FREQUENCY DOMAIN BASED CONTROL RELEVANT BILINEAR MODEL ESTIMATION

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Abstract

In this paper we propose a control relevant bilinear model estimation methodology using the generalized frequency response function defined for nonlinear systems. The control relevant identification problem is defined as a weighted multiplicative error minimization function. This is achieved by synthesizing a prefilter having bilinear structure to be used for prefiltering the identification data so as to bias the frequency content of the data from closed loop performance point of view. By fitting a first order bilinear model to the prefiltered identification data, a control relevant bilinear model is estimated. The closed loop performance obtained using nonlinear internal model control strategy for a representative nonlinear system is shown to exhibit specified closed loop performance, and hence validates the approach.

Keywords: Generalized frequency response function, bilinear model, non linear internal model control, control relevant identification.

1 Introduction

Most plants existing in real life exhibit nonlinear dynamics. In recent years, lot of research work is being carried out on the identification of reduced order model for such nonlinear plants, see, for example, Billings [1], Haber and Unbehauen [2], Pearson [3], Ling and Rivera [4, 5] and the references cited there in. The commonly used structures for the nonlinear models are the Volterra models, the nonlinear auto regressive with eXogenous input(NARX) models, block-oriented models, such as the Hammerstein and the Weiner models, and neural network based models. The bilinear model is a special class of NARX model, which is shown to capture the dynamics of many physical systems in a simple structure, especially in chemical engineering systems. Bilinear models can approximate the nonlinear processes to an accuracy restricted by the order of the model. They represent a mathematically tractable structure over Volterra models, for a nonlinear plant. Also, bilinear model can obviously represent the dynamics of a nonlinear plant more accurately than that represented by a linear model. Hence, modeling and control of nonlinear systems in a bilinear framework has been extensively studied by Hsu and Mohler [6], Yeo and Williams [7, 8] and Bartee and Georgakis [9].

The frequency domain descriptions for nonlinear systems is developed by Peyton-Jones and Billings [10–12]. In this paper we propose to use this frequency domain description for NARX/Bilinear systems to express the model plant mismatch in the frequency domain. We propose here to estimate a first order bilinear model in a control relevant manner. This can be obtained by minimizing the weighted multiplicative error in the frequency range of interest from closed loop performance view point. For achieving this, a prefilter in bilinear structure is proposed to be used, to bias the identification data to contain frequency content in the desired frequency band of interest. The weight function is obtained based on the closed loop performance requirements. The closed loop performance is analyzed using nonlinear internal model control (NIMC) strategy and is shown to be satisfactory.

This paper is organized in the following manner: The methodology proposed for control relevant bilinear model estimation is presented in Section 2. The algorithm for estimation of the control relevant bilinear

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model and analysis of NIMC strategy based system is outlined in Section 3. In Section 4 the simulation results obtained from a representative nonlinear plant is presented. The important findings of this work and scope of future work are outlined in Section 5.

2 Control relevant model reduction methodology

The proposed methodology for control relevant model reduction is presented in this Section. The overall system performance requirements can be specified in terms of the complementary sensitivity function $\tilde{\eta}(z)$, which determines the frequency range of interest from closed loop performance view point. This can be assigned similar to that for linear systems, based on the desired closed loop performance requirements. The frequency response of $\tilde{\eta}(z)$ is evaluated over the frequency range of interest and is represented as $\tilde{\eta}(\omega)$. Now, using $\tilde{\eta}(\omega)$ the frequency domain representation of the sensitivity function $\tilde{\epsilon}(\omega)$ is defined as is done for one degree of freedom linear systems. That is,

$$\tilde{\epsilon}(\omega) = 1 - \tilde{\eta}(\omega) \tag{1}$$

Let the closed loop system will be subjected to a set point/disturbance signal represented as $v(z)$. Based on the nature of the set point/ disturbance signals the frequency domain representation of $v(z)$ is estimated and is represented as $\mathbf{v}(\omega)$. The three terms defined by $\tilde{\eta}(\omega)$, $\tilde{\epsilon}(\omega)$ and $\mathbf{v}(\omega)$ actually define the closed loop performance requirements and the frequency band of importance from closed loop performance point of view. Hence using these a weighting factor is defined as follows:

$$\mathbf{W}(\omega) = |\tilde{\eta}(\omega). * \tilde{\epsilon}(\omega). * \mathbf{v}(\omega)| \tag{2}$$

Using the interpretation of the generalized frequency response function proposed by Peyton-Jones and Billings [10–12] for NARX and Bilinear models, the model plant mismatch(multiplicative error) between the NARX plant and the reduced order Bilinear model can be estimated in the frequency domain as,

$$\mathbf{e}_m(\omega) = \left| \frac{(\hat{p}(e^{j\omega}) - \tilde{p}(e^{j\omega})).}{\tilde{p}(e^{j\omega})} \right| \tag{3}$$

where,
$\hat{p}(e^{j\omega})$ is the generalized frequency response function of the NARX plant,
$\tilde{p}(e^{j\omega})$ is that of the bilinear model.
Using the above defined weight function and the multiplicative error description, we define the control relevant model estimation problem as

$$min[\mathbf{W}(\omega). * \mathbf{e_m}(\omega)]^T[\mathbf{W}(\omega). * \mathbf{e_m}(\omega)] \tag{4}$$

In the following Section, an algorithm for implementing the above defined control relevant bilinear model estimation problem is outlined.

