2. Review of Basic Concepts - *Professor J R Lucas*

2.1 Single Phase Power

2.1.1 Active Power

In direct current theory, *Power* $\mathbf{P}_{dc}$ is defined as the product of the *Voltage (difference)* $\mathbf{V}_{dc}$ and the *Current* $\mathbf{I}_{dc}$ as

$$\mathbf{P}_{dc} = \mathbf{V}_{dc} \,.\, \mathbf{I}_{dc}$$

However, in alternating current theory, as the *voltage* **v(t)** and the *current* **i(t)** are *instantaneous*ly varying, what we would get is an *instantaneous* value of *power* **p(t)** given by

$$\mathbf{p(t) = v(t) \,.\, i(t)}$$

Under normal steady state conditions, the *instantaneous* variations of *voltage* and *current* are sinusoidal with time at an angular frequency ω and differing in phase by an angle θ, so that

$$\mathbf{v(t)} = \mathbf{V_{max}\sin\omega t} \quad \text{and} \quad \mathbf{i(t)} = \mathbf{I_{max}\sin(\omega t - \theta)}$$

giving an *instantaneous power* of

$$\mathbf{p(t)} = \mathbf{V_{max}\sin\omega t \,.\, I_{max}\sin(\omega t - \theta)}$$

p(t) will be positive when both **v(t)** and **i(t)** have the same sign, and become negative when they have opposite signs. It is periodic and would have an average value $\mathbf{P_{av}}$ given by

$$\begin{aligned}
\mathbf{P_{av}} &= \frac{1}{T}\int_0^T \mathbf{V_{max}\,.\,I_{max}\sin\omega t\,.\,\sin(\omega t-\theta)\,dt} \\
&= \frac{\mathbf{V_{max}I_{max}}}{2T}\int_0^T \left[\cos\theta - \cos(2\omega t-\theta)\right]dt \\
&= \frac{\mathbf{V_{max}I_{max}}}{2}\cos\theta \quad = \quad \frac{\mathbf{V_{max}}}{\sqrt{2}}\,\frac{\mathbf{I_{max}}}{\sqrt{2}}\cos\theta \\
&= \mathbf{V_{rms}I_{rms}}\cos\theta
\end{aligned}$$

The phase angle θ arises out of the impedance angle θ of the circuit. In the form written, θ is *positive* when the *current is lagging the voltage* and *negative* when the *current is leading the voltage*. θ is also the phase angle difference between the voltage and current waveforms.

When considering a pure resistance, the voltage drop and the current through it are in phase and $\theta = 0^0$. The voltage and current would always have the same sign, giving a *positive* value for the *instantaneous power* at all times. Thus the *average power* $\mathbf{P_{av}}$ would have a maximum value of $\mathbf{V_{rms}\, I_{rms}}$.

When considering a pure inductance on the other hand, the current through it would *lag* the voltage drop across it by a phase angle of 90^0 so that $\theta = 90^0$. The voltage and current would have the same sign only half of the time, giving equal *positive* and *negative* values for the *instantaneous power.* Thus the *average power* $\mathbf{P_{av}}$ would have the minimum value of *0.*

Similarly when considering a pure capacitance, the current through it would *lead* the voltage drop across it by a phase angle of 90^0 so that $\theta = -90^0$. The voltage and current would again have the same sign only half of the time, giving equal *positive* and *negative* values for the *instantaneous power.* Thus the *average power* $\mathbf{P_{av}}$ would again have the minimum value of *zero.*

The above analysis shows that $\mathbf{P_{av}}$ is no longer equal to the product of $\mathbf{V_{rms}\,.\,I_{rms}}$ for alternating currents.

$\mathbf{P_{av}}$ is defined as the *Active Power* (also called *Real Power*) **P**.

$\mathbf{V_{rms}}$ and $\mathbf{I_{rms}}$ are commonly written as **V** and **I** and are understood to be the *rms values* of voltage and current in a.c. theory unless otherwise defined.

