Analysis of Pre-filter Based Closed-loop Control-relevant Identification Methodologies

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Control-relevant identification strategies have been variously proposed for open loop model building. In this paper, issues related to control-relevant model building in closed-loop schemes are discussed. Various important aspects such as pre-filter design and plant friendliness of the perturbation signals have been examined. Simulations involving representative problems have been considered from chemical engineering literature to highlight the applicability of the proposed methods.

Différentes stratégies d'identification pertinentes ont été proposées pour la construction de modèles de régulation en boucle ouverte. Dans cet article, on examine les problèmes liés à la construction de modèles pertinents de régulation pour les schémas en boucle fermée. Plusieurs aspects importants comme la conception des pré-filtres et le réalisme industriel des signaux de perturbation ont été examinés. On a tenu compte des simulations faisant intervenir des problèmes représentatifs de la littérature du domaine du génie pour souligner l'applicabilité des méthodes proposées.

Keywords: closed-loop identification, control-relevant identification, pre-filter, plant friendliness index.

From the perspective of achieving an enhanced and superior closed-loop performance using a model based control scheme, accurate estimation of a plant model is the most important step. Towards this end, empirical model identification methods, that use plant data, obtained after appropriate excitation of the plant, have been recommended. Typically, the excitation and model building has been proposed in open loop schemes. However, for open loop unstable plants as well as in situations involving time varying plant characteristics, excitation and model building are also proposed in closed-loop schemes. The added advantage of using closed-loop data for identification is that the data in general, implicitly reflects the intended operating condition of the plant, and may thus yield a better plant model than that obtained from open loop identification. Irrespective of whether identification is carried out in open loop or closed loop schemes, the influence of the model order on the model accuracy in terms of the bias and variance errors is an important consideration. This is particularly relevant when a parsimonious model and ease of controller structure and design is sought, subsequent to the identification process. The task of obtaining a parsimonious yet accurate model requires a compromise between achieving a good match at certain control-relevant frequencies versus large mismatch at other frequencies and has generated considerable interest under the area of joint identification and control or control-relevant identification.

Often times, operating plants cannot afford the expensive task of open loop identification as this violates the primary operating objective. Furthermore, in the presence of nonlinearities, time varying behaviour or changes in the operating regions, the model needs to be updated or adapted under closed-loop conditions. Huang and Shah (1997), Van den Hof and Schrama (1993), Vishwanathan and Rangaiah (2000), MacGregor et al. (1995), Zang et al. (1992) have proposed various approaches towards model identification under closed-loop conditions. Vishwanathan and Rangaiah (2000) have proposed an optimization-based approach to estimate a second order plus dead time model of the process using closed-loop response. Zang et al. (1992) have proposed an online refinement methodology for controllers, for disturbance rejection using closed-loop modeling. Schrama and co-workers (1993,1992a, 1992b) have proposed an iterative identification and control design scheme for adaptive performance enhancement using closed-loop identification in the frequency domain with co-prime factor perturbations. Various other techniques have been proposed for closed-loop identification from the point of view of bias and variance error minimization (Forssell and Ljung, 1999; Gevers et al., 2001; Gilson and Van den Hof, 2001). Another area of interest wherein closed-loop identification is useful, is direct controller order reduction proposed by Landau et al. (2001) and Rivera and Morari (1992a). Wang et al. (2001) have proposed a robust closed-loop identification scheme with application to auto tuning. Lakshminarayanan et al. (2001a) have used the canonical variate analysis method for closed-loop identification and control loop reconfiguration in chemical plants.

When identification is carried out under closed-loop conditions, a number of additional considerations need to be looked into. The first is the issue of persistent excitation via a dither signal, which although necessary is contrary to the controller objective of minimizing the output variability. This dither signal needs to be

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carefully designed and frugally implemented to preserve system stability and maintain the output variability at the minimum necessary level. Needless to say the nominal controller in the closed-loop also influences, through sensitivity function, the amplitude and the frequency of the dither signals that could be employed, in the context of overall system stability. This is also directly related to the issue of plant friendliness proposed by Parker et al. (2001), Yang and Rivera (2001).

The second issue is related to the re-distribution of bias (model-plant mismatch) error by data pre-filtering (Huang and Shah, 1997; MacGregor and Fogal, 1995). In the presence of closed-loop, Huang and Shah (1997) have shown that the sensitivity function plays a role in the design of the pre-filter. They propose a two-step method in which the effect of the sensitivity function is decoupled through appropriate filtering of the data in the first step, followed by the regular open loop based pre-filtering methods in the second step to shape the bias. Yang and Rivera (2001), Rivera and Bhatnagar (1993) and Bhatnagar and Rivera (1994) have presented a systematic design of the pre-filter, by incorporating the effect of sensitivity function explicitly in the pre-filter design. Their method is iterative and uses the new pre-filter synthesized with the new controller acting on the closed-loop system in each iteration, until a satisfactory performance is achieved. Lee et al. (1992) have proposed an adaptive robust control design method in the frequency domain using closed-loop identification in a control-relevant manner.