3 Algorithm for Control relevant model estimation

The algorithm for control relevant bilinear model estimation can be outlined as follows:
In the following analysis, the set point is assumed to be of step type of signal. Assuming the condition that $\hat{p}(e^{j\omega}) = \tilde{p}(e^{j\omega})$ is valid in the frequency band of importance, the closed loop performance specification can be represented as,

$$\tilde{\eta} \; = \; \tilde{p} * q \; = \; \tilde{p} * \tilde{p}^{-1} * f_{imc} \; = \; f_{imc} \tag{5}$$

where, f_{imc} is the frequency domain description of the linear filter required to be used for achieving robustness to the nonlinear IMC based system in the presence of model plant mismatch, and can be defined as,

$$f_{imc}(z) = \frac{(1 - e^{\frac{-T_s}{\tau_{cl}}})z}{z - e^{\frac{-T_s}{\tau_{cl}}}} \tag{6}$$

The complementary sensitivity function, as given by Equation (5) explicitly incorporates the desired closed loop performance specification (τ_{cl}), through the IMC filter (Equation (6)). T_s is the sampling time chosen. Using Equation (6) the frequency domain description of $\eta(\omega)$ is evaluated by estimating the frequency response of $f_{imc}(z)$ over the frequency range of importance. Using Equation (1) the sensitivity function $\tilde{\epsilon}(\omega)$ is obtained. $\mathbf{v}(\omega)$ is defined using the frequency spectrum of the step signal. Using these the weight function $\mathbf{W}(\omega)$ is synthesized, as defined in Equation(2). Once the weight function is synthesized, the further steps in control relevant model reduction are outlined as follows:

1. The nonlinear plant to be modeled has to be perturbed about the given operating point by an appropriately designed perturbation signal and the open loop input/output data is obtained. In the analysis followed here, multilevel(3- level) pseudo random signal, of appropriate variance, suitably designed from *a priori* knowledge of the plant are used as perturbation signals.

2. The generalized frequency response function of the given NARX plant is obtained by following the procedure given in, [10–14]. The frequency response is obtained in the output frequency domain. The frequency response is recorded as a vector $\hat{p}(e^{j\omega})$ as a function of frequency.

3. A bilinear prefilter is initialized.

4. The open loop input/output data is prefiltered by the prefilter. Also, data set generated by taking the product of the open loop input/output data sequence obtained from the plant with unit lag, has to be prefiltered. This is used as a second input in the bilinear model estimation.

5. Using the prefiltered data consisting obtained after filtering of the input, the output and their product, a first order bilinear model is estimated by assuming an auto regressive with eXogenous (ARX) input like structure.

6. The generalized frequency response function of the estimated bilinear model is obtained and recorded as a vector $\tilde{p}(e^{j\omega})$, in the chosen frequency band of interest.

7. The multiplicative error between the NARX plant and the estimated bilinear model in the chosen frequency band has to be estimated as specified in Equation (3).

8. The objective function defined by Equation (4) in Section 2 has to be minimized as a function of the prefilter coefficients so that the identification data prefiltered by the optimal prefilter synthesized gives a control relevant model.

The optimization routine is terminated with a set of optimal bilinear prefilter coefficients, and a control relevant bilinear model. Now, using this control relevant bilinear model, NIMC based controller is synthesized in the following manner:
Let the bilinear model estimated is represented as,

$$\begin{aligned} y(i) &= a_m y(i-1) \\ &+ b_m u(i-1) + c_m y(i-1)u(i-1) \end{aligned} \quad (7)$$

The Equation (7) can be rearranged using one step ahead predictor form as,

$$u(i) = \frac{y(i+1) - a_m y(i)}{b_m + c_m y(i)} \quad (8)$$

Equation (8) is a formal representation of the inverse of a bilinear system, which is required for the implementation of NIMC strategy based closed loop system. However, it is to be noted here that the controller (model inverse) output $u(i)$ is dependent on the next sample of the controller input, that is, the control error, $e(i+1)$(replacing output y with control error e in Equation (8)). This is similar to arriving at non causal structure for the inverse of linear models. By considering $u(i)$ as $u(i+1)$ on left hand side of Equation (8), the estimation of inverse becomes causal. This is similar to partitioning the linear model into all pass and minimum phase elements and writing the linear model inverse as the inverse of the all pass element. Thus the model inverse can be defined as,

$$u(i+1) = \frac{e(i+1) - a_m e(i)}{b_m + c_m e(i)} \quad (9)$$

where, u is the output of model inverse and e is the input to the model inverse(after replacing output of model y with input to the controller, that is controller error e). This model inverse given by Equation (9) is augmented by the IMC filter, $f_{imc}(z)$, as defined in Equation (6). The linear filter $f_{imc}(z)$ is introduced for robustness and realizability. Also, when the control error e does not belong to the output space of model (which is input space for the model inverse), it is assumed that $f_{imc}(z)$ will project e into the appropriate space. Using the above proposed bilinear model based controller, the true plant performance is analyzed.