2.1.2 Power Factor and Reactive Power

The product **V . I** is defined as the *Apparent Power* in alternating current work. Since the *apparent power* is no longer equal to the *Active Power* the *Power Factor* (p.f. for short) is defined as the ratio of these quantities.

$$Power\,Factor = \frac{Active\ Power}{Apparent\ Power}$$

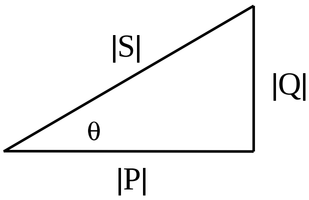

Figure 2.1 Power Triangle

In the case of sinusoidal waveforms, we can write

$$Power\,Factor = \frac{V\ I\ \cos\theta}{V\ I} = \cos\theta$$

so that the *power factor* is commonly defined as $\cos\theta$, and the term *lag* or *lead* is associated with it depending on whether the current considered is *lagging* or *leading* the respective voltage. θ is known as the power factor angle.

The relation between *apparent power* ***S*** and the *active power* ***P*** can be represented by the Power triangle shown in the figure 2.1.

$$|S| = \sqrt{|P|^2 + |Q|^2}$$

$$\tan\theta = \frac{|Q|}{|P|}$$

The quantity $\boldsymbol{Q}$ is defined as the *Reactive Power.* They have the dimension of Power and are measured in the unit *var*. *Apparent Power* is measured in the unit *VA*. In power systems, since the quantities measured are large, it is usual to express *active power* in *MW*, *reactive power* in *Mvar* and *apparent power* in *MVA.*

In the case of sinusoidal waveforms, *Reactive Power* may also be expressed as

$$\boldsymbol{Q} = \boldsymbol{V} . \boldsymbol{I} \ \sin \theta$$

Like *active power* occuring when the voltage and the current are in phase, *reactive power* occurs when they are out of phase by 90°. This quadrature, or 90° out of phase can occur either when the current is lagging the voltage (as in an ideal inductor) or leading the voltage (as in an ideal capacitor). As these are opposite forms of the reactive power, the usual convention is to define the *reactive power* absorbed by an *inductive load* as *positive*. Thus the *reactive power* absorbed by an *capacitive load* is *negative*. *[Note:* A purely *Resistive load* does not absorb any *reactive power*].

2.1.3 Complex Power

Complex numbers (either in polar form or cartesian form) are used to represent r.m.s. voltages and currents as

$$\underline{\boldsymbol{V}} = |\boldsymbol{V}| \angle \theta_V \qquad \text{or} \qquad \underline{\boldsymbol{V}} = \boldsymbol{V}_x + \mathrm{j}\boldsymbol{V}_y$$

$$\underline{\boldsymbol{I}} = |\boldsymbol{I}| \angle \theta_I \qquad \text{or} \qquad \underline{\boldsymbol{I}} = \boldsymbol{I}_x + \mathrm{j}\,\boldsymbol{I}_y$$

Thus the *apparent power* $\boldsymbol{S}$, *active power* $\boldsymbol{P}$ and *reactive power* $\boldsymbol{Q}$ will become

$$\boldsymbol{S} = |\boldsymbol{V}|\,|\boldsymbol{I}|, \qquad \boldsymbol{P} = |\boldsymbol{V}|\,|\boldsymbol{I}|\cos(\theta_V - \theta_I) \quad \text{and} \quad \boldsymbol{Q} = |\boldsymbol{V}|\,|\boldsymbol{I}|\sin(\theta_V - \theta_I)$$

However in the complex form, the direct product of $\underline{\boldsymbol{V}}\,\underline{\boldsymbol{I}}$ does not give the correct components of $\boldsymbol{P}$ and $\boldsymbol{Q}$. This is easily seen by considering the polar form which would give $\boldsymbol{P}$ and $\boldsymbol{Q}$ corresponding to an addition of the angles ($\theta_V + \theta_I$) and not to the required difference of the angles ($\theta_V - \theta_I$).

