i \in T_j} x_i$.

We will differentiate between 2 types of crystallization: (1) When the full structure of the players (the partition) is common knowledge and (2) Each coalition knows about its own formation but knows nothing about the other coalitions. The first model will be called *public crystallization*, whereas the second will be called *private crystallization*. If the knowledge is partial, it will be called *semi-private crystallization*.

3 Conjectures

Some questions which we will cope with in our research refer to outcomes of a crystallization. In particular here are typical questions:

1. Whether an equilibrium in a crystallization with a simple distribution rule is an

equilibrium in the original game?

2. Is there a strategy for a private crystallization, with a simple distribution rule, where by joining coalitions each agent will (weakly) increase his utility?

3. Is there a distribution rule, where joining in a grand coalition gives each agent (weakly) more utility?