MISO-Structure-Based Control-Relevant Identification of MIMO Systems with Steady-State Gain Matching

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The prefilter-based control-relevant identification (CRI) scheme proposed by Rivera et al. [*IEEE Trans. Autom. Control* **1992**, *37* (7), 964−974] for single-input/single-output (SISO) systems estimates a model of the system that matches the true system dynamics in a range of frequencies that are important and relevant for closed-loop performance. In this paper, an extension of this approach is proposed for the CRI of multi-input/multi-output (MIMO) systems by partitioning the overall system into individual multi-input/single-output (MISO) structures. Furthermore, CRI methodologies proposed hitherto have not examined the issue of accurate identification of the steady-state gain. In this paper, the importance of achieving good steady-state gain match is emphasized, and a method is proposed to identify a control-relevant model that has a good match, at steady state (zero frequency), as well as in the control-relevant frequency band, with the true plant. The effect of the proposed control-relevant prefiltering on the identification of the high- and low-gain directions is also analyzed in this paper. The proposed methodology is validated using simulations on representative problems from the literature and on a paper machine problem.

1. Introduction

The emergence of model-based control has given an impetus to the idea of estimating models whose end use is for the design of the controller. This controller, when applied to the true system, is expected to give desired closed-loop performance as specified. The branch of system identification that deals with such strategies is called control-relevant identification (CRI). The focus of research publications by Rivera and co-workers,[1−12] Kwok and Shah,[13] Shook et al.,[14] and Van den Hof and Schrama[15,16] is mainly centered on CRI. In traditional system identification, the objective is the minimization of bias and variance error of the estimated model, whereas in CRI, the aim is to minimize the model−plant mismatch in the frequency band that is most relevant from a closed-loop-performance viewpoint. Rivera and Gaikwad[9] have discussed the modeling issues for achieving satisfactory closed-loop performance using a model-based controller. Rivera and co-workers[1,5−7,9] have focused on prefilter-based methods for the CRI of SISO systems for feedback and combined feed-forward/feedback control design and have also discussed issues such as input design.[8,10,12] Kwok and Shah[13] proposed a method wherein the control-relevant model is estimated with a terminal matching condition that makes computation easy in a generalized predictive control scheme. Van den Hof and Schrama[16] showed that, when the estimated model exhibits minimum mismatch with the true system in the closed-loop frequency range of interest, the resulting closed-loop system with the model-based controller exhibits performance as specified. Schrama and Bosgra[15] proposed an iterative identification and control design methodology in the frequency domain, employing co-prime factor perturbations. Using an approach similar to that employed for SISO systems, Rivera and Gaikwad[9] proposed a methodology for the CRI of MIMO systems through the prefiltering route, wherein the prefilter is assumed to be a diagonal matrix consisting of identical prefilters as its elements. They also proposed a CRI scheme for MIMO systems using a multivariable frequency-response curve-fitting technique[3] and orthogonal multifrequency input perturbations.[10] Input signal design is also very important in accurate system identification and has been addressed by Davies, Gorfrey, Koung and MacGregor, and Srinivasan and Rengasamy.[17−21]

When compared with methodologies proposed for SISO systems, the CRI of MIMO systems requires that the following critical issues be addressed: The first issue is that of accurate identification of gain directionality, which is important from a closed-loop-performance viewpoint. Toward this end, Koung and MacGregor[19,20] proposed a suitable experiment design for estimating MIMO systems with accurate characterization of the low-gain direction at the steady-state condition. They proposed a method based on singular-value decomposition for the design of correlated, nonbinary signals.

Li and Lee[22,23] have sought to achieve accurate characterization of the low- and high-gain directions by minimizing the error between the plant and model, as well as their inverses. However, the minimization requires a proxy plant and its inverse, which they developed by fitting a high-order autoregressive with exogenous input (ARX) model to each input/output pair. These proxy models are used to generate frequency-domain data, and minimization is performed in a least-squares sense. However, in their method, as the size of

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the MIMO problem increases, the number of experiments to be performed on the plant also increases.

Prefiltering of the input/output data through an appropriate filter structure also plays an important role in the accurate characterization of gain directionality. Toward this end, Andersen et al.[24] proposed that MIMO models be estimated by combining MISO models fitted to the low-pass-filtered data obtained with one loop closed. They also showed that the resulting MIMO model yielded a reasonable characterization of the low-gain plant direction.

The second issue that is hitherto unaddressed even for SISO systems is the accurate identification of steady-state gain, which is an important requirement from the viewpoints of closed-loop robustness and performance. Banerjee and Shah[25] showed that, at steady state, the small-gain condition for stability will never be violated if the steady-state gain of model exceeds one-half of the steady-state gain of the plant. Thus, accurate estimation of the steady-state gain of the system within certain bounds is important from a closed-loop stability and performance perspective. From a practitioners viewpoint as well, it is desirable to estimate a model that not only matches the frequency response of the true system in the control-relevant frequency range but also gives a good match at the steady-state condition. Toward this end, Inouye and Kojima[26] proposed methods for the model reduction of linear systems, whose steady-state values of the step responses are identical to those of the original system, based on constrained-least-squares and modified-least-squares approximation methods. Also, Ludlage et al.[27] used a priori knowledge of static gains in MIMO process identification and estimated models that exhibit steady-state gain match with the original system by forcing the model steady-state gain to be the same as that of the original system in the identification process.

From an implementation viewpoint, the CRI method of Rivera and co-workers,[1,5−7] which is based on prefiltering of input/output data to bias or highlight the frequency content of interest, is an elegant approach for SISO systems. For MIMO systems, the frequency-response curve-fitting methods proposed by them,[2,3,8−10,12] although practicable, could perhaps lead to computational complexity as the dimension of the MIMO system increases. An approach based on prefiltering that is computationally less intensive is therefore desirable. Furthermore, the assumption of a diagonal prefilter consisting of identical elements made in their work[3,9] could be restrictive if the control specifications in the various channels are different.

This paper analyzes the role of CRI in the context of the issues discussed above. Here, an alternate methodology is proposed for performing CRI of MIMO systems by prefiltering input/output open-loop data. The methodology proposed in this paper extends the CRI strategy proposed by Rivera and Gaikwad in refs 6 and 7 for combined feed-forward/feedback control design to MIMO systems. The approach proposed here is based on partitioning of the $n \times n$ MIMO system into n MISO subsystems, which are then designed for good tracking and regulatory performances. The interactions from all other branches to each single loop, including decoupler effects, are considered as measured structured disturbances along with the regular output disturbances. By considering a 2×2 system as an illustration, two separate prefilter expressions for each channel are

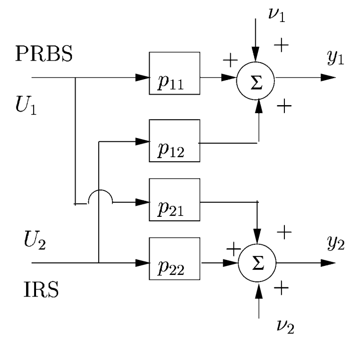

Figure 1. General 2×2 system representation.

designed by specifying different desired speeds of response for closed-loop performance in each channel. To achieve a good match of the steady-state gain of the identified models with that of the true plant, a modified input design and prefiltering strategy is proposed. It is shown here that a full matrix of prefilter transfer function results as a solution to the CRI problem and that this approach can accommodate differing control specifications in each channel relatively easily. The proposed prefiltering methodology is then used to prefilter the input/output data by the appropriate prefilters, and MISO models are identified by fitting to the prefiltered data. It has been shown that the resulting MISO models, when combined into a MIMO framework, exhibit good gain directionality properties. Using these representative models, a multiloop internal model control (IMC) scheme with a decoupler is proposed. Evaluation of the proposed approach is carried out through closed-loop simulation for tracking and disturbance rejection involving representative systems taken from the literature.

The paper is structured as follows: In section 2, the design of prefilters for each of the SISO channels of the partitioned 2×2 system is presented, along with an extension for a general $n \times n$ system. A method for obtaining the steady-state gain match of the control-relevant model with that of the true plant is presented in section 3. Some issues about gain directionality and the role of prefiltering for accurate characterization of the high- and low-gain directions are discussed in section 4. The methodology for estimating control-relevant models of the direct branch and the interacting branch elements in each of the channels of the plant is proposed in section 5, along with the requirement of steady-state gain matching. The utility of the overall identification methodology is demonstrated through simulations in section 6.

2. Prefilter Design for CRI of MIMO Systems

In the following section, a methodology is proposed for the design of prefilters for each of the channels of the MIMO systems. These prefilters are used for CRI of the direct and interacting branch elements of the respective channels. The development of the proposed methodology follows the definition of prefilters provided by Rivera and Gaikwad[6,7] for combined feed-forward/feedback control design. The methodology is illustrated here for a general 2×2 system as shown in Figure 1. Assuming that the choice of input/output pairing for control is made appropriately, each input/output combination can be treated as an independent MISO subsystem. In doing so, all other branches interacting with each of the single loops are treated as measured

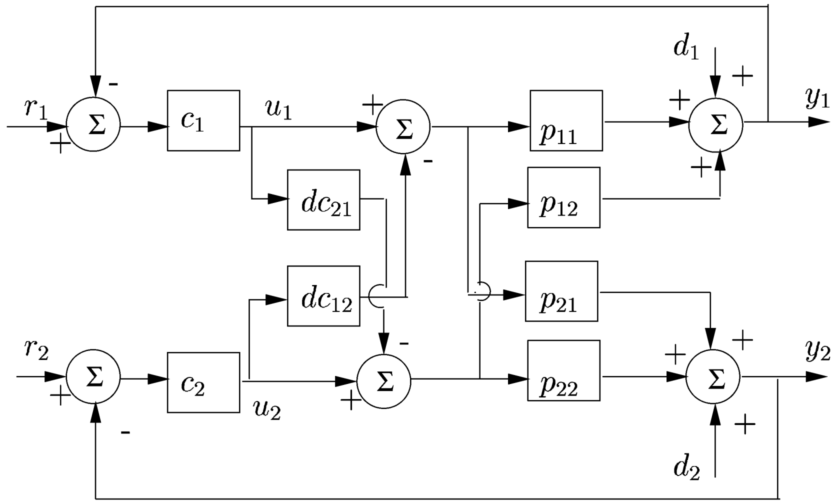

Figure 2. Closed-loop system with decoupler.

structured disturbances acting on the respective loops, in addition to the regular disturbance that affects the particular loop under consideration. A decoupler-based multiloop IMC strategy is proposed as the control structure. This representation is shown schematically in Figure 2. Thus, in addition to good servo properties, each individual loop will also have specifications for the rejection of disturbances that affect the output directly and through the interacting branches. The prefilter expressions for the control-relevant estimation of the direct branch and the interacting branch elements of each of the channels are derived as follows.