This can be easily corrected by considering either the complex conjugate of $\boldsymbol{V}$ or $\boldsymbol{I}$ in the product. Thus

$$\underline{\boldsymbol{V}}\,\underline{\boldsymbol{I}}^* = |\boldsymbol{V}||\boldsymbol{I}|\angle(\theta_V - \theta_I) = |\boldsymbol{V}||\boldsymbol{I}|\cos(\theta_V - \theta_I) + \mathrm{j}\,|\boldsymbol{V}||\boldsymbol{I}|\sin(\theta_V - \theta_I)$$

i.e. $$\underline{\boldsymbol{V}}\,\underline{\boldsymbol{I}}^* = \boldsymbol{P} + \mathrm{j}\,\boldsymbol{Q}$$

and

$$\underline{\boldsymbol{V}}^*\,\underline{\boldsymbol{I}} = |\boldsymbol{V}||\boldsymbol{I}|\angle(-\theta_V + \theta_I) = |\boldsymbol{V}||\boldsymbol{I}|\cos(-\theta_V + \theta_I) + \mathrm{j}\,|\boldsymbol{V}||\boldsymbol{I}|\sin(-\theta_V + \theta_I)$$

i.e. $$\underline{\boldsymbol{V}}^*\,\underline{\boldsymbol{I}} = \boldsymbol{P} - \mathrm{j}\,\boldsymbol{Q}$$

[These expressions can also be shown using the cartesian form of the complex numbers, however the derivation is not as elegant].

In alternating current circuits, since both voltage and current are instantaneously changing sign, it is not always obvious whether at a particular port power is delivered to the circuit or being absorbed from it. It is best understood by considering the circuit shown in figure 2.2.

If **P** + j **Q** is calculated from **V** and **I** as indicated using **V I*** then, if **P** > **0** active power is absorbed by the circuit and if **P** < **0** active power is supplied by the circuit. Similarly if **Q** > **0** reactive power is absorbed by the circuit and if **Q** < **0** reactive power is supplied by the circuit .

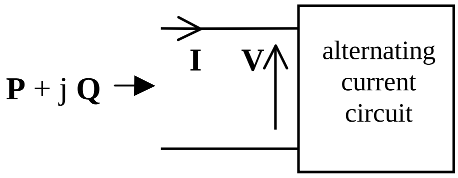

Figure 2.2 Direction of Power Flow

2.2 Three Phase Systems

2.2.1 Balanced Three Phase Systems - Phase and Line Quantities

For reasons of economics, electric power is usually supplied from balanced three phase generators. Ideally we would like the loads also to be equally distributed among the three phases giving balanced loads.

While balanced voltages and currents each have equal magnitudes in their three phases and differing in phase angle by 120° from each other, balanced loads would have equal impedances equal not only in magnitude but also in phase angle.

Balanced three phase systems may either consist of star connected generators and/or loads and delta connected generators and/or loads. In the case of both being star, a neutral wire may or may not be present.

Consider a balanced star connected source connected to a balanced star connected load by a 4-wire line as shown in figure 2.3.

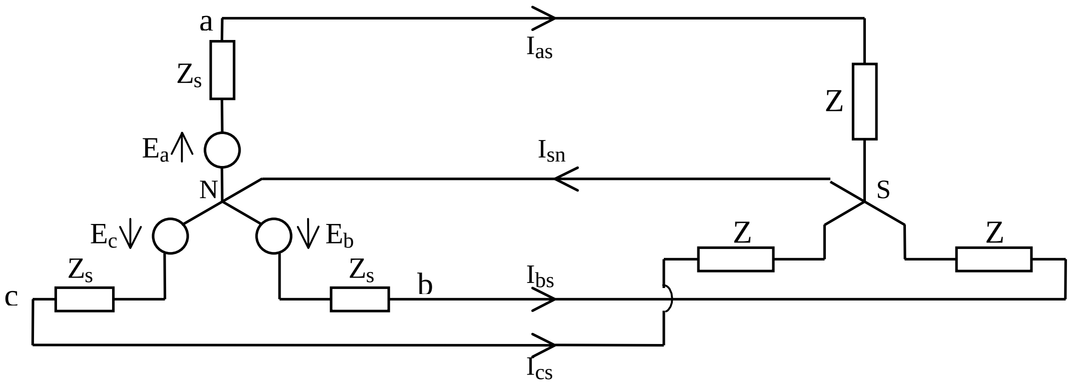

Figure 2.3 - Balanced 4-wire Star Connected System

For a balanced system, the phase currents add up to zero so that the neutral current will always be zero.

$$I_{sn} = I_{as} + I_{bs} + I_{cs} = 0 \quad \text{for a balanced system}$$

Also the potential difference across *SN* would also be zero. Thus the analysis of the balanced 3-wire and the balanced 4-wire systems would be identical.

