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MISO Structure Based Control-Relevant Identification of MIMO Systems [1]

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Abstract

The prefilter based open loop control-relevant identification scheme proposed for single input single output (SISO) systems by Rivera *et al.*[1] is extended here for multi input multi output (MIMO) systems using a multi input single output (MISO) structural framework. By selecting input output pair for control, the MIMO system is partitioned into individual SISO systems with the interacting branches acting on each SISO loop being treated as structured measured disturbances (in addition to the regular disturbance at each output). With this consideration, a methodology is proposed for design of separate prefilters for the individual channels, taking into account the performance specifications for each loop. The use of an uncorrelated set of inputs is proposed for obtaining accurate estimation of individual channel elements. The prefiltering for each individual loop has been shown to give good estimates of the control-relevant model for the direct as well as the interacting branches. Closed loop simulations, using decoupled internal model controllers, involving representative processes taken from literature demonstrate the validity of the approach.

Keywords : Control-relevant identification, MIMO systems, MISO structure based multi loop identification, internal model control.

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1 Introduction

System identification is the most important step in the design of model based controller for the plant. This controller should guarantee safe, reliable, efficient, economical and satisfactory performance from the system. To derive a simple, implementable, and tight controller based on a model of the system, an accurate yet parsimonious model of the system is sought. To obtain such parsimonious models, under modeling of the system is inevitable, and hence there could be a degradation in the closed loop performance. The identification methodology to obtain good closed loop performance, despite parsimony in the model structures, is termed as *control-relevant identification* and has been the focus of a series of papers by Rivera and co-workers[1, 2, 3, 4, 5, 6, 7, 8, 9], Shah and co-workers[10, 11], and Van den Hof and Schrama [12, 13]. In control-relevant identification, the aim is to model the system with an intention of using the estimated model for the design of the controller, that gives satisfactory performance from the system. Here, the minimization of the model plant mismatch around the closed loop, control-relevant frequencies is more important than matching the model frequency response characteristics with that of open loop system characteristics.

In their work Rivera *et al.*[1], propose methods for control-relevant identification of SISO systems with a focus on design of input signal, and open loop prefilter based identification. A control-relevant identification strategy for generalized predictive control(GPC) is presented by Shook *et al.*[11], by proposing an identification objective function that is dual of the control objective function. The same work is extended and a long-range predictive control with terminal matching condition is proposed by Kwok and Shah[10]. Some of the results by Van den Hof and Schrama[12], reveal that the model plant mismatch should be minimum around the frequencies which are relevant from the closed loop performance point of view. A frequency domain closed loop iterative identification and control design scheme is proposed by Schrama *et al.*[13], where in identification, control design, and new data collection is carried out iteratively, till an enhanced control performance is achieved, with co-prime factor perturbations. The importance of gain directionality in MIMO system control-relevant identification is discussed by Li and Lee[14, 15]. They have shown that when the frequency response of the model and that of the inverse of the model fits the frequency response of the MIMO system and its inverse respectively, the resulting model of the MIMO system captures both the high and low gain directionality of the system. Based on an approach similar to that for SISO systems, Rivera *et al.*[4], have proposed a methodology for control-relevant identification of MIMO systems, based on prefiltering input output open loop data by a diagonal prefilter. In [9], they have proposed multi-frequency curve fitting method for MIMO system control-relevant identification. Input signal design is also very important in accurate system identification and has been addressed in [16, 17, 18].

This paper proposes an alternate methodology for performing control-relevant identification for MIMO systems, by prefiltering input output open loop data. The approach proposed here is based on partitioning of the n×n MIMO system into n single loops which are then designed for good tracking and regulatory performances. The interaction from all other branches to each SISO loop is considered as measured structured disturbances along with the regular output disturbances. Separate prefilters are designed for each channel by specifying the respective speeds of response for tracking and measured structured disturbance rejection. These prefilters are used to filter the input output data to get control-relevant MISO models where in the direct and interacting branch transfer functions are identified. Using these representative models, a multi loop internal model control (IMC) scheme with decoupler is proposed. Evaluation of the proposed approach is carried out through closed loop simulation for tracking and disturbance rejection, involving representative systems taken from literature.

The paper is organized as follows. In section 2 of the paper, the expression for the prefilter design is derived for each channel of MIMO system, by a MISO framework. This derivation follows a similar approach as proposed by Rivera *et al.*[1] for SISO systems. The estimation methodology for the control-relevant identification of the MISO structure, is proposed in section 3. Simulation results demonstrating the validity of the proposed method are described in section 4. Section 5 provides a summary of the approach proposed in this paper.