Consider channel 1 in Figure 1. The prediction error, $e_1(t)$, for the MISO system between y_1, U_1, and U_2 can be written as

$$e_1(t) = \tilde{p}_{e_1}^{-1}(z)\{[p_{11}(z) - \tilde{p}_{11}(z)]U_1(t) + [p_{12}(z) - \tilde{p}_{12}(z)]U_2(t) + \nu_1(t)\} \quad (1)$$

where $p_{11}(z)$ and $p_{12}(z)$ are the true system transfer function elements; $\tilde{p}_{11}(z)$ and $\tilde{p}_{12}(z)$ are the respective nominal model elements estimated; $\tilde{p}_{e_1}(z)$ is the noise model associated with first MISO model; $U_1(t)$ and $U_2(t)$ are the inputs to the system and model, respectively; and $\nu_1(t)$ is a stationary noise sequence having power spectrum ϕ_{ν_1}.

Equation 1 can alternatively be written as

$$e_1(t) = \tilde{p}_{e_1}^{-1}(z)\, \tilde{p}_{11}(z)\, e_{m_{11}}(z)\, U_1(t) + \tilde{p}_{e_1}^{-1}(z)\, e_{a_{12}}(z)\, U_2(t) + \tilde{p}_{e_1}^{-1}(z)\, \nu_1(t) \quad (2)$$

where $e_{m_{11}}(z)$ is the multiplicative error of the nominal model $\tilde{p}_{11}(z)$ with the plant element $p_{11}(z)$ and $e_{a_{12}}(z)$ is the additive error of the interacting branch nominal model $\tilde{p}_{12}(z)$ with the respective plant element $p_{12}(z)$.

These errors are expressed as

$$e_{m_{11}}(z) = \frac{p_{11}(z) - \tilde{p}_{11}(z)}{\tilde{p}_{11}(z)} \quad \text{and} \quad e_{a_{12}}(z) = p_{12}(z) - \tilde{p}_{12}(z) \quad (3)$$

The error due to model−plant mismatch in the estimations of the direct branch element and the interacting branch element contributes to the prediction error given by eq 2, in addition to the contribution due to measurement noise $\nu_1(t)$. Let the contribution to the prediction error due to the model−plant mismatch of the direct branch element be $e_{11}(t)$ and that due to the interacting branch element be $e_{12}(t)$. Using these definitions for the individual contributions to the total prediction error, eq 2 can be written as

$$e_1(t) = e_{11}(t) + e_{12}(t) + \tilde{p}_{e_1}^{-1}(z)\, \nu_1(t) \quad (4)$$

Let $f_{11}(z)$ and $f_{21}(z)$ be the prefilters that must be used for filtering of the two channel inputs to estimate the control-relevant model of the elements associated with the first channel. These prefilters have to be designed such that the identification errors $e_{m_{11}}(z)$ and $e_{a_{12}}(z)$ are biased in a user-specified manner. That is, the individual terms contributing to the prediction error, as given by eq 4, need to be minimized over a desired frequency range, specified by the filters $f_{11}(z)$ and $f_{21}(z)$. The objective function for CRI is to design the prefilters $f_{11}(z)$ and $f_{21}(z)$ based on (i) the desired closed-loop specification, (ii) the assumed model structure, (iii) the nature of the set-point/disturbance signals to which the closed-loop system will be subjected, and (iv) the control strategy to be used. Now, using prefilters $f_{11}(z)$ and $f_{21}(z)$, the expression for the filtered prediction error can be written as

$$e_{1_f}(t) = f_{11}(z)\, e_{11}(t) + f_{21}(z)\, e_{12}(t) \quad (5)$$

In writing eq 5 above, it is assumed that the measurement noise, $\nu_1(t)$, makes a negligible contribution to the prediction error (that is, the signal-to-noise ratio at the output is $\gg 1$ (**cf. Remark 3**). It must be noted here that only the total prediction error $e_1(t)$ is actually measurable in practice and not the constituent parts $e_{11}(t)$ and $e_{12}(t)$. However, we assume for the purposes of derivation that the latter quantities are measurable and can be filtered through the prefilters. This assumption, however, does not affect the analysis and implementation of the resultant prefilters. The identification objective is to minimize the mean squared filtered prediction error, V_1, defined as

$$V_1 = \lim_{N\to\infty} \frac{1}{N}\sum_{t=0}^{N}[e_{1_f}(t)]^2 \quad (6)$$

Using eqs 2, 5, and 6, the frequency-domain expression for the mean squared filtered prediction error

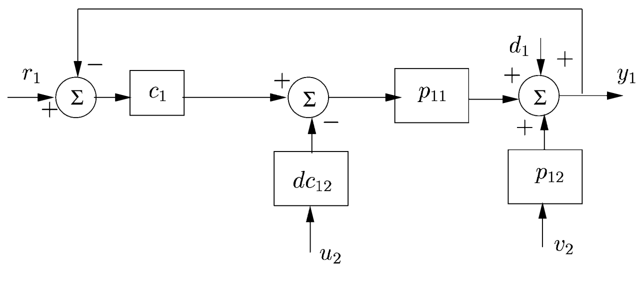

Figure 3. Equivalent SISO loop for channel 1.

minimization function can be written as

$$\min_{\tilde{p}_{11},\tilde{p}_{12}} V_1 = \min_{\tilde{p}_{11},\tilde{p}_{12}} \frac{1}{2\pi}\int_{-\pi}^{+\pi} \frac{|f_{11}(\mathrm{e}^{j\omega})|^2}{|\tilde{p}_{e_1}(\mathrm{e}^{j\omega})|^2}|\tilde{p}_{11}(\mathrm{e}^{j\omega})|^2|e_{\mathrm{m}_{11}}(\mathrm{e}^{j\omega})|^2\phi_{U_1}(\omega)\,\mathrm{d}\omega + \frac{1}{2\pi}\int_{-\pi}^{+\pi}|e_{\mathrm{a}_{12}}(\mathrm{e}^{j\omega})|^2\phi_{U_2}(\omega)\frac{|f_{21}(\mathrm{e}^{j\omega})|^2}{|\tilde{p}_{e_1}(\mathrm{e}^{j\omega})|^2}\,\mathrm{d}\omega \quad (7)$$

where ϕ_{U_1} and ϕ_{U_2} are the power spectra of the perturbation inputs. In writing eq 7, perturbation inputs U_1 and U_2 are assumed to be uncorrelated. The objective function given by eq 7 is to be minimized by minimizing $e_{\mathrm{m}_{11}}$ and $e_{\mathrm{a}_{12}}$, that is, by suitably estimating the models $\tilde{p}_{11}$ and $\tilde{p}_{12}$.

Now, consider loop 1 of the 2 × 2 system, as shown in Figure 3. This loop is acted upon by the disturbances due to the interaction effect of the second loop and the regular disturbance at the output. Also, the effect of the decoupler element is considered in a manner similar to that of the feed-forward controller. The expression for the control error e_{c_1} $(= r_1 - y_1)$ for the first channel with the feedback controller and the decoupler is given by[28]

$$e_{\mathrm{c}_1}(z) = \frac{\tilde{\epsilon}_1(z)}{1 + \tilde{\eta}_1(z)e_{\mathrm{m}_{11}}(z)}\{[r_1(z) - d_1(z)] - p_{12}(z)\, v_2(t) + dc_{12}(z)\, p_{11}(z)\, u_2(t)\} \quad (8)$$

where $\tilde{\epsilon}_1(z)$ and $\tilde{\eta}_1(z)$ are the nominal sensitivity function and nominal complementary sensitivity function of loop 1, respectively, given by

$$\tilde{\epsilon}_1(z) = \frac{1}{1 + \tilde{p}_{11}(z)\, c_1(z)} \quad \text{and} \quad \tilde{\eta}_1(z) = \frac{\tilde{p}_{11}(z)\, c_1(z)}{1 + \tilde{p}_{11}(z)\, c_1(z)} \quad (9)$$

and $dc_{12}(z) = \tilde{p}_{12}(z)/\tilde{p}_{11}(z)$ is the decoupler element between channel 1 and channel 2.