For a balanced star connected system,

V_{an} , V_{bn} , V_{cn} are the phase-to-neutral or *Phase Voltages* $\boldsymbol{V_P}$

V_{ab} , V_{bc} , V_{ca} are the line-to-line or *Line Voltages* $\boldsymbol{V_L}$

I_{an} , I_{bn} , I_{cn} are the currents in the phases or *Phase Currents* $\boldsymbol{I_P}$

I_{an} , I_{bn} , I_{cn} are the are also the currents in lines or *Line Currents* $\boldsymbol{I_L}$

Using Phasor diagrams it is easily seen that the magnitudes

$$V_L = \sqrt{3}\, V_P \qquad \text{and} \qquad I_L = I_P$$

There is also a 30° phase difference between the phase voltage and the line voltage for the star connection.

In the case of the Delta connection, the voltage across a phase would be the line voltage and the corresponding currents in the line and phase would differ by a magnitude of $\sqrt{3}$ and a phase angle difference of 30°.

Unless otherwise stated, the terms *phase voltage* and *phase current* would be usually taken as those corresponding to the *balanced star connection.*

A balanced system with a phase sequence of ***a-b-c*** would mean that *phase a* leads *phase b* by 120° which in turn leads *phase c* by 120°. Since the phasors are cyclic, it would also mean that *phase c* leads *phase a* by 120°. Thus sequences ***a-b-c***, ***b-c-a*** and ***c-a-b*** are the same while ***a-c-b***, ***c-b-a*** and ***b-a-c*** are opposite.

2.2.2 Power in Balanced Three Phase Circuits

The power delivered by a three phase supply, or delivered to a three phase load is obtained by adding the power in each of the three phases. Active power can be algebraically added with other active powers, and reactive power can be algebraically added with other reactive powers. However, apparent power can only be added using the power triangle (i.e. adding the active and reactive powers individually and then obtaining the resultant).

In a balanced system, the power associated with each phase is the same, so that we may obtain the total power by multiplying that of one phase by the factor 3. Thus if V_P and I_P are the phase quantities of voltage and current, with phase power factor angle ϕ then the active power P_T in a balanced three phase circuit is given by $P_T = 3\, V_P\, I_P \cos\phi$ and the reactive power Q_T is given by $Q_T = 3\, V_P\, I_P \sin\phi$.

The terms phase voltage and phase current are dependant on the type of load (star or delta), whereas the line voltage and line current are independant. Thus we usually specify the line voltage V_L and line current I_L. Let us see how these line quantities can be made use of to calculate the total power for both star-connected and delta-connected systems.

For the *Star-connected system*

$$V_P = V_L/\sqrt{3} \quad \text{and} \quad I_P = I_L \text{ so that}$$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\phi \text{ and } Q_T = \sqrt{3}\, V_L\, I_L \sin\phi. \text{ Also } S_T = \sqrt{3}\, V_L\, I_L$$

Similarly for the *Delta-connected system*

$$V_P = V_L \quad \text{and} \quad I_P = I_L/\sqrt{3} \quad \text{so that again}$$

$$P_T = \sqrt{3}\, V_L\, I_L \cos\phi \text{ and } Q_T = \sqrt{3}\, V_L\, I_L \sin\phi. \text{ Also } S_T = \sqrt{3}\, V_L\, I_L$$

Thus we get a unique set of expressions which does not depend on the type of connection, but only on the line quantities. It must be noted that ϕ is the phase angle by which the phase current lags the phase voltage (i.e. the power factor) and not the angle between the line voltage and line current.

2.2.3 Equivalent Circuits - Single phase and Equivalent Single Phase Circuits

Since the information relating to one single phase gives the information relating to the other two phases as well in a balanced three phase circuit, it is sufficient to do calculations in a single phase circuit. There are two common forms used.

