# CONTROL RELEVANT IDENTIFICATION WITH STEADY STATE GAIN MATCHING

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**Keywords:** Linear Systems, Identification Methods, Identification for Control, Multi variable Control, Process Control

## Abstract

The prefilter based control relevant identification scheme proposed by Rivera *et al.*[14], for single input single output(SISO) system gives a model of the system which matches the true system in a range of frequencies that are important for closed loop performance. However, it may also be desirable to have small mismatch at the steady state condition as well. This is specifically true when the desired closed loop speed of response is much higher than the open loop speed of response. Here a method is proposed to estimate a control relevant model that, in addition to having a small mismatch in the control relevant frequencies, also has a good steady state gain match, with that of the true system. With such a control relevant model having steady state gain match, the closed loop internal model control (IMC) based system is shown to give satisfactory response even with higher desired speed of response. Also, a control relevant identification scheme for multi input single output(MISO) system, having steady state gain match is proposed, using uncorrelated inputs. Application of these identified models to a feed forward control scheme has been considered to demonstrate superior performance with these models. Similarly, with combined feed forward/feedback control, the model having steady state gain match gives an improved performance even with higher desired speed of closed loop response.

## 1 Introduction

The emergence of model based control has given impetus to the idea of estimating models whose end use is for the design of the controller. This controller when applied to the true system is expected to give satisfactory performance from the system. The branch of system identification which deals with such a strategy is called *control relevant identification*. The focus of research publications from Rivera and co-workers [3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19], Kwok and Shah[6], Shook *et al.*[21], and Van den Hof and Schrama[20, 22] are all mainly centered on control relevant identification. In traditional system identification, the objective is the minimization of bias and variance error of the estimated model with that of the true system; whereas in control relevant identification, the interest is to minimize the model plant mismatch at those frequencies which are most relevant from closed loop performance. Rivera and Gaikwad[16] have discussed the modeling issues for obtaining satisfactory closed loop performance using the controller designed based on an estimated model. Rivera and co-workers [11, 12, 13, 14, 15, 17] have focused on prefilter based methods for control relevant identification of SISO systems for the design of feedback and combined feed forward/feedback control, and also discussed issues related to input design. Kwok and Shah[6] have proposed a method wherein the control relevant model is estimated with terminal matching condition which makes computation easy for a generalized predictive control scheme. Van den Hof and Schrama[22] have shown that when the estimated models exhibits minimum mismatch with the true system in the closed loop frequencies of interest, the resulting model based controller gives a better closed loop performance. Schrama and Bosgra have proposed[20] an iterative identification and control design methodology in frequency domain, using closed loop data with co-prime factor perturbations.

Banerjee and Shah[1] have shown that the small gain condition for stability will never be violated if the steady state gain of model exceeds half of the steady state gain of the system. Thus the accurate estimation of the steady state gain of the system within certain bounds is important from closed loop stability and performance point of view. From a practitioners view point as well, it is desirable to estimate a model, which not only matches the frequency response of the true system at control relevant frequencies but also gives a good match at steady state. This paper examines issues related to the role of steady state gain (mis)match on the closed loop performance in a control relevant context. Examples are presented to show that the control relevant identification methodology without regard to steady state gain match could yield models, that makes the

### 3.1 SISO feedback control

To estimate the control relevant model having a steady state gain match with that of the true system, it is desired that the data used for estimation also contain low frequency components in addition to the control relevant frequencies. Hence, a low frequency signal, of appropriate gain is applied to the plant, along with the filtered regular input. The resultant input to the plant and the output from the plant are recorded. For generating an uncorrelated low frequency signal that needs to be added to the regular input, a delayed version of same regular input, fed through a low pass filter (lpf), is used. A first order discrete model with unit delay is fitted to this effective input and the output, recorded from the plant. It is observed that, proper choice of gain and cutoff frequency of the low frequency signal, makes the steady state gain of the model estimated nearly equal to that of the true system.

### 3.2 SISO combined feed forward/feedback control

In the following a methodology for control relevant identification of plant and disturbance transfer functions is presented, for the design of feed forward and combined feed forward/feedback control. To get accurate identification of plant and disturbance transfer functions, the perturbation inputs for the two elements must be uncorrelated. Here, the use of a pseudo random binary sequence(PRBS) and an inverse repeat sequence(IRS)[2, 5, 9] signals is proposed as the inputs to the plant and disturbance transfer functions respectively. To estimate control relevant models of the plant and the disturbance transfer functions the following steps are followed: The inputs to the plant and disturbance elements are filtered by the respective prefilters given by Equations (2) and (3). The resulting prefiltered signals are applied to the system and the effective output from the system is recorded. A MISO model is fitted for this filtered inputs and the output, and control relevant models of plant and disturbance functions are obtained.