Assuming that the small-gain theorem given by

$$|\tilde{\eta}(\mathrm{e}^{j\omega})\, e_{\mathrm{m}}(\mathrm{e}^{j\omega})| \ll 1, \quad \forall -\pi \le \omega \le \pi \quad (10)$$

is valid in the control-relevant frequency range,[1] the term $[1 + \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)]^{-1}$ in eq 8 can be approximated as $[1 - \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)]$, considering only the linear term in the Taylor series expansion of $[1 + \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)]^{-1}$. Using eq 3 and the above approximation, eq 8 can be written as

$$e_{\mathrm{c}_1}(z) \approx \tilde{\epsilon}_1(z)[1 - \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)]\{[r_1(z) - d_1(z)] - \tilde{p}_{12}(z)\, v_2(t) - e_{\mathrm{a}_{12}}(z)\, v_2(t) + dc_{12}(z)\, \tilde{p}_{11}(z)\, u_2(t) + dc_{12}(z)\, \tilde{p}_{11}(z)\, e_{\mathrm{m}_{11}}(z)\, u_2(t)\} \quad (11)$$

Expanding eq 11 and neglecting terms containing the product of model uncertainties [i.e., $e_{\mathrm{m}_{11}}(z)\, e_{\mathrm{m}_{11}}(z)$ and $e_{\mathrm{m}_{11}}(z)\, e_{\mathrm{a}_{12}}(z)$], eq 11 can be expressed as

$$\begin{aligned} e_{\mathrm{c}_1}(z) \approx{}& \tilde{\epsilon}_1(z)[r_1(z) - d_1(z)] - \tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)[r_1(z) - d_1(z)] - \tilde{\epsilon}_1(z)\, \tilde{p}_{12}(z)\, v_2(t) \\ &+ \tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)\, \tilde{p}_{12}(z)\, v_2(t) - \tilde{\epsilon}_1(z)\, e_{\mathrm{a}_{12}}(z)\, v_2(t) \\ &+ \tilde{\epsilon}_1(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, u_2(t) + \tilde{\epsilon}_1(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, e_{\mathrm{m}_{11}}(z)\, u_2(t) \\ &- \tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, u_2(t) \end{aligned} \quad (12)$$

The above control error can be shown to consist of two components as follows

$$\begin{aligned} e_{\mathrm{c}_1}(z) \approx{}& \{\tilde{\epsilon}_1(z)[r_1(z) - d_1(z)] - \tilde{\epsilon}_1(z)\, \tilde{p}_{12}(z)\, v_2(t) + \tilde{\epsilon}_1(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, u_2(t)\} \\ &+ \{-\tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)[r_1(z) - d_1(z)] + \tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)\, \tilde{p}_{12}(z)\, v_2(t) \\ &- \tilde{\epsilon}_1(z)\, e_{\mathrm{a}_{12}}(z)\, v_2(t) + \tilde{\epsilon}_1(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, e_{\mathrm{m}_{11}}(z)\, u_2(t) \\ &- \tilde{\epsilon}_1(z)\, \tilde{\eta}_1(z)\, e_{\mathrm{m}_{11}}(z)\, dc_{12}(z)\, \tilde{p}_{11}(z)\, u_2(t)\} \end{aligned} \quad (13)$$

The terms in the first braces contribute to the control error even in the nominal case (assuming $\tilde{p}_{11} = p_{11}$ and $\tilde{p}_{12} = p_{12}$). In an IMC context, this component of the control error could arise as a result of performance degradation from perfect control due to the introduction of IMC filter, which is essential from the realizability and robustness point of view, even when model–plant mismatch is zero. The second component essentially quantifies the performance degradation due to model identification. It is this component that we seek to minimize in the CRI through appropriate prefilter and model identification. Hence, the contribution to the control error from an identification point of view is written by considering only the component in the second braces of eq 13, i.e.

$$e_{\mathrm{c}_1}(z) \approx -\tilde{\epsilon}_1(z)(\tilde{\eta}_1(z)(r_1 - d_1) - \{\tilde{\eta}_1(z)\, \tilde{p}_{12}(z)v_2 + dc_{12}(z)\, \tilde{p}_{11}(z)[1 - \tilde{\eta}_1(z)]u_2\})e_{\mathrm{m}_{11}}(z) - \tilde{\epsilon}_1(z)\, e_{\mathrm{a}_{12}}(z)v_2 \quad (14)$$

$$\approx -\tilde{\epsilon}_1(z)\{\tilde{\eta}_1(z)(r_1 - d_1) - [\tilde{\eta}_1(z)\, \tilde{p}_{12}(z)v_2 + dc_{12}(z)\, \tilde{p}_{11}(z)\, \tilde{\epsilon}_1(z)u_2]\}e_{\mathrm{m}_{11}}(z) - \tilde{\epsilon}_1(z)\, e_{\mathrm{a}_{12}}(z)v_2 \quad (15)$$

The objective function for control error minimization by better model identification can be expressed as

$$\min_{\tilde{p}_{11},\tilde{p}_{12}} ||e_{c_1}(z)||_2 \approx \min_{\tilde{p}_{11},\tilde{p}_{12}} ||-\tilde{\epsilon}_1(z)\{\tilde{\eta}_1(z)(r_1 - d_1) - [\tilde{\eta}_1(z)\, \tilde{p}_{12}(z)v_2 + dc_{12}(z)\, \tilde{p}_{11}(z)\, \tilde{\epsilon}_1(z)u_2)]e_{m_{11}}(z) - \tilde{\epsilon}_1(z)\, e_{a_{12}}(z)v_2|_2 \quad (16)$$

Using the triangular inequality property of the norms, i.e., $||A - B||_2 \approx ||A||_2 + ||B||_2$,[29] the expression above can be decomposed into terms containing $e_{m_{11}}$ and $e_{a_{12}}$ as

$$\min_{\tilde{p}_{11},\tilde{p}_{12}} ||e_{c_1}(z)||_2 \approx \lim_{\tilde{p}_{11}} ||\tilde{\epsilon}_1(z)\{\tilde{\eta}_1(z)(r_1 - d_1) - [\tilde{\eta}_1(z)\, \tilde{p}_{12}(z)v_2 + dc_{12}(z)\, \tilde{p}_{11}(z)\, \tilde{\epsilon}_1(z)u_2]\}e_{m_{11}}(z)||_2 + \min_{\tilde{p}_{12}} ||\tilde{\epsilon}_1(z)\, e_{a_{12}}(z)v_2||_2 \quad (17)$$

Using Parseval's theorem, the frequency-domain representation of the CRI objective function given by eq 17 can be written as

$$\min_{\tilde{p}_{11},\tilde{p}_{12}} ||e_{c_1}(e^{j\omega})||_2 \approx \min_{\tilde{p}_{11}} \frac{1}{2\pi}\int_{-\pi}^{\pi}|\tilde{\epsilon}_1(e^{j\omega})|^2|\{\tilde{\eta}_1(e^{j\omega})[r_1(e^{j\omega}) - d_1(e^{j\omega})] - [\tilde{\eta}_1(e^{j\omega})\, \tilde{p}_{12}(e^{j\omega})\, v_2(e^{j\omega}) + dc_{12}(e^{j\omega})\, \tilde{p}_{11}(e^{j\omega})\, \tilde{\epsilon}_1(e^{j\omega})\, u_2(e^{j\omega})]\}|^2|e_{m_{11}}(e^{j\omega})|^2\, d\omega + \min_{\tilde{p}_{12}} \frac{1}{2\pi}\int_{-\pi}^{\pi}|\tilde{\epsilon}_1(e^{j\omega})\, e_{a_{12}}(e^{j\omega})\, v_2(e^{j\omega})|^2\, d\omega \quad (18)$$

By comparing separately the terms associated with $e_{m_{11}}$ and $e_{a_{12}}$ in the frequency-domain prediction error minimization function given by eq 7 and the control-relevant parameter estimation expression given by eq 18, two separate prefilter expressions can be defined. These prefilters are used for the CRI of the plant and of the disturbance functions, respectively. In doing so, it is assumed that the power spectra of the inputs ϕ_{U_1} and ϕ_{U_2} are unity and that the perturbation inputs U_1 and U_2 are uncorrelated sequences. Thus, the resulting expressions for the two prefilters are

$$f_{11}(z) = \tilde{p}_{e_1}(z)\, \tilde{p}_{11}(z)^{-1}\, \tilde{\epsilon}_1(z)\{\tilde{\eta}_1(z)(r_1 - d_1)(z) - [\tilde{\eta}_1(z)\, \tilde{p}_{12}(z)\, v_2(z) + \tilde{\epsilon}_1(z)\, \tilde{p}_{11}(z)\, dc_{12}(z)\, u_2(z)]\} \quad (19)$$

$$f_{21}(z) = \tilde{p}_{e_1}(z)\, \tilde{\epsilon}_1(z)\, v_2(z) \quad (20)$$

Similarly, the two prefilters [that is, $f_{12}(z)$ and $f_{22}(z)$] associated with the CRI of the interacting and direct branch elements associated with the second channel can be written as

$$f_{12}(z) = \tilde{p}_{e_2}(z)\, \tilde{\epsilon}_2(z)\, v_1(z) \quad (21)$$

$$f_{22}(z) = \tilde{p}_{e_2}(z)\, \tilde{p}_{22}(z)^{-1}\, \tilde{\epsilon}_2(z)\{\tilde{\eta}_2(z)(r_2 - d_2)(z) - [\tilde{\eta}_2(z)\, \tilde{p}_{21}(z)\, v_1(z) + \tilde{\epsilon}_2(z)\, \tilde{p}_{22}(z)\, dc_{21}(z)\, u_1(z)]\} \quad (22)$$

Thus, for each loop, two separate prefilters based on (i) the performance specifications, (ii) the assumed model structure, (iii) the chosen controller structure, and (iv) the nature ofthe set-point/disturbance signal to which the loop will be subjected can be designed to identify the control-relevant models of the direct and interacting elements associated with the respective loops. As can be seen, the above-derived prefilter expression for each channel has an explicit characterization of the disturbance and the sensitivity function associated with the respective channel, as well as the "disturbance" and decoupler effects due to interactions from the other channel.

The method proposed in this paper is an alternate approach to that proposed by Rivera and Gaikwad[3,9] for the CRI of MIMO systems. In their approach, the scalar prefilter amplitude ratio is iteratively estimated by minimizing the MIMO control error minimization objective function. The diagonal matrix with its elements as the estimated scalar prefilter is used to prefilter the input/output data to obtain the control-relevant model. They define the MIMO control-relevant parameter estimation problem following an approach similar to that of the SISO case by adapting the matrix fraction description (MFD) technique. The method proposed in our paper is a straightforward extension of the methodology proposed by Rivera and Gaikwad[6,7] for the CRI of SISO systems for combined feed-forward/feedback control design. In the method proposed here, it is seen that a separate prefilter expression results for the CRI of each of the plant elements. Hence, the resulting prefilter obtained by following the method proposed in our paper is, in general, nondiagonal and nonscalar.