(i) Single Phase Circuit

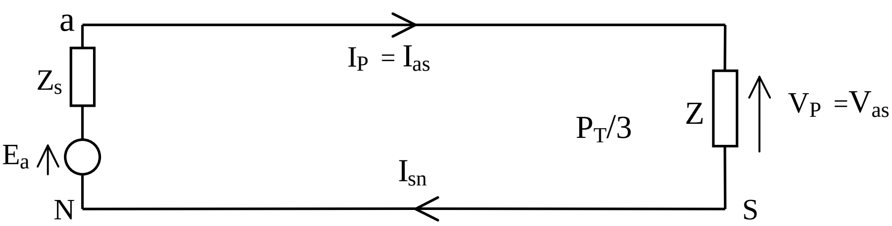

Figure 2.4 - Single Phase Circuit

This is one single phase of the three phase circuit. There is no potential drop across the neutral wire as the system is balanced.

In this cirucit, $I = I_P = I_L$, $V = V_P = V_L/\sqrt{3}$ and $S = S_P = S_T/3$

(ii) Equivalent Single Phase Circuit

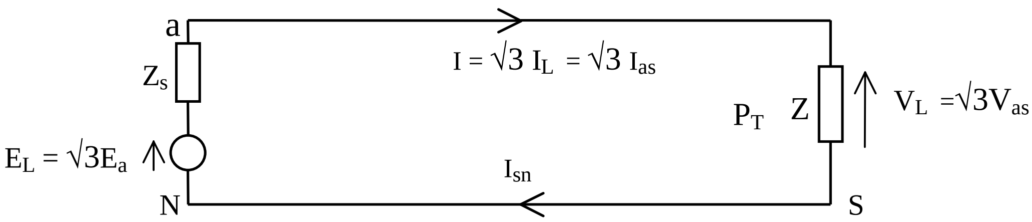

Figure 2.5 - Equivalent Single Phase Circuit

Of the parameters in the single phase circuit shown in figure 2.4, the Line Voltage and the Total Power are the most important quantities. It would be useful to have these quantities obtained directly from the circuit rather than having conversion factors of $\sqrt{3}$ and 3 respectively. This is achieved in the Equivalent Single Phase circuit by multiplying the voltage by a factor of $\sqrt{3}$ to give line voltage directly. However as the impedance is left as the phase impedance, the line current gets artificially amplified by $\sqrt{3}$. This also increases the power by a factor of $(\sqrt{3})^2$, which is the required correction to get the total power.

2.3 Per Unit Quantities

In Power Systems calculations, it is common to use per unit (and sometimes per cent) quantities. Per unit quantities are actually fractional quantities of a reference quantity. These have a lot of importance as per unit quantities tend to have similar values even when the system voltage and rating change drastically. The per unit system is very similar to the percent system, except that when percentage quantities are to be multiplied or divided additional factors of 100 must be brought in which are not in the original equations.

$$\mathbf{S_{pu} = S/S_{base},\ V_{pu} = V/V_{base},\ I_{pu} = I/I_{base}} \text{ and } \mathbf{Z_{pu} = Z/Z_{base}}$$

Expressions such as Ohm's Law can be applied directly in per unit quantities as well. Since Voltage, Current, Impedance and Power are related, only two Base or reference quantities can be independently defined. The Base quantities for the other two can be derived therefrom. Since Power and Voltage are the most important, they are usually chosen to define the independent base quantities.

2.3.1 Calculation for Single Phase Systems

If $\mathbf{VA_{base}}$ and $\mathbf{V_{base}}$ are the selected base quantities of *power* (complex, active or reactive) and *voltage*, then

Base current $\mathbf{I_{base} = VA_{base}/V_{base}}$

Base Impedance $\mathbf{Z_{base} = V_{base}/I_{base} = V_{base}^2/VA_{base}}$

Voltages and power are usually expressed in *kV* and *MVA*, thus it is usual to select a $\mathbf{MVA_{base}}$ and $\mathbf{kV_{base}}$ and to express

Base current $\mathbf{I_{base} = MVA_{base}/kV_{base}}$ in *kA*

Base Impedance $\mathbf{Z_{base} = kV_{base}^2/MVA_{base}}$ in Ω

In these expressions, all the quantities are single phase quantities.

2.3.2 Calculations for Three Phase Systems

In three phase systems the line voltage and the total power are more important than the per phase quantities. It is thus usual to express base quantities in terms of these.