To achieve good feed forward control, the plant and the disturbance transfer functions need to be estimated with accurate steady state gain in addition to control relevant sense. In the proposed MISO identification scheme, a low frequency signal of appropriate gain is added to both of the regular inputs, as shown in Figure 1. These low frequency signals need to be uncorrelated from each other and with the respective inputs into which they are added. These are therefore generated from the delayed version of each of the respective inputs. The resulting inputs to the plant and the disturbance functions and the output from the system ($U_{1df}, U_{2df}$, and $y_{df}$) are used to fit a MISO model. The estimated plant and the disturbance models exhibit good steady state gain match with that of the true plant and the disturbance transfer functions.

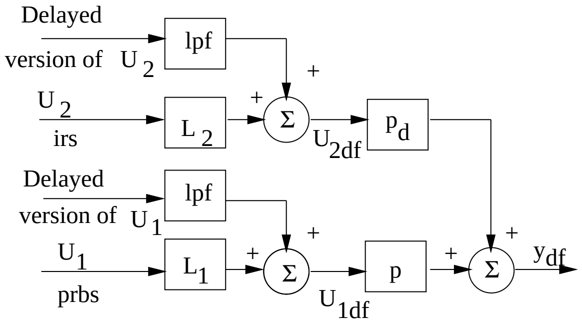

Figure 1: Schematic for control relevant estimation with gain match, MISO systems

## 4 Simulation results

To illustrate the significance of steady state gain match of the estimated models, results from two case studies are presented in this section. The first case study concerns the feedback control performance analysis of a SISO system based on i)nominal model, and ii) control relevant models with and without steady state gain match for two different desired speeds of response. In the second illustration, plant and disturbance models are estimated in a MISO structure. Here, the feed forward and the combined feed forward/ feedback control performances are compared using a controller designed based on i) nominal model, and ii) control relevant model with and without steady state gain match, for a higher desired speed of closed loop response (1 second).

### 4.1 Example 1

The SISO system considered for illustration of the proposed method is given as,

$$p(s) = \frac{2s+1}{50s^3+65s^2+16s+1}$$

This system has an open loop speed of response(dominant time constant) of 16 seconds, and a steady state gain of unity. The sampling time assumed is 0.1 second. The input output data are collected from the system by exciting the plant with a suitably designed PRBS signal. The nominal model estimated by directly fitting a first order with unit delay model to this input output data is given by, $p_{nom}(z) = \frac{0.2047\times10^{-3}}{z-0.9999}$. This model has a steady state gain of 2.1151. In the following the results obtained for the two desired speeds of response are presented.

1. **Desired speed of response equal to 5 seconds:** For a specified closed loop speed of response of 5 seconds, a prefilter based control relevant model is estimated. The control relevant model estimated is given by $p_{cr}(z) = \frac{0.0056}{z-0.9977}$ which has a steady state gain of 2.4375. The closed loop performance using the nominal and control relevant models exhibit stable responses, with the control relevant model exhibiting better response which is closer to the specification. A model is estimated as proposed in this paper, which has its steady state gain close to the true system. The resulting model for this specification, is estimated as $p_{gm}(z) = \frac{0.0059}{z-0.9947}$ which has a steady

state gain of 1.1193. The Bode and the Nyquist plot comparison of all the three models with that of the true system, as well as closed loop performance evaluation with these three models are shown in Figure 2. However, for this specification, the steady state gain match does not give any significant improvement in the performance, and the control relevant model estimation alone is sufficient.

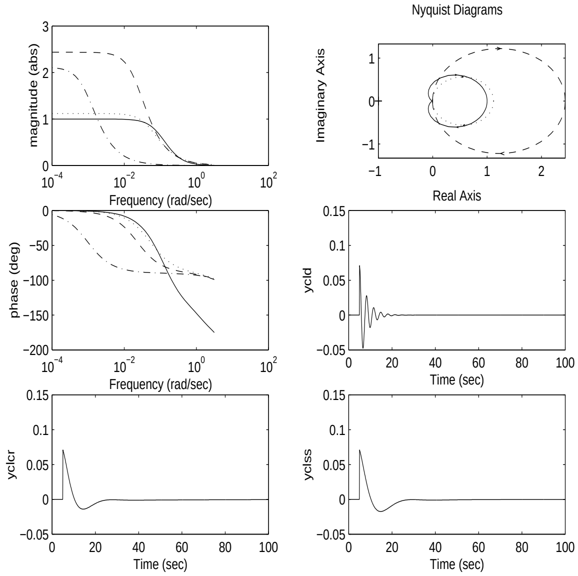

Figure 2: Results for 5 seconds specification: i) left top: Bode magnitude comparison; ii) left middle: Bode phase comparison; iii) right top: Nyquist plot comparison; a) 'solid' True plant; b) 'dot-dot' control relevant model with Steady state match; c) 'dash-dash' control relevant model without steady state gain match; d) 'dash-dot' Direct estimated model; and Closed loop response comparison: iv) right middle: using direct estimated model; v) left bottom: using control relevant model without steady state gain match; vi) right bottom: using control relevant model having steady state gain match.