2.1. Extension to the General $n \times n$ Case. The above derivation was considered specifically for the 2×2 case for simplicity. Likewise for the general case one could arrive at the expressions for the prefilter by equating the relevant terms in the prediction error minimization problem with the corresponding terms in the control-relevant parameter estimation problem.

The expression for the prediction error minimization can be written analogously to eq 7 as

$$\min_{\tilde{p}_{ij}} V_i = \min_{\tilde{p}_{ij}} \frac{1}{2\pi}\int_{-\pi}^{+\pi}\frac{|f_{ii}(e^{j\omega})|^2}{|\tilde{p}_{e_i}(e^{j\omega})|^2}|\tilde{p}_{ii}(e^{j\omega})|^2|e_{m_{ii}}(e^{j\omega})|^2\phi_{U_i}(\omega)\, d\omega + \sum_{j=1,j\neq i}^{n}\left[\frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|e_{a_{ij}}(e^{j\omega})\right|^2\phi_{U_j}(\omega)\frac{|f_{ji}(e^{j\omega})|^2}{|\,\tilde{p}_{e_i}(e^{j\omega})|^2}\, d\omega\right] \quad (23)$$

The expression for the control error minimization function can be developed for the $n \times n$ case (refer to eq 18) and written as

$$\min_{\tilde{p}_{ij}} ||e_{c_i}(e^{j\omega})||_2 \leq \min_{\tilde{p}_{ii}} \frac{1}{2\pi}\int_{-\pi}^{\pi}|\tilde{\epsilon}_i(e^{j\omega})|^2|(\tilde{\eta}_i(e^{j\omega})[r_i(e^{j\omega}) - d_i(e^{j\omega})] + \sum_{j=1,j\neq i}^{n}\{-[\tilde{\eta}_i(e^{j\omega})\, \tilde{p}_{ij}(e^{j\omega})\, v_j(e^{j\omega}) + dc_{ij}(e^{j\omega})\, \tilde{p}_{ii}(e^{j\omega})\, \tilde{\epsilon}_i(e^{j\omega})\, u_j(e^{j\omega})]\})|^2|e_{m_{ii}}(e^{j\omega})|^2\, d\omega + \min_{\tilde{p}_{ij}} \sum_{j=1,j\neq i}^{n}\frac{1}{2\pi}\int_{-\pi}^{\pi}|\tilde{\epsilon}_i(e^{j\omega})\, e_{a_{ij}}(e^{j\omega})\, v_j(e^{j\omega})|^2\, d\omega \quad (24)$$

By comparing separately the terms associated with $e_{m_{ii}}$ and $e_{a_{ij}}$ in the frequency-domain prediction error minimization function given by eq 23 and the control-

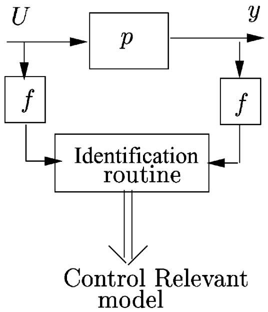

Figure 4. Implementation of prefilters.

relevant parameter estimation expression given by eq 24, one can arrive at the prefilter expressions as follows

Diagonal prefilter

$$f_{ii}(z) = \tilde{p}_{e_i}(z)\, \tilde{p}_{ii}(z)^{-1}\, \tilde{\epsilon}_i(z)\{\tilde{\eta}_i(z)(r_i - d_i)(z) + \sum_{j=1,j\neq i}^{n} - [\tilde{\eta}_i(z)\, \tilde{p}_{ij}(z)\, v_j(z) + \tilde{\epsilon}_i(z)\, \tilde{p}_{ii}(z)\, dc_{ij}(z)\, u_j(z)]\} \quad (25)$$

Off-diagonal prefilters

$$f_{ji}(z) = \sum_{j=1,j\neq 1}^{n} [\tilde{p}_{e_i}(z)\, \tilde{\epsilon}_i(z)\, v_j(z)] \quad (26)$$

In eqs 23−26, V_i is the prediction error minimization objective function for channel i; $e_{\mathrm{m}_{ii}}$ is the multiplicative error in the estimation of the direct branch element associated with channel i; $e_{\mathrm{a}_{ij}}$ is the additive error in the estimation of the elements between output i and input j; f_{ii} is the prefilter for estimating the control-relevant model of the direct branch element associated with channel i; f_{ji} is the prefilter for estimating the control-relevant model of element p_{ij}; $\phi_{U_i}(\omega)$ is the power spectrum of perturbation input i; $\tilde{p}_{ii}(z)$ is the nominal model of the direct branch element associated with channel i; $\tilde{p}_{ij}(z)$ is the nominal model of the interacting branch element between output i and input j; $\tilde{p}_{e_i}(z)$ is the nominal noise model associated with channel i; $\tilde{\epsilon}_i(z)$ and $\tilde{\eta}_i(z)$ represent the nominal sensitivity function and complementary sensitivity function, respectively, associated with channel i, which implicitly incorporate the speed of closed-loop response desired for this channel; v_j and u_j are the disturbance signals from channel j to channel i due to plant interaction and to use of the decoupler, respectively; and $dc_{ij}(z)$ is the decoupler element associated with output i and input j.

Generalization to the $n \times n$ case is therefore relatively easy, as it involves only the specifications of the nominal models in the prefilter expressions and the desired performance specifications for each of the channels, along with the nature of the set-point/disturbance signal to which each channel would be subjected. Furthermore, the approach proposed here explicitly accommodates the dynamic nature of the decoupler and its effect on the CRI of the plant.

Remark 1: Implementation of Prefilters. For SISO systems, the prefilters can be implemented in the identification methodology proposed by Rivera et al.[1] by suitably prefiltering the plant output y and input U by the prefilter $f(z)$ as shown in Figure 4. The prefilter $f(z)$ can be synthesized by following either the iterative or the single-step procedure of Rivera et al.[1] Assuming that the measurement noise affecting the output has negligible effects, i.e., assuming that the signal-to-noise ratio is greater than 1, and that the measurement noise is also uncorrelated with the perturbation input U, its effect on the prediction error can be neglected. Hence, an alternate block diagram of implementing the same prefiltering is proposed here, as shown in Figure 5. In this case, the input is filtered first, and the filtered input is applied to the plant. The resulting input/output data from the plant can then be shown to yield a control-relevant model. The latter method shown in Figure 5 can be interpreted as shaping the frequency content of the input by the prefilter toward biasing of the control-relevant frequencies in the plant input/output data. Also, the latter alternate method of prefiltering is feasible only for the noniterative version of prefilter design, as each iteration in the iterative design requires plant perturbation. However, if the dominant time constants of the dynamic elements are known, the single-step method of prefilter design could be used.[1] The prefiltering methodology depicted in Figure 5 is employed in this paper in view of the MISO nature of the problem involved and the nondiagonal structure of the prefilter.

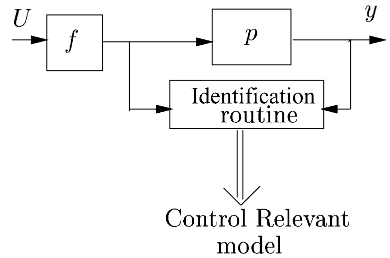

Figure 5. Alternate method of prefilter implementation.

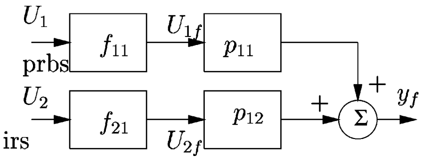

Figure 6. Prefilter implementation for MISO systems.

Remark 2. The above-developed nondiagonal, nonscalar methodology of prefiltering for MIMO systems can then be implemented using the single-step design methodology by using the block diagram shown in Figure 6 (for the MISO case of y_1, U_1, and U_2). The control-relevant models of the direct branch and interacting branch elements of the selected channel are obtained by fitting a MISO model to the data $U_{1\mathrm{f}}$, $U_{2\mathrm{f}}$, and $y_{1\mathrm{f}}$.

Remark 3. If the disturbances affecting the plant during data collection, identification of the dynamics of the plant from the input/output data would be corrupted by the disturbances. As has been shown by Rivera et al.,[1] the effects of noise on the CRI can be attenuated by the presence of the noise model $[p_e(z)]$ in the prefilter expression.

Remark 4. The derivation of the prefilters is critically dependent on the assumption indicated in eq 10, which is also made by Rivera et al.[1] Although it is easy to verify this assumption when the plant is known, in practical terms, only an estimate of the uncertainty can be obtained via the use of signal processing tools as indicated by Banerjee and Shah.[25] The only other justification for the assumption is to check that the closed-loop performance satisfies the specification, which we show in a subsequent section (section 6) appropriately.

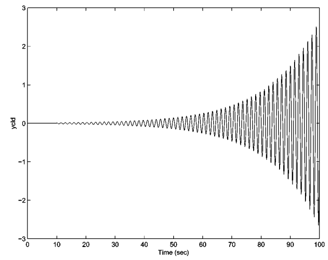

Figure 7. Closed-loop response using direct estimated model (G_{md}).