2 Prefilter Design for MISO Structure Based Control-Relevant Identification

Consider a general 2×2 system as shown in Figure 1. Assuming that the choice of input output pairing for control is made appropriately, each input output combination can be treated as an independent SISO loop. In doing so all the other branches interacting with each of the SISO loops are treated as measured structured disturbance acting on the respective SISO loops, in addition to the regular disturbance that the particular loop is subjected. This representation is shown schematically as in Figure 2. Thus, in addition to good servo properties, each individual loop will also have specifications for rejection of disturbances that affect the output directly and through the interacting branch. Here it is proposed to derive a prefilter for each of the SISO loops. To derive an expression for the prefilter for each SISO loop, the given 2×2 system to be identified is represented as two MISO systems. Assuming that the same prefilter is used for filtering both the inputs and the output of the individual MISO structure, the expression for the prefilter can be derived. As shown by Rivera *et al.*[1], for the SISO case, the prefilter for each SISO loop is obtained by equating the objective function for prefiltered prediction error minimization with the objective function for minimization of control error. The extension of the derivation to the MISO case is carried out as follows:

The prediction error $e_1(t)$ for the MISO system between y1, U1, and U2 can be written as,

$$\begin{aligned} e_1(t) &= \tilde{p}_{e_1}^{-1}(z)[(p_{11}(z)-\tilde{p}_{11}(z))U_1(t) \\ &+ (p_{12}(z)-\tilde{p}_{12}(z))U_2(t)+\nu_1(t)] \end{aligned} \quad (1)$$

where, $p_{11}(z)$ and $p_{12}(z)$ are the true system transfer functions, $\tilde{p}_{11}(z)$ and $\tilde{p}_{12}(z)$ are the respective nominal models estimated, $\tilde{p}_{e_1}(z)$ is the noise model, U1(t) and U2(t) are the inputs to the system and model and $\nu_1(t)$ is a stationery noise sequence of power spectrum ϕ_{ν_1}. Let $f_1(z)$, be the prefilter to be used for this MISO structure control-relevant estimation. Hence, the prefiltered prediction error is given as,

$$e_{1_f}(t) = f_1(z)e_1(t)$$

The identification task is the minimization of the squared filtered prediction error,

$$V_1 = \frac{1}{N}\sum_{t=1}^{N}[e_{1_f}(t)]^2 \quad (2)$$

Assuming that U1 and U2 are uncorrelated, the frequency domain expression for the filtered prediction error given by Equation (2) can be written as[19],

$$\lim_{N\to\infty} V_1 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} [|p_{11}(e^{j\omega}) - \tilde{p}_{11}(e^{j\omega})|^2 \phi_{U1}(\omega) + |p_{12}(e^{j\omega}) - \tilde{p}_{12}(e^{j\omega})|^2 \phi_{U2}(\omega) + \phi_{\nu_1}(\omega)].\frac{|f_1(e^{j\omega})|^2}{|\tilde{p}_{e_1}(e^{j\omega})|^2} d\omega \quad (3)$$

where, ϕ_{U1} and ϕ_{U2} are the power spectrum of the inputs. The objective function given by Equation (3) is to be minimized by estimating the models $\tilde{p}_{11}$ and $\tilde{p}_{12}$. Now, consider loop 1 of the 2×2 system, with the interaction effect of other loop being considered as disturbance on the main loop as shown in Figure 2. The controller $c_1(z)$ designed based on the model $\tilde{p}_{11}(z)$ when applied to the plant results in a control error, $e_{c_1}(= r_1 - y_1)$. This control error can be written as[20],

$$e_{c_1}(z) = \left[\frac{\tilde{\epsilon}_1(z)}{1 + \tilde{\eta}_1(z)e_{m_{11}}(z)}\right][r_1 - d_1 - p_{12}(z)u_2] \quad (4)$$

where, $\tilde{\epsilon}_1$ and $\tilde{\eta}_1$ are respectively the sensitivity function and complementary sensitivity function of loop 1, and are given by,

$$\tilde{\epsilon}_1(z) = \frac{1}{1+\tilde{p}_{11}(z)c_1(z)}; \qquad \tilde{\eta}_1(z) = \frac{\tilde{p}_{11}(z)c_1(z)}{1+\tilde{p}_{11}(z)c_1(z)}$$