2. **Desired speed of response equal to 1 second:** The control relevant model and the model having a steady state gain match is now estimated for a higher desired speed of closed loop response(1 second). The estimated models are given as,

$$p_{cr}(z) = \frac{0.0035}{z - 1.0027}, \quad p_{gm}(z) = \frac{0.0059}{z - 0.9947}$$

The steady state gain of the control relevant model is -1.2967 and also its pole is located outside the unit circle. The model with steady state gain match, has a steady state gain of 1.1193 and is stable. The Bode and the Nyquist plots comparison of different models with that of the true plant, as well as closed loop performance evaluation with these models are shown in Figure 3. It can be seen that both nominal and control relevant models exhibit an unstable closed loop response. However, the closed loop performance using the model having its steady state gain nearly matching that of the true system is stable and satisfactory as per specification.

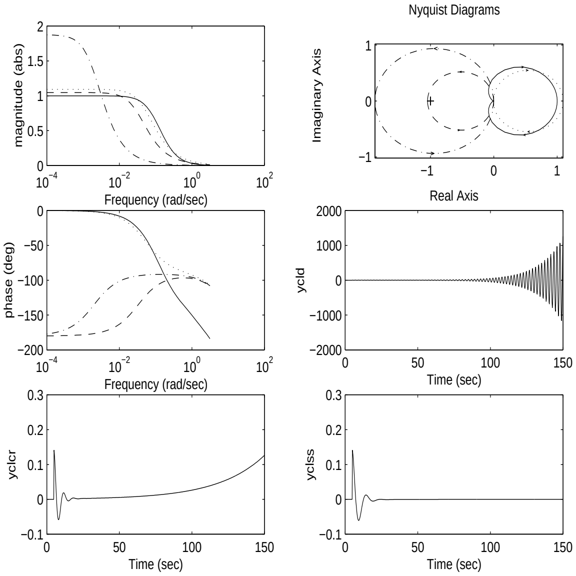

Figure 3: Results for 1 second specification: i) left top: Bode magnitude comparison; ii) left middle: Bode phase comparison; iii) right top: Nyquist plot comparison; a) 'solid' True plant; b) 'dot-dot' control relevant model with Steady state match; c) 'dash-dash' control relevant model without steady state gain match; d) 'dash-dot' Direct estimated model; and Closed loop response comparison: iv) right middle: using direct estimated model; v) left bottom: using control relevant model without steady state gain match; vi) right bottom: using control relevant model having steady state gain match.

Thus, control relevant identification without steady state gain match resulted in unstable performance when higher closed loop speed of performance is specified; however, incorporating steady state gain in control relevant identification yielded stable and satisfactory performance even at higher desired speed of closed loop response.

### 4.2 Feed forward control and combined feed forward/feedback control performance analysis

For illustration of the significance of steady state gain match of the plant and the disturbance transfer functions for feed forward control, an example with the following plant and disturbance transfer functions is considered.

$$p(s) = \frac{2s + 1}{50s^3 + 65s^2 + 16s + 1}$$

$$p_d(s) = \frac{s}{40s^3 + 38s^2 + 11s + 1}$$

The open loop speed of response of this system is approximately 16 seconds. The steady state gain of the plant is unity and the disturbance is zero. The sampling time chosen is 0.2 seconds. The excitation signals used for the two inputs are a PRBS and an IRS signal designed appropriately. With these suitably designed inputs, the out put is recorded and the nominal plant and the disturbance models are estimated using MISO identification routines. The nominal models for the plant and the disturbance transfer functions estimated are given as,

$$p_{nom}(z) = \frac{0.00076}{z - 0.9998}, \quad p_{dnom}(z) = \frac{0.00049}{z - 0.9998}$$

It can be seen in the above that, the steady state gain of the nominal plant model is 3.5128 and that of the nominal disturbance model is 2.2567. The estimation of the control relevant models and the models having steady state gain match resulted the following plant and disturbance models, with a specification for closed loop response of 1 second.