3. Analysis of Steady-State Gain Match for SISO Systems

It has been shown that the control-relevant models estimated by fitting to prefiltered open-loop data exhibit an excellent match with the true system characteristics in the control-relevant frequency band of interest. As has been the focus of such approaches, relatively lesser importance is accorded to frequencies other than the control-relevant frequencies. However, as shown by Banerjee and Shah,[25] the role of the steady-state gain match (that is, the match at zero frequency) is important from the perspective of stability and performance at steady state. In the following section, a SISO example having plant dynamics defined by

$$G_p(s) = \frac{2s + 1}{50s^3 + 65s^2 + 16s + 1}$$

is considered to illustrate this issue. Assume that a stationary disturbance having a dominant time constant of 7 s acts at the output of the closed-loop system and we seek a control strategy for disturbance regulation, having a specified speed of response of 1 s, with a sampling time of 0.1 s. The control-relevant prefilter for this problem can be derived using an approach proposed by Rivera et al.[1] and is obtained as

$$f(z) = \frac{0.0013 - 0.0027z^{-1} + 0.0013z^{-2}}{1 - 3.7872z^{-1} + 5.375z^{-2} - 3.3883z^{-3} + 0.8004z^{-4}}$$

Model G_{md}, estimated by fitting to input/output data generated from a appropriately designed pseudorandom binary sequence (PRBS) perturbation and the control-relevant model G_{mf}, fitted to the prefiltered input/output data, can be written as

$$G_{md}(z) = \frac{0.2047 \times 10^{-3}z^{-1}}{1 - 0.9999z^{-1}}, \quad G_{mf}(z) = \frac{0.0035z^{-1}}{1 - 1.0027z^{-1}}$$

The steady-state gains of the true plant (k_p) and the direct estimated (k_{md}) and control-relevant (k_{mf}) models are given by

$$k_p = 1, \quad k_{md} = 2.1151, \quad k_{mf} = -1.2967$$

The IMC-based closed-loop system using controllers based on models G_{md} and G_{mf} exhibit unstable responses, as shown in Figures 7 and 8. Whereas the unstable response of the system based on the direct estimated model (G_{md}) is attributed to its lack of a match with the true system dynamics in the control-relevant frequency band, the closed-loop system based on model G_{mf} exhibits unstable performance that is due to both the opposite sign of the steady-state gain and the presence of its pole outside the unit circle. Also, the Nyquist plot of the control-relevant model without steady-state gain match, shown in Figure 13 below, exhibits a large mismatch in the phase at low frequency and an encirclement of the point -1 of the Nyquist plot. Hence, this model-based system results in unstable closed-loop performance, as shown in Figure 8. In general, it has been observed that such a performance degradation is seen for the control-relevant case whenever the desired closed-loop speed of response is much higher than the open-loop speed of response. Hence, to overcome this problem, we propose that a control-relevant model be obtained that shows a good match in the control-relevant frequency band, as well as at the steady-state condition. In the methodology proposed here, the latter alternate method of implementation of the prefilter (noniterative), as shown in Figure 5, is utilized to ease the task of steady-state gain matching. To estimate the control-relevant model having a steady-state gain match with the true system, it is desirable that the estimation data also be weighted appropriately at low frequency, in addition to being weighted the same by the chosen prefilter in the control-relevant frequency band. Hence, a low-frequency signal of an appropriate gain is applied to the plant, along with the filtered regular input, as shown in Figure 9. To generate this low-frequency signal that is approximately uncorrelated with the original signal (i.e., the regular input), a cyclic shift can be performed on the original signal by using a different seed so as to obtain a cross correlation that is finite only at large lags. The shifted version of the regular input generated in this way can then be filtered through a first-order Butterworth filter having a low cut-off frequency of the order of 10^{-2} to 10^{-4} rad/s. This signal, which is rich in low-frequency content, is then added to the original signal. It is important to note that, because the sequence length is not altered, this procedure does not increase the experiment time. The resulting input to the plant and output from the plant (U_{df}

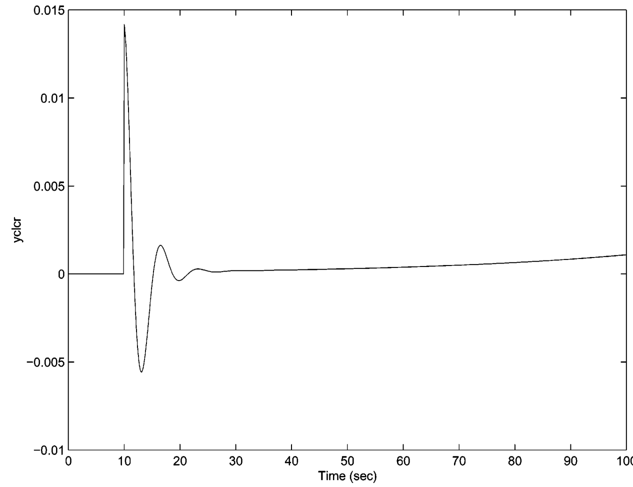

Figure 8. Closed-loop response using control-relevant model without steady-state gain match (G_{mf}).

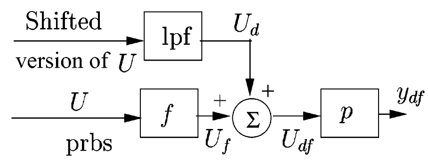

Figure 9. Schematic of control-relevant identification with steady-state gain match.

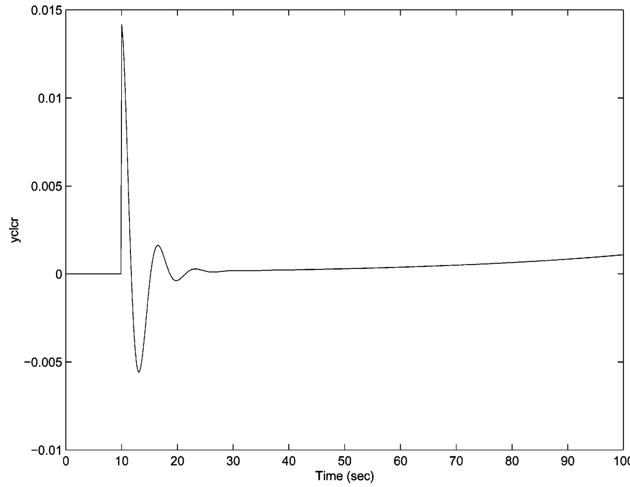

Figure 10. Closed-loop response using control-relevant model with steady-state gain match (G_{ms}).

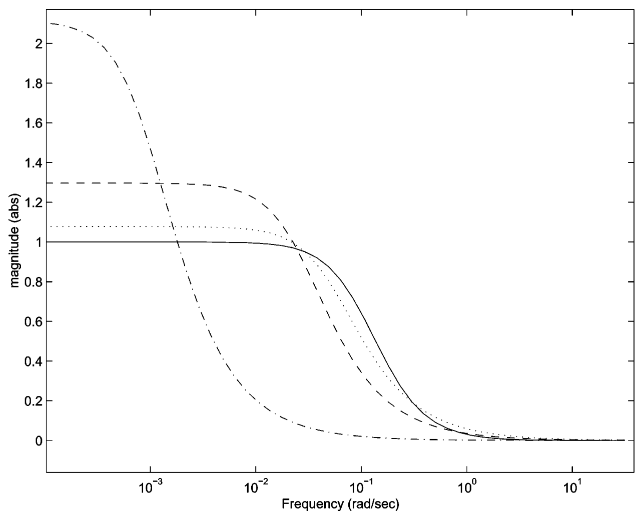

Figure 11. Bode magnitude response comparison: —, true plant (G_p); – · – · –, direct estimated model (G_{md}); – – –, control-relevant model without gain match (G_{mf}); · · ·, control-relevant model with gain match (G_{ms}).

and y_{df}, respectively) are used to fit a control-relevant model with a steady-state gain match for the above example given by

$$G_{ms}(z) = \frac{0.0059z^{-1}}{1 - 0.9945z^{-1}}$$

This model has a steady-state gain of $k_{ms} = 1.0773$. The closed-loop response obtained using the controller based on this model [$G_{ms}(z)$] is stable and satisfactory, as shown in Figure 10. The Bode and Nyquist plots of all three models are shown in Figures 11–13, respectively, along with that of the true plant. From these figures, it is evident that, whereas the magnitude plot of the control-relevant model estimated using prefiltered data (G_{mf}) exhibits a good match in the control-relevant frequency band with that of the true plant dynamics, the phase plot has a significant mismatch with that of the true plant at all frequencies. This is because the gain of the control-relevant model without steady-state gain matching (G_{mf}) has the wrong sign. However, the control-relevant model with the steady-state gain match (G_{ms}) matches the true plant dynamics with respect to

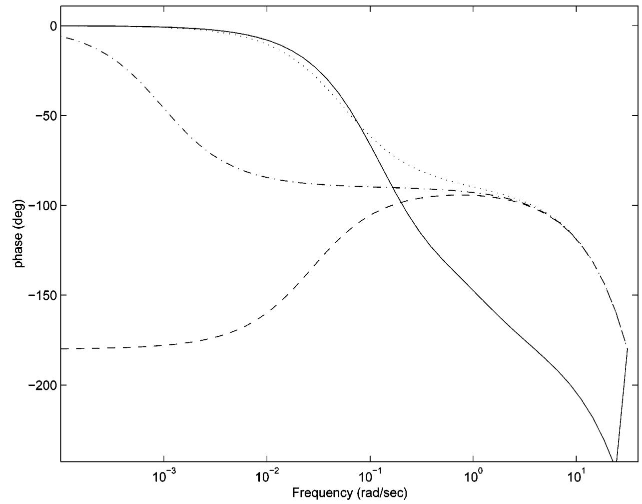

Figure 12. Bode phase response comparison: —, true plant (G_p); – · – · –, direct estimated model (G_{md}); – – –, control-relevant model without gain match (G_{mf}); · · ·, control-relevant model with gain match (G_{ms}).

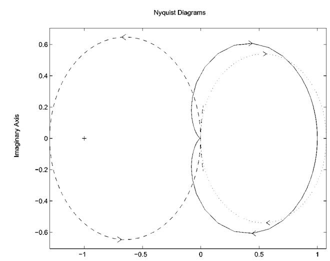

Figure 13. Nyquist response comparison: —, true plant (G_p); – · – · –, direct estimated model (G_{md}); – – –, control-relevant model without gain match (G_{mf}); · · ·, control-relevant model with gain match (G_{ms}).

the gain and phase in the control-relevant frequency band and at the steady-state condition as well. The extension of the above-proposed method for the estimation of a control-relevant model with steady-state gain matching for MIMO systems is outlined in section 5.