The stability of any system is guaranteed if the small gain theorem is satisfied. That is,

$$|\tilde{\eta}_1(z)e_{m_{11}}(z)| \leq 1 \quad \forall -\pi \leq \omega \leq \pi$$

where, ω is frequency in radians per seconds. For systems obeying the small gain theorem, using the Taylors series expansion, the expression given by Equation (4), can be approximately written as,

$$e_{c_1}(z) \approx \tilde{\epsilon}_1(z)(1 - \tilde{\eta}_1(z)e_{m_{11}}(z))[r_1 - d_1 - \tilde{p}_{12}(z)(1 + e_{m_{12}}(z))u_2] \quad (5)$$

where, $e_{m_{11}}$ and $e_{m_{12}}$ are the multiplicative error with respect to models p_{11} and p_{12}. The frequency domain representation of Equation (5) is written as,

$$||e_{c_1}(e^{j\omega})||_2^2 \approx \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1(e^{j\omega})|^2 |1 - \tilde{\eta}_1(e^{j\omega}) \times e_{m_{11}}(e^{j\omega})|^2 |r_1 - d_1 - \tilde{p}_{12}(e^{j\omega})(1 + e_{m_{12}}(e^{j\omega}))u_2|^2 d\omega \quad (6)$$

Using the triangle inequality property of the norm, Equation (6) becomes,

$$||e_{c_1}(e^{j\omega})||_2^2 \leq \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |r_1 - d_1|^2 - \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{\eta}_1 e_{m_{11}}|^2 |r_1 - d_1|^2 d\omega - \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{p}_{12}(1 + e_{m_{12}})|^2 |u_2|^2 d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{\eta}_1 e_{m_{11}}|^2 |\tilde{p}_{12}|^2 \times |1 + e_{m_{12}}|^2 |u_2|^2 d\omega \quad (7)$$

Expanding Equation (7) a term with product of multiplicative error $e_{m_{11}}e_{m_{12}}$ appears. This product of $e_{m_{11}}e_{m_{12}}$ can be assumed to have a negligible contribution to the control error and hence can be ignored. Similarly, there are terms independent of multiplicative error. These terms, which do not consist multiplicative error term explicitly, contribute to control error even when the model plant mismatch is zero, due to introduction of IMC filter, which detunes perfect control condition. Hence, the control error minimization objective function simplifies as follows:

$$\min_{\tilde{p}_{11},\tilde{p}_{12}} \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{\eta}_1 e_{m_{11}}|^2 |r_1 - d_1|^2 d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{p}_{12}(e_{m_{12}})|^2 |u_2|^2 d\omega - \frac{1}{2\pi}\int_{-\pi}^{\pi} |\tilde{\epsilon}_1|^2 |\tilde{\eta}_1 e_{m_{11}}|^2 |\tilde{p}_{12}|^2 |u_2|^2 d\omega \quad (8)$$

To obtain an expression for the prefilter $f_1(z)$, an explicit nonlinear minimization of a) the terms containing the expression for multiplicative uncertainty in the frequency domain minimization function as given by Equation (3) and b) the control- relevant parameter estimation expression given by Equation (8) needs to be done. However, as the stability and performance of this loop under consideration is decided by the sensitivity function, complementary sensitivity function and the multiplicative uncertainty $e_{m_{11}}$, here the comparison of the terms consisting $e_{m_{11}}$ in Equation (3) and Equation (8) is carried out to obtain an explicit expression for the prefilter $f_1(z)$. In doing so, it is assumed that ϕ_{U1}, ϕ_{U2} are assumed as unity and ν_1 is assumed to be zero. Thus, the resulting expression for the prefilter is written as follows:

$$f_1(z) = \tilde{\epsilon}_1(z)\tilde{\eta}_1(z)\tilde{p}_{e_1}(z)\tilde{p}_{11}^{-1}(z) \times [r_1(z) - d_1(z) - \tilde{p}_{12}(z)u_2(z)] \quad (9)$$

As can be seen the prefilter Equation (9) has an explicit characterization of the disturbance and the sensitivity function and also, the "disturbance" of the other channel transfer function $\tilde{p}_{12}(z)$ explicitly appears in the filter expression given by Equation (9).