4 Illustrative example

The control relevant bilinear model estimation methodology proposed in this paper is validated by applying to a representative nonlinear plant having NARX structure, given by:

$$\begin{aligned} y(k) &= 0.6y(k-1)+0.5y(k-2) \\ &- 0.3y(k-3)+2u(k-1) \\ &+ 0.5u(k-2)+0.6y(k-1)u(k-1) \\ &+ 0.2y(k-2)u(k-2) \end{aligned} \tag{10}$$

The rise time for closed loop response is specified to be 8 seconds and a sampling time of 1 second is chosen. This plant is perturbed by 3-level pseudo random signal, and the input/output data are recorded. Using this data, new set of data is generated with product of input and output vector, which is used as a second input in fitting bilinear element. Multi input single output ARX routine is used to fit the bilinear model. Using this set of data, a bilinear model is estimated which can be written as,

$$\begin{aligned} y_d(k) &= 0.9014y_d(k-1)+2.0529u_d(k-1) \\ &+ 0.5899y_d(k-1)u_d(k-1) \end{aligned} \tag{11}$$

A prefilter in bilinear structure is initialized, and using the algorithm proposed in Section 3, the prefilter is synthesized as,

$$\begin{aligned} y_f(k) &= -0.1711y_f(i-1)+0.1760y(i-1) \\ &- 0.0066y_f(i-1)y(i-1) \end{aligned} \tag{12}$$

Similarly, the input and the product of input/output are also filtered. The control relevant bilinear model estimated using the prefiltered data set can be written as,

$$\begin{aligned} y_c(k) &= 0.8518y_c(k-1)+1.9514u_c(k-1) \\ &+ 0.4569y_c(k-1)u_c(k-1) \end{aligned} \tag{13}$$

The frequency response characteristics of the NARX plant, direct estimated bilinear model, control relevant bilinear model and the complementary sensitivity function is shown in Figure 1. This shows the good match of the dynamics of the control relevant bilinear model with that of the NARX plant in the closed loop frequency band of interest. The frequency band of interest is up to the bandwidth of the complementary sensitivity function. The closed loop performance exhibited by direct and control relevant bilinear model based system are shown in Figure 2. The response from the control relevant bilinear model based system is superior and as per specification. These simulation results validates the proposed method.

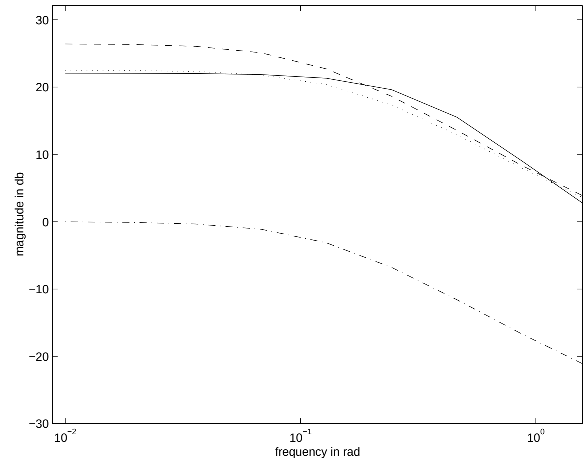

Figure 1: Generalized frequency response function comparison: 'solid' True plant, 'dash dash' nominal bilinear model, 'dot dot' Control relevant bilinear model, 'dash dot' nominal complementary sensitivity function

5 Conclusion

A prefilter based approach for the control relevant bilinear model estimation is proposed. A prefilter having bilinear structure is synthesized so as to minimize the model plant mismatch expressed in terms of the generalized frequency response characteristics of the NARX plant and the bilinear model. The model plant mismatch is weighted by closed loop performance requirement so as to bias the identification data from closed loop performance point of view. A bilinear model based NIMC strategy is employed and the simulation results validate the proposed approach.

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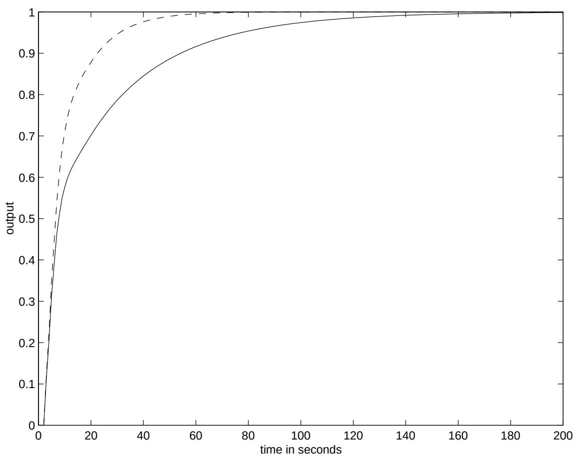

Figure 2: Closed loop performance comparison: ‘solid’ using nominal bilinear model, ‘dash dash’ using control relevant bilinear model’

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