4. Issues of Gain Directionality

Multivariable systems generally exhibit strong interactions and ill-conditioning. The frequency response of an $l \times m$ multivariable system described by $\mathbf{G}(s)$ results in a constant complex matrix, $\mathbf{G}(j\omega)$, at any given fixed frequency ω. Let $\mathbf{G}(j\omega)$ evaluated at the frequency ω be denoted as $\mathbf{G}$. This matrix ($\mathbf{G}$) can be decomposed into

$$\mathbf{G} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \tag{27}$$

where $\mathbf{U}$ is an $l \times l$ unitary matrix of output singular vectors $\mathbf{u}_i$; $\mathbf{V}$ is an $m \times m$ unitary matrix of input singular vectors $\mathbf{v}_i$; and $\boldsymbol{\Sigma}$ is an $l \times m$ matrix with $k = \min\{l, m\}$ nonnegative singular values, σ_i, arranged in descending order along its main diagonal and with other entries equal to zero.

The column vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ are orthogonal and of unit length (orthonormal). These vectors represent the output and input directions, respectively. That is, when input in a direction represented by the unit vector $\mathbf{v}_i$ is applied to the plant, the plant output will be in a direction represented by the unit vector $\mathbf{u}_i$. The amplification or gain the system provides to the input $\mathbf{v}_i$ is given by σ_i. That is

$$\sigma_i(\mathbf{G}) = \frac{||\mathbf{G}\mathbf{v}_i||_2}{||\mathbf{v}_i||_2} = \sigma_i\frac{||\mathbf{u}_i||_2}{||\mathbf{v}_i||_2}, \quad \mathbf{v}_i \neq 0 \tag{28}$$

Thus, at any fixed frequency, the plant offers different amplifications ($\sigma_1 - \sigma_k$) for inputs of unit norm having different directions ($\mathbf{v}_1 - \mathbf{v}_k$). The largest gain for any input direction is equal to the maximum singular value

$$\bar{\sigma}(\mathbf{G}) \equiv \sigma_1(\mathbf{G}) = \frac{||\mathbf{G}\mathbf{v}_1||_2}{||\mathbf{v}_1||_2}, \quad \mathbf{v}_1 \neq 0 \tag{29}$$

and the smallest gain for any input direction is equal to the minimum singular value

$$\underline{\sigma}(\mathbf{G}) \equiv \sigma_k(\mathbf{G}) = \frac{||\mathbf{G}\mathbf{v}_k||_2}{||\mathbf{v}_k||_2}, \quad \mathbf{v}_k \neq 0 \tag{30}$$

When the ratio of the maximum singular value to the minimum singular value is very large, then the system is said to have a large condition number, and the system is ill-conditioned. That is, the condition number is given by[29]

$$\gamma(\mathbf{G}(j\omega)) = \frac{\bar{\sigma}[\mathbf{G}(j\omega)]}{\underline{\sigma}[\mathbf{G}(j\omega)]} \tag{31}$$

For dynamical systems, the singular values and associated directions vary with frequency. Evaluation of these quantities in a frequency range corresponding to the closed-loop bandwidth is of importance from the perspective of closed-loop performance. The maximum and minimum singular values are plotted similarly to the plot of the Bode's magnitude for SISO systems.

From a control-relevant viewpoint, one could analyze the characteristics of the minimum and maximum singular values as follows: Let ($\bar{\sigma}$ and $\bar{\sigma}'$) and ($\underline{\sigma}$ and $\underline{\sigma}'$) denote the maximum and minimum singular values of the plant and the control-relevant model, respectively, at any given fixed frequency. The problem of minimization of the error between the maximum singular values of the plant and the control-relevant model, as well as between the minimum singular values, is a multiobjective optimization problem in the space of the filter parameters. Because the low-gain directions are relatively more important in feedback control,[30,31] this problem can be formulated as a weighted optimization problem with relatively higher weights accorded to the matches in the low-gain directions as

$$\min[w_1(\bar{\sigma} - \bar{\sigma}')^2 + w_2(\underline{\sigma} - \underline{\sigma}')^2] \tag{32}$$

The above minimization has to be carried out at each frequency in the control-relevant band, and it can be carried out iteratively in the space of the prefilter parameters. However, this requires the assumption of a plant proxy, as was done by Li and Lee.[22,23] The role of accurate characterization of the low-gain direction in a feedback control context was emphasized by Jacobsen and Skogestad[30,31] and Skogestad and Morari.[32] Furthermore, evaluation of the estimated gain directionality was also discussed in Andersen and Kummel.[33,34] In this paper, because the control-relevant prefilter is designed on the basis of the control-relevant parameter estimation problem with the objective of minimization of the control error, we examine the role of the prefilter only in the accurate characterization of the gain directionality. By means of an illustrative example, we also show that imposing the requirement of steady-state gain match (in section 6) as an addition to the control-relevant parameter estimation problem does indeed result in a more accurate characterization of the high- and low-gain directions, at least in the control-relevant region.

In the control-relevant model estimation method proposed in this paper, even though it is based on a MISO structure, the data are still collected by simultaneous uncorrelated perturbations of all of the inputs. Moreover, the weighting of the data by control-relevant prefilters results in a good match of both the low- and high-gain directions of the model with the true plant in the control-relevant frequency band. This is illustrated by the singular-value plot of the true plant and the control-relevant model as a function of frequency, presented in section 6. This match can be attributed to the filtering of the input/output data by the two prefilters designed for each channels, similarly to low-pass filtering of the closed-loop data, as was shown by Andersen and Kummel.[24]

5. Overview of Proposed CRI Procedure for 2 × 2 Systems

The proposed method for control-relevant model estimation of 2 × 2 system is summarized in this section.

1. To achieve good estimation in the MISO framework, the inputs must be uncorrelated. For this purpose, a set of signals corresponding to an uncorrelated sequence, such as a pseudorandom binary sequence (PRBS) or an inverse repeat sequence (IRS), with a length equal to 2.5 times the settling time of the process, is used as perturbation inputs for the two channels of the plant to be modeled, and the plant input/output data are recorded. These perturbation inputs are designed on the basis of a priori knowledge of the dynamics of the plant, as specified in refs 12, 18, and 21. The prefilters $f_{11}(z)$ and $f_{21}(z)$, which must be used for the CRI of the direct and interacting branch elements of the plant associated with the first loop of the MISO structure are synthesized as follows:

2. The IMC filter associated with the feedback controller of the first channel is defined as

$$f_{\mathrm{imc}_1}(z) = \frac{(1 - \mathrm{e}^{-T_s/\tau_{\mathrm{cl1}}})z}{z - \mathrm{e}^{-T_s/\tau_{\mathrm{cl1}}}}$$

where T_s is the sampling time chosen and τ_{cl_1} is the desired speed of response for the first channel.

3. The terms appearing in eqs 19 and 20 are initialized as

$$\tilde{p}_{11}(z) = \frac{1}{z - \mathrm{e}^{-T_s/T_{\mathrm{dom1}}}}$$

$$\tilde{p}_{12} = \frac{1}{z - \mathrm{e}^{-T_s/T_{\mathrm{dom1}}}}$$

$$\eta_1(z) = z^{-nk} f_{\mathrm{imc}_1}(z)$$

where nk is the delay in the number of sample units in the estimated models, which is assumed to be unity.

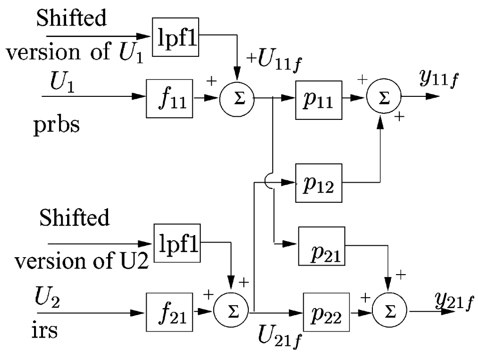

Figure 14. Schematic of control-relevant identification with steady-state gain match of channel 1 elements of MIMO systems.

T_{dom1} is the open-loop dominant time constant as measured from the channel 1 output, obtained from the step test on the system.

4. The noise model is initialized according to the model structure used to fit the model. It is taken as unity for the output error (OE) model structure, and in the case of an ARX model structure, it is initialized as

$$\tilde{p}_{e_1}(z) = \frac{z}{z - e^{-T_s/T_{dom1}}}$$

5. The signals $(r_1 - d_1)(z)$, $u_2(z)$, and $v_2(z)$ are assumed to be step-type signals.

6. According to the above steps, prefilters $f_{11}(z)$ and $f_{21}(z)$ are obtained using eqs 19 and 20. The inputs are filtered by the respective prefilters, and the plant is perturbed with these prefiltered signals as inputs, as shown in Figure 6. (Channel 1 is shown only for illustration.) A discrete MISO model having a first-order with unit delay structure is fitted to the resulting inputs and the output in channel 1 of the system (that is, U_{1f}, U_{2f}, and y_{1f}).

7. A similar approach is followed, and the control-relevant models for the direct and interacting branch elements associated with channel 2 are estimated.

8. Thus, the control-relevant model of the plant is obtained by combining the two MISO control-relevant model sets estimated.

By extending the methodology proposed in section 3, the control-relevant model with steady-state gain match can be obtained for the MIMO system. The methodology is described in the following steps, again for the case of 2 × 2 systems:

1. The prefilter expressions $f_{11}(z)$, $f_{21}(z)$ and $f_{12}(z)$, $f_{22}(z)$ for each of the channels are obtained as per the above procedure.

2. The shifted versions of the regular inputs to the two channels are generated. These shifted versions of the regular perturbation inputs are passed through an appropriately designed first-order Butterworth low-pass filter (lpf1) with a cut-off frequency in the range of 10^{-2} to 10^{-4} rad/s. These low-pass-filtered, shifted versions of the regular perturbation inputs are superimposed on the regular inputs of the respective channels filtered by the prefilters $f_{11}(z)$ and $f_{21}(z)$, respectively, as shown in Figure 143. The resulting inputs to the plant and the output in channel 1 of the plant (U_{11f}, U_{21f}, and y_{11f}) are recorded.