$$p_{cr}(z) = \frac{0.0072}{z - 0.9974}, \quad p_{gm}(z) = \frac{0.0111}{z - 0.9899}$$

$$p_{dcr}(z) = \frac{-0.00095}{z - 0.9974}, \quad p_{dgm}(z) = \frac{0.00006}{z - 0.9899}$$

The steady state gain estimated for the control relevant model of the plant is 2.7618 and that of the plant model estimated to have a steady state gain match is 1.1064. The corresponding values for the disturbance model are respectively, -0.3662 and 0.0062. The Nyquist plots for different plant and disturbance models are compared with those of the true plant and disturbance functions, are shown in Figure 4. With feed forward control, and combined feed forward/feedback control the performance evaluation is carried out using i) the nominal models and ii)the control relevant models with and without steady state gain match. The performance with different models are obtained as shown in Figure 5. The simulation results obtained clearly show that in case of feed forward control, the model having steady state gain match gives superior performance than that obtained with the other two models. Similarly, with the combined feed forward/feedback control, the control relevant models with steady state gain match gives superior performance. However, unlike in illustration 1, the control relevant model which do not have steady state gain match, gave stable performance.

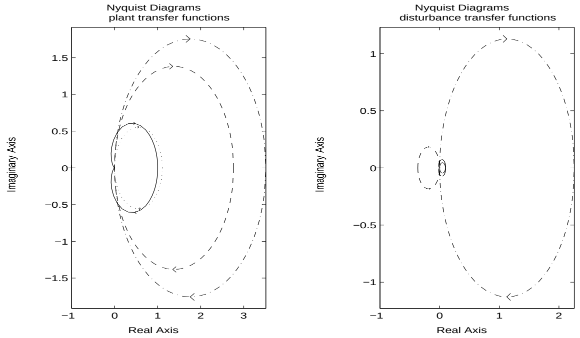

Figure 4: Nyquist plot comparison i) left: plant transfer function and ii) right: disturbance transfer function; a) 'solid' full order, b)'dash dot' nominal model, c)'dash dash' model estimated using prefiltered data, d)'dot dot' model with steady state match

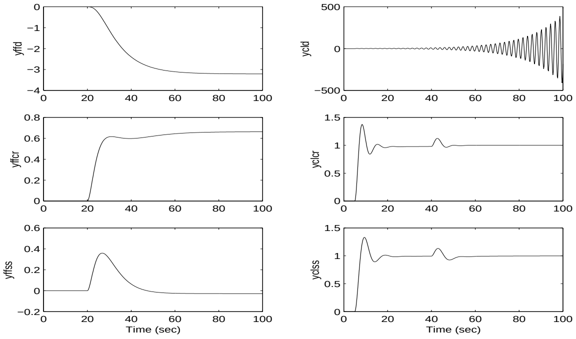

Figure 5: i) left: Feed forward control performance and ii) right: Combined feed forward/feedback control performance a) top: using direct estimated model, b) middle: using prefilter based model, c) bottom: using model with steady state gain match

## 5 Conclusion

The control relevant identification technique based on prefiltering input output data is seen to give large differences in the steady state gain of the estimated model. However, as long as the desired speed of response is not too fast, when compared with open loop speed of response, even with this difference, the IMC based closed loop system is shown to exhibit acceptable response with the control relevant models. In this paper, it is shown that if the desired specification for closed loop speed of response is much higher than the open loop speed of response, then the steady state gain of the model could be significantly different from that of true system and thus, the prefilter based control relevant models, may not yield adequate closed loop control performance. Here, a procedure is proposed which enables matching of the steady state gain of the estimated model with that of true system. It is demonstrated that the model having steady state gain match exhibits an IMC response which is stable and meets the desired specifications. Similarly, when feed forward control is used to compensate for measured disturbances, the control relevant model obtained with no regard to steady state gain, resulted in large offset. Relatively smaller offsets are obtained by using models having steady state gain match.

The methodology proposed is based on the addition of a appropriately scaled delayed versions of the regular perturbation signal passed through a low pass filter, to the filtered regular perturbation signal, and using the resulting superimposed signal as input to the plant. A systematic approach that suitably biases the model to reflect control relevant as well as lower frequency(steady state) properties of the plant would require further study. Also, the MISO structure based control relevant identification of plant and disturbance models proposed here is useful in extending a similar approach for control relevant estimation of multi input multi output systems, for decoupler based multi loop control, considering the interacting branches on each SISO loop as measured disturbance elements.

## Acknowledgements

This research work is funded under project no. III-5(27)/98-ET, Department of Science and Technology, Govt. of India.

The first author thank the management of N. M. A. M. Institute of Technology, Nitte, Udupi, Karnataka, 574110, India, for the financial assistance.

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