An alternate expression for the same prefilter could also have been derived by comparison of Equation (3) and Equation (8), with respect to $e_{m_{12}}$. While the choice amongst these expressions for the prefilter needs further careful analysis and is not addressed

here, it must be mentioned that the prefilter expression given in Equation (9) does actually reflects the specification on the performance in terms of the complementary sensitivity function, sensitivity function, and disturbance regulation.
To some extent this also justifies the use of a diagonal prefilter by Rivera and Gaikwad[4], in their MIMO formulation. They have proposed a method for the control-relevant identification directly for the MIMO case. Their approach is based on the use of matrix fraction descriptions(MFDs) and requires the frequency response curve fitting to obtain the prefilter. The method proposed in this paper is an alternate approach that extends the control-relevant formulation for the SISO case to MIMO systems. Generalization of the approach proposed here is easily done for systems with input dimensions greater than 2 as it involves only the specifications of the nominal models in the prefilter expression given by Equation (9).

3 MISO structure based control-relevant identification

The estimation procedure using the prefilters derived is given below.

1. To achieve good estimation in the MISO framework, the inputs must be uncorrelated. For the 2×2 system considered here, a set of pseudo random binary sequence(PRBS) and inverse repeat sequence(IRS) signals are used as perturbation inputs for the two channels respectively and the outputs are recorded. These perturbation inputs are designed based on *a priori* knowledge of the dynamics as specified by [17, 18].

2. The prefilters $f_i(z), i = 1, 2$ are derived as follows: The IMC filters for the two channels are obtained as

$$f_{imc_i}(z) = \frac{(1 - e^{\frac{-T_s}{\tau_{cl_i}}})z}{z - e^{\frac{-T_s}{\tau_{cl_i}}}}$$

   where, T_s is the sampling time chosen, and τ_{cl_i} is the desired speed of response for each of the channels.

3. Initialize the unknown elements in the prefilter expression as,

$$\tilde{p}_{ii} = \frac{1}{z-1}; \quad \eta_i = z^{-nk} f_{imc_i}(z); \quad \tilde{p}_{ij} = \frac{1}{z-1}$$

   where, nk is the delay in number of sample units in the estimated models, which is assumed to be unity, and $j = 1, 2$ but $j \neq i$.

4. The noise model is initialized based on model structure used to fit the model. It is taken as unity for output error(OE) models in all the iterations, and for auto regressive with exogenous input(ARX) model structure, it is initialized as, $\tilde{p}_{e_i} = \frac{z}{z-1}$

5. The $(r_i(z) - d_i(z))$ and $u_j(z)$ is assumed as a step signal.

6. Thus the expression for the prefilters $f_1(z)$ and $f_2(z)$ are obtained for first iteration. Following the iterative procedure as proposed by Rivera *et al.*[1] control-relevant models for the 2×2 system considered is obtained as a combination of the individual MISO structure control-relevant models. In the case of ARX model structure the noise model in subsequent iteration has to be taken as $\tilde{p}_{e_i}(z) = \frac{z}{z-\alpha_{ii}}$ where the denominator $(z - \alpha_{ii})$ is the same as that of control-relevant model estimated in previous iteration. Likewise, the control-relevant models estimated for $\tilde{p}_{ii}(z)$ and $\tilde{p}_{ij}(z)$ in the previous iteration replace nominal models in the prefilter expression in subsequent iteration.

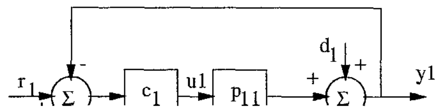
4 Simulation Results

In the following the paper machine control design problem, taken from [21], is used for validating the proposed method. The state space model of the plant is derived from the plant dynamical equations. The parameters of the state space model are as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} -0.2 & 0.1 & 1 \\ -0.05 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 1 \\ 0 & 0.7 \\ 1 & 0 \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}$$

Based on the relative gain array (RGA) analysis, the pairing of control and manipulated variables for control are given as $(1, 1)$ and $(2, 2)$. The sampling time taken is 0.2 seconds and the closed loop specifications are chosen to be 2 seconds for channel 1 and 8 seconds for channel 2. With these known specifications, the control-relevant models are estimated based on the steps given in earlier sections. The nominal model for the same system is also estimated directly by fitting MISO model structure to the input output data. The magnitude plot of all the four transfer function matrix elements, are shown in Figure 3. Good match between the true plant dynamics and the control-relevant models is seen at the control-relevant frequencies, than with the direct estimated nominal models. The closed loop simulation results, using IMC based decoupled multi loop control with the control-relevant model and direct estimated nominal model are shown in Figure 4. It is clear from these results that the control-relevant



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