4. Similarly, both inputs are filtered by prefilters $f_{12}(z)$ and $f_{22}(z)$ corresponding to the second channel. The shifted versions of each of the regular inputs are filtered with a second first-order digital Butterworth low-pass filter (lpf2) with a cut-off frequency in the range of 10^{-2} to 10^{-4} rad/s. The prefiltered and low-pass-filtered signals of the respective channels are superimposed, and the plant is perturbed again with the resulting signals, as shown in Figure 15.

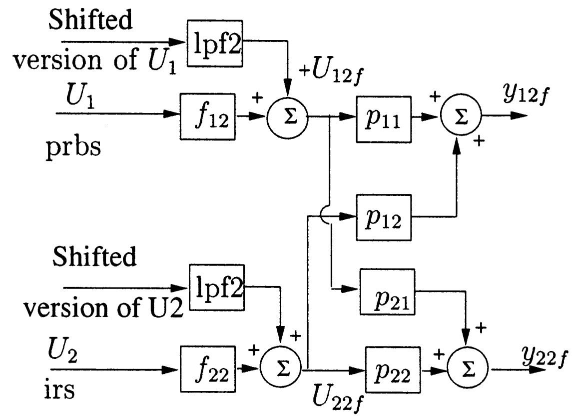

Figure 15. Schematic of control-relevant identification with steady-state gain match of channel 2 elements of MIMO systems.

4. The corresponding inputs and output for channel 2 (U_{12f}, U_{22f}, and y_{22f}) are recorded.

5. A discrete MISO model having a first-order with unit delay structure is fitted for each combination of input/output data specified above.

6. The MISO models thus obtained are suitably combined to generate a model of the given 2 × 2 system. The resulting model is expected to exhibit a good match of the steady-state gain of each of the model elements with that of the true system, in addition to having a minimum mismatch with the true plant dynamics in the control-relevant frequency band.

6. Simulation Results

In the following subsections, the proposed methods for the MISO-structure-based control-relevant identification of MIMO systems and the steady-state gain match of the estimated control-relevant models are validated by application to two 2 × 2 illustrative examples.

6.1. Paper Machine Problem. The paper machine control design problem given in ref 35 is used to validate the proposed method for the CRI of a 2 × 2 system. The state space model of the plant is derived from the plant dynamical equations. The parameters of the state space model are as follows

$$\dot{\mathbf{x}} = \begin{bmatrix} -0.2 & 0.1 & 1 \\ -0.05 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \\ 1 & 0 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{x}$$

From the open-loop step response of the system, the longest settling time is measured as 175 s, and the corresponding dominant time constant is 75 s. On the basis of the relative gain array (RGA) analysis, the pairing of controlled and manipulated variables for control is chosen as (1, 1) and (2, 2). The specifications for the closed-loop speeds of response are chosen to be 2 s for channel 1 and 8 s for channel 2. In accordance with these specifications, the sampling time is chosen as 0.2 s (one-fifth to one-tenth of the fastest closed-loop speed of response), and with these known specifications,

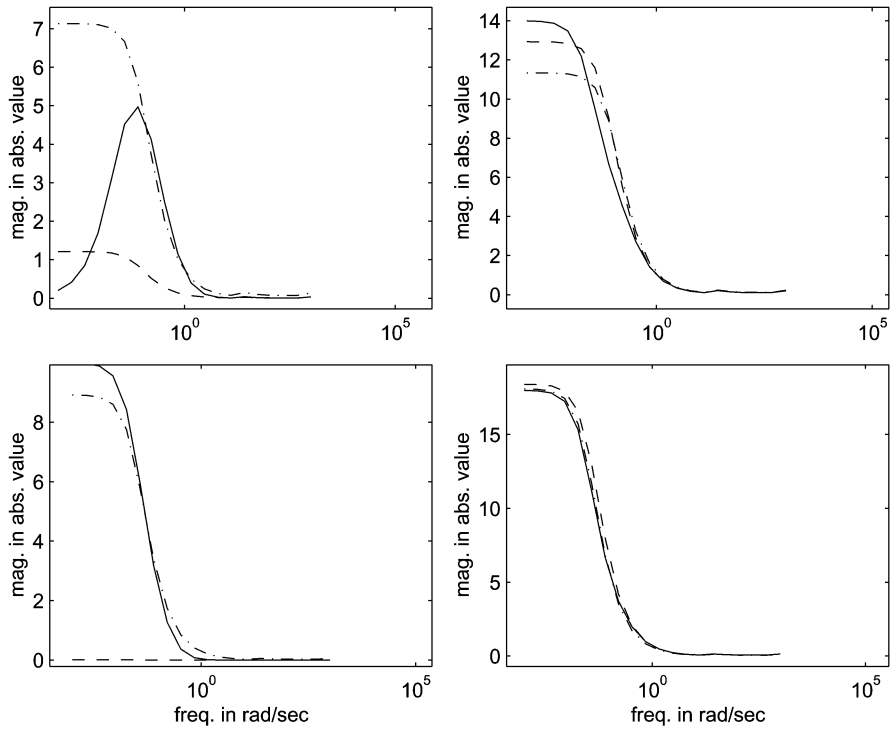

Figure 16. Amplitude ratio comparison for paper machine problem: —, true plant; – · – · –, control-relevant model; – – –, direct estimated nominal model.

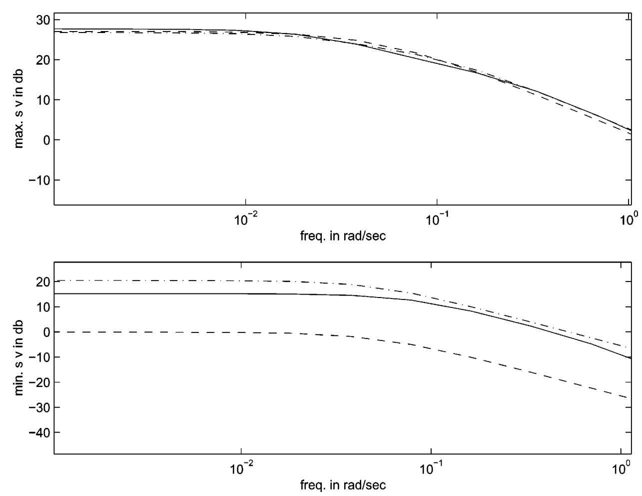

Figure 17. Singular-value comparison of paper machine problem: —, true plant; – · – · –, control-relevant model; – – –, direct estimated model.

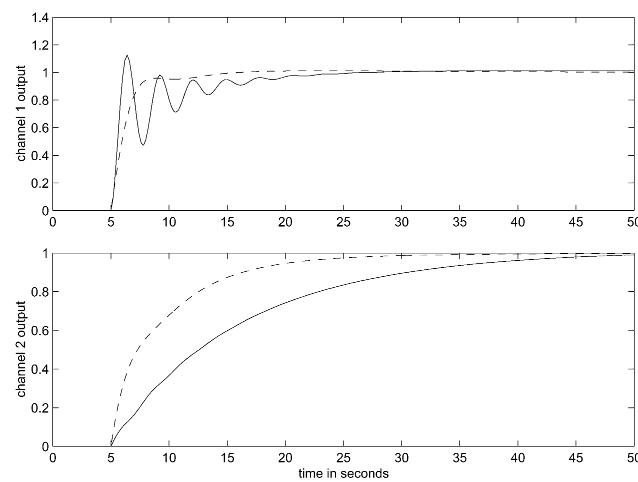

Figure 18. Closed-loop tracking performance comparison: —, direct estimated nominal model; – – –, control-relevant models.

the control-relevant models are estimated according to the steps given in section 5. The nominal model for the same system is also estimated directly by fitting MISO models to the input/output data. The magnitude response of all four transfer function matrix elements of the direct estimated and control-relevant models are plotted along with those of the true plant elements in Figure 16. A good match between the true plant dynamics and the dynamics of the control-relevant model is exhibited in the control-relevant frequency band, when compared with that of the direct estimated nominal model. Plots of the singular values of the control-relevant model, direct estimated model, and the true plant are shown in Figure 17. A good match of the minimum and maximum singular values of the control-relevant model with those of the true plant can be seen in the control-relevant frequency range. The closed-loop simulation results obtained using IMC-based decoupled multiloop control with the control-relevant model and the direct estimated nominal model are shown in Figure 18. The closed-loop performance obtained from the control-relevant model-based system is better as per specifications, when compared with that obtained using the direct estimated model.

With the same experimental conditions, the above procedure is also carried out with the reverse pairing of controlled and manipulated variables. Whereas the direct estimated model-based system exhibited unstable closed-loop performance, the control-relevant model-based system performance was similar to the performance obtained with the original pairing.