If $\mathbf{VA_{3\phi base}}$ and $\mathbf{V_{LLbase}}$ are the three phase base power and line to line voltage respectively,

Base current $\mathbf{I_{base} = VA_{base}/V_{base} = 3VA_{base}/3V_{base} = VA_{3\phi base}/\sqrt{3}V_{LLbase}}$

Base Impedance $\mathbf{Z_{base} = V_{base}^2/VA_{base} = 3V_{base}^2/3VA_{base} = V_{LLbase}^2/VA_{3\phi base}}$

and in terms of $\mathbf{MVA_{3\phi base}}$ and $\mathbf{kV_{LLbase}}$

Base current $\mathbf{I_{base} = MVA_{3\phi base}/\sqrt{3}\, kV_{LLbase}}$ in *kA*

Base Impedance $\mathbf{Z_{base} = kV_{LLbase}^2/MVA_{3\phi base}}$ in Ω

Thus in three phase, the calculations of per unit quantities becomes

$\mathbf{S_{pu} = S_{actual}}(MVA)\mathbf{/MVA_{3\phi base}}$,

$\mathbf{V_{pu} = V_{actual}}(kV)\mathbf{/kV_{LLbase}}$,

$\mathbf{I_{pu} = I_{actual}}(kA)\mathbf{.\sqrt{3}\, kV_{LLbase}/MVA_{3\phi base}}$ and

$\mathbf{Z_{pu} = Z_{actual}}(\Omega)\mathbf{.\ MVA_{3\phi base}/kV_{LLbase}^2}$

P and *Q* have the same base as *S*, so that $P_{pu} = P_{actual}/MVA_{3\phi base}$, $Q_{pu} = Q_{actual}/MVA_{3\phi base}$. Similarly, *R* and *X* have the same base as *Z*, so that $R_{pu} = R_{actual}(\Omega)\,.\ MVA_{3\phi base}/kV_{LLbase}^2$ and $X_{pu} = X_{actual}(\Omega)\,.\ MVA_{3\phi base}/kV_{LLbase}^2$. The *power factor* remains unchanged in *per unit*.

2.3.3 Conversions from one Base to another

It is usual to give data in per unit to its own rating. As different components can have different ratings, it is necessary to convert all quantities to a common base to do arithmetic operations. Additions, subtractions, multiplications and divisions will give meaningful results only if they are to the same base. This can be done for three phase systems as follows.

$\mathbf{S_{puNew} = S_{puGiven}\ .\ MVA_{3\phi baseGiven}/MVA_{3\phi baseNew}}$,

$\mathbf{V_{puNew} = V_{puGiven}\ .\ kV_{LLbaseGiven}/kV_{LLbaseNew}}$, and

$\mathbf{Z_{pu} = Z_{puGiven}\ .\ (MVA_{3\phi baseNew}/\ MVA_{3\phi baseGiven})\ .\ (kV_{LLbaseGiven}/kV_{LLbaseNew})^2}$

Example:

A 200 MVA, 13.8 kV generator has a reactance of 0.85 p.u. and is generating 1.15 pu voltage. Determine (a) the actual values of the line voltage, phase voltage and reactance, and (b) the corresponding quantities to a new base of 500 MVA, 13.5 kV.

(a) Line voltage = 1.15 * 13.8 = 15.87 kV

Phase voltage = $1.15 * 13.8/\sqrt{3}$ = 9.16 kV

Reactance = $0.85 * 13.8^2/200$ = 0.809 Ω

(b) Line voltage = 1.15 * 13.8/13.5 = 1.176 pu

Phase voltage = $1.15 * (13.8/\sqrt{3})/(13.5/\sqrt{3})$ = 1.176 pu

Reactance = $0.85 * (13.8/13.5)^2/(500/200)$ = 0.355 pu

2.3.4 Per Unit Quantities across Transformers

Although the power rating on either side of a transformer remains the same, the voltage rating changes, and so does the base voltage across a transformer. [This is like saying that full or100% (or 1 pu) voltage on the primary of a 220/33 kV transformer corresponds to 220 kV while on the secondary it corresponds to 33 kV.] Since the power rating remains unchanged, the impedance and current ratings also change accordingly.