6.2. Example 2: Results on Steady-State Gain Match. In the following subsection, the results obtained from the control-relevant estimation of a MIMO system with a steady-state gain match requirement are presented. A good match of both of the singular values of the resulting control-relevant model with steady-state

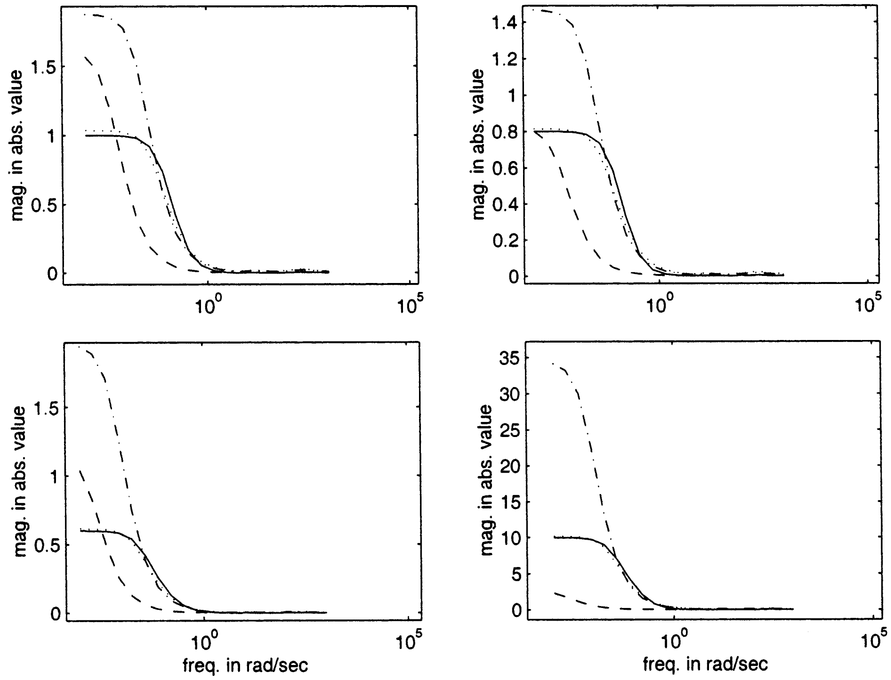

Figure 19. Amplitude ratio comparison for example 2: —, true plant; – · – · –, control-relevant model without steady-state match; · · ·, control-relevant model with steady-state match; – – –, direct estimated model.

gain match with those of the true plant is illustrated here at steady state (zero frequency), as well as in the control-relevant frequency range, which ensures that the proposed method estimates a model that captures both the high- and low-gain directionalities of the plant very well. The following MIMO system is considered for an illustration of the method

$$\mathbf{P(s)} = \begin{bmatrix} \dfrac{2s+1}{50s^3+65s^2+16s+1} & \dfrac{-1-0.8}{50s^3+65s^2+16s+1} \\ \dfrac{1+0.6}{80s^3+100s^2+30s+1} & \dfrac{s+10}{80s^3+100s^2+30s+1} \end{bmatrix}$$

The open-loop step response yields settling times of 44 s for channel 1 and 106 s for channel 2. The open-loop plant has dominant time constants of 25 s in channel 1 and 58 s in channel 2. The steady-state gains of the plant elements are given by the gain matrix **Gp**

$$\mathbf{Gp} = \begin{bmatrix} 1 & -0.8 \\ 0.6 & 10 \end{bmatrix}$$

With a sampling time of 0.4 s and desired closed-loop specifications of 2 s for channel 1 and 4 s for channel 2, the models are estimated using input/output data and prefiltered data and with the proposed method for the steady-state gain match requirement. The steady-state gains of the direct estimated model **Gd**, control-relevant model **Gcr**, and model with steady-state gain match **Gss** are given as follows

$$\mathbf{Gd} = \begin{bmatrix} 1.6078 & -0.8207 \\ 1.1636 & 2.5673 \end{bmatrix}$$

$$\mathbf{Gcr} = \begin{bmatrix} 1.8759 & -1.467 \\ 1.9601 & 34.4323 \end{bmatrix}$$

$$\mathbf{Gss} = \begin{bmatrix} 1.0363 & -0.8138 \\ 0.6166 & 10.1866 \end{bmatrix}$$

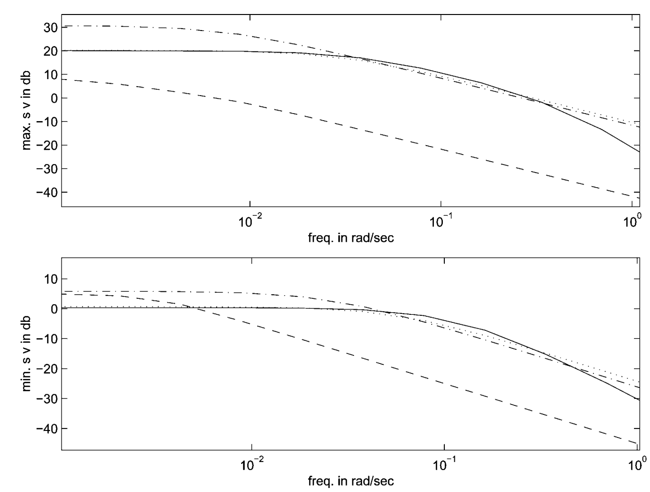

Figure 20. Singular-value comparison for example 2: —, true plant; – · – · –, control-relevant model without steady-state match; · · ·, control-relevant model with steady-state match; – – –, direct estimated model.

The amplitude ratio plots of the individual elements of the three models are compared with those of the true plant elements as shown in Figure 19. The frequency response of the control-relevant model having the steady-state gain match exhibits good agreement with the plant at low frequencies, as well as in the control-relevant frequency range. This highlights the fact that the signal design procedure developed in section 3 supplements the filter design procedure developed in section 2. The singular values of the three models and the true system are plotted in Figure 20. This figure shows that the control-relevant model without steady-state gain match exhibits a good match of its singular values with those of the true system at the closed-loop frequency band of interest. However, the control-relevant model with the steady-state gain match requirement exhibits a better match of its singular values, especially the minimum singular value, with those of the true plant not only at the control-relevant frequency

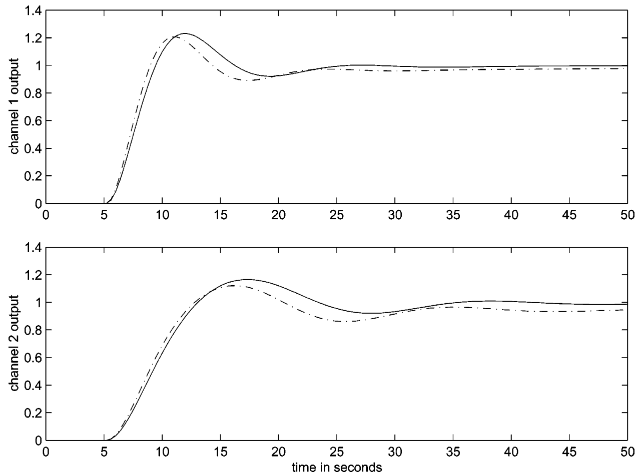

Figure 21. Closed-loop response comparison of Example 2: —, control-relevant model with steady-state match; – · – · –, control-relevant model without steady-state match.

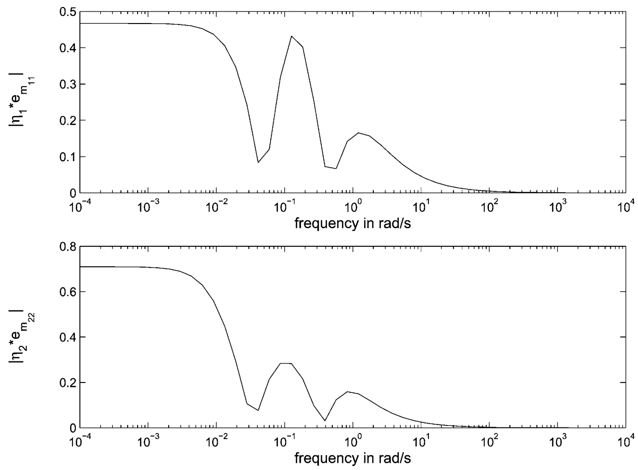

Figure 22. Magnitude plots of $\tilde{\eta} e_m$ for the two channels for example 2.

band but also at zero frequency. The closed-loop performances obtained using the control-relevant models with and without steady-state gain match requirement are shown in Figure 21. It is observed from this figure that the control-relevant model with steady-state gain match does exhibit marginally better control performance than the control-relevant model without steady-state gain match. Although the present illustration does indicate the role of the steady-state gain match in achieving an accurate characterization of the low-gain directionality, it does not adequately justify the improvements in control that result from this characterization. The significance of the steady-state gain match, importantly when the specifications are tight, is illustrated in section 3. Issues related to the proposed steady-state gain matching methodology in accurately characterizing the gain directionality and their role in enhancing the feedback control performance are currently under investigation. Finally, one of the key assumptions made during the derivation of the prefilter was that the magnitude of the product ηe_m is smaller than 1 at all frequencies. As stated in remark 4, it is possible to verify this assumption for the simulation case. Figure 22 shows the magnitude plot of the product ηe_m as a function of frequency for the two channels, respectively. It can be seen that the magnitude of the product is substantially smaller than 1 at all frequencies, thus justifying the assumption made in the prefilter derivation.

7. Conclusions

The prefilter-based control-relevant identification scheme of SISO systems is extended here for $n \times n$ systems. Once the pairing of manipulated and controlled variables is done, the system is decomposed into n MISO subsystems. The interaction from all other channels is treated as a measured structured disturbance on each of the loops. With this consideration, the method proposed by Rivera and Gaikwad[1,5−7] for the control-relevant estimation of SISO systems for combined feedforward/feedback control is extended here for the CRI of multivariable systems. In addition, a method to achieve the steady-state gain match of the control-relevant models with that of true plant is also proposed. The methods are seen to work satisfactorily on various case studies.

The validity of the methods is demonstrated by application to different problems of varying complexity. As the control-relevant identification method proposed here is based on partitioning the $n \times n$ system into n MISO subsystems with appropriate disturbance transfer functions, the validity of the approach is also therefore critically dependent on the nature of decoupling that is employed. This issue needs to be further analyzed. Also, the control-relevant model estimated using the proposed approach of fitting to the prefiltered data is shown to exhibit good matches of both the maximum and minimum singular values over the frequency range of relevance from closed-loop performance. The method can be extended to systems having more than two inputs and outputs in a relatively simpler way by considering the interaction of various branches on the particular single-loop system properly as additive measured structured disturbances and by superimposing different interacting branch elements.

The methodology proposed here to obtain a control-relevant model with steady-state gain match is based on the addition of an appropriately scaled shifted version of the regular perturbation signal passed through a low pass filter, to the prefiltered regular perturbation signal. The same technique is extended to estimate MIMO systems with steady-state gain match. In the case of MIMO systems, it is observed that the control-relevant model with steady-state gain match accurately captures the low-gain direction characteristics of the true plant not only in the control-relevant frequency band, but also at the low-frequency (steady-state) condition. This is an added advantage of the proposed method over conventional MISO estimation routines. A systematic approach such as the use of multifrequency input signals[36] that suitably biases the model to reflect the control-relevant as well as lower-frequency (steady-state) properties of the plant will be the focus of further study.

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