While a common $\mathbf{MVA_{3\phi base}}$ can be selected for a power system, a common $\mathbf{V_{LLbase}}$ must be chosen corresponding to a particular location and changes in proportion to the nominal voltage ratio whenever a transformer is encountered. The current base changes inversely as the ratio. Hence the impedance base changes as the square of the ratio.

For a transformer with turns ratio $N_P{:}N_S$, base quantities change as follows.

Quantity	Primary Base	Secondary Base
Power (S, P and Q)	$\mathbf{S_{base}}$	$\mathbf{S_{base}}$
Voltage (V)	$\mathbf{V_{1base}}$	$\mathbf{V_{1base} \cdot N_S/N_P = V_{2base}}$
Current (I)	$\mathbf{S_{base}/\sqrt{3}V_{1base}}$	$\mathbf{S_{base}/\sqrt{3}V_{1base} \cdot N_P/N_S = S_{base}/\sqrt{3}V_{2base}}$
Impedance (Z, R and X)	$\mathbf{V_{1base}^2/S_{base}}$	$\mathbf{V_{1base}^2/S_{base} \cdot (N_S/N_P)^2 = V_{2base}^2/S_{base}}$

Example :

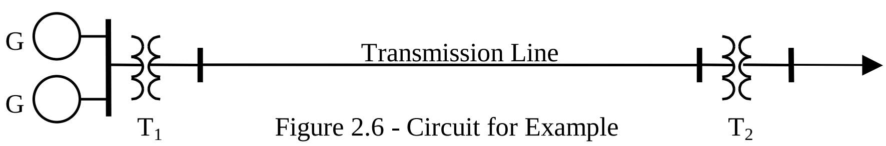

Figure 2.6 - Circuit for Example

In the single line diagram shown, each three phase generator G is rated at 200 MVA, 13.8 kV and has reactances of 0.85 pu and are generating 1.15 pu. Transformer T_1 is rated at 500 MVA, 13.5 kV/220 kV and has a reactance of 8%. The transmission line has a reactance of 7.8 Ω. Transformer T_2 has a rating of 400 MVA, 220 kV/33 kV and a reactance of 11%. The load is 250 MVA at a power factor of 0.85 lag. Convert all quantities to a common base of 500 MVA, and 220 kV on the line and draw the circuit diagram with values expressed in pu.

Solution:

The base voltage at the generator is (220*13.5/220) 13.5 kV, and on the load side is (220*33/220) 33 kV. [Since we have selected the voltage base as that corresponding to the voltage on that side of the transformer, we automatically get the voltage on the other side of the transformer as the base on that side of the transformer and the above calculation is in fact unnecessary.

Generators G

Reactance of 0.85 pu corresponds 0.355 pu on 500 MVA, 13.5 kV base (see earlier example)

Generator voltage of 1.15 corresponds to 1.176 on 500 MVA, 13.5 kV base

Transformer T_1

Reactance of 8% (or 0.08 pu) remains unchanged as the given base is the same as the new chosen base.

Transmission Line

Reactance of 78 Ω corresponds to 7.8 * 500/220^2 = 0.081 pu

Transformer T_2

Reactance of 11% (0.11 pu) corresponds to 0.11 * 500/400 = 0.1375 pu

(voltage base is unchanged and does not come into the calculations)

Load

Load of 250 MVA at a power factor of 0.85 corresponds to 250/500 = 0.5 pu at a power factor of 0.85 lag (power factor angle = 31.79°)

∴ resistance of load = 0.5 * 0.85 = 0.425 pu

and reactance of load = 0.5 * sin 31.79° = 0.263 pu

The circuit may be expressed in per unit as shown in figure 2.7.

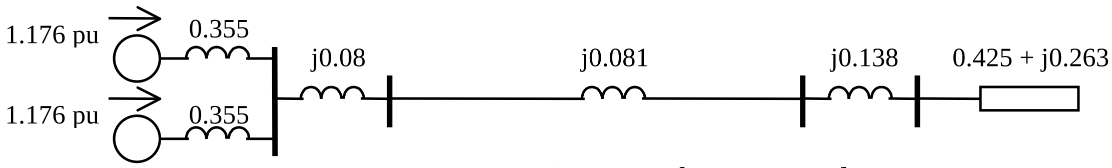

Figure 2.7 - Circuit with per unit values