The objective of this paper is to analyze the issues related to prefilter design and implementation in the context of closed-loop, control-relevant identification. In particular, we propose the use of the two-step approach of Huang and Shah (1997) to decouple the influence of sensitivity function on the bias error. The first step of this approach is critically dependent on the accurate identification of the sensitivity function especially in the presence of disturbances affecting the closed-loop. Therefore we analyze the careful design of the dither signal to obtain accurate characterization of the sensitivity function. This first step transforms the closed-loop identification to open loop identification between the dither signal and system output that is filtered by inverse of sensitivity function. In the subsequent step of data pre-filtering to shape the bias error in a control-relevant manner, we propose the use of the control-relevant pre-filtering method of Rivera et al. (1992a) that has been developed for open loop identification. We also compare the proposed methodology with the direct closed-loop, iterative, control-relevant model identification method proposed by Yang and Rivera (2001) especially in the context of plant friendliness.

In this paper a closed-loop control-relevant identification methodology is proposed by extending the two-step identification method of Huang and Shah (1997) using closed-loop data. The control-relevant identification methodology proposed in this paper is compared with the iterative control-relevant identification methodology using closed-loop data, proposed by Yang and Rivera (2001), Rivera and Bhatnagar (1993) and Bhatnagar and Rivera (1994). Later, the simulation results obtained in different operating conditions by applying the methods presented in this paper are discussed along with the comparison of the merits and drawbacks of each of the methods. The paper is concluded with summary of the important results and the scope for future work.

Pre-filter Based Methodologies

In the following, the pre-filter based methodologies for estimation of control-relevant models using closed-loop data are presented. The two-stage identification method proposed by Huang and Shah (1997) and our extension of this method to control-relevant identification is presented first. Then the single step iterative pre-filter design methodology proposed by Yang and Rivera (2001), Rivera and Bhatnagar (1993) and Bhatnagar and Rivera (1994) under closed-loop condition is presented.

Closed-loop Identification by Two Stage Approach

Consider the closed-loop SISO system represented in Figure 1. The system can be defined as,

$$y(t) = p(z)v(t) + p_d(z)d'(t) + d(t)$$

that is,

$$y(t) = p(z)v(t) + \upsilon(t)$$

where, $\upsilon(t) = p_d(z)d'(t) + d(t)$ (1)

By assuming the set point signal $r(t)$ to be zero, the manipulated variable $u(t)$ can be expressed as,

$$u(t) = w(t) - c(z)y(t) \tag{2}$$

In the closed-loop case, consistent estimate of $p(z)$ cannot be obtained by using plant input/output data via prediction error method, due to the correlatedness of $u(t)$ with $y(t)$ through the sensitivity function $\varepsilon(z)$. In this section, we first look at the approach of Huang and Shah (1997) to decouple the effect of sensitivity function $\varepsilon(z)$, in the estimation of the model using the closed-loop data.

The sensitivity function of the system represented in Figure 1, can be written as,

$$\varepsilon(z) = \frac{1}{1 + p(z)c(z)} \tag{3}$$

Equation (2) can therefore be written as,

$$v(t) = \varepsilon(z)w(t) - \varepsilon(z)c(z)\upsilon(t) \tag{4}$$

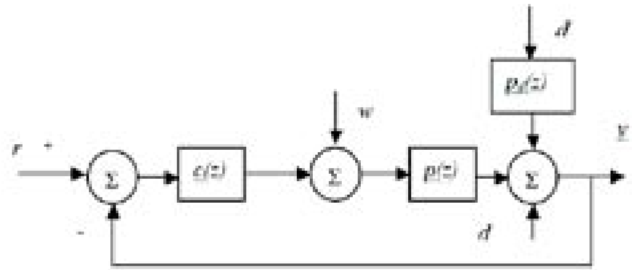

Figure 1. The closed-loop system configuration for identification.

Using Equation (4) and assuming $w(t)$ and $\upsilon(t)$ are uncorrelated signals, and also the signal to noise ratio is large, a consistent estimate of the sensitivity function $\varepsilon(z)$ can be obtained by a straightforward open loop identification strategy using $u(t)$ and $w(t)$. Estimation of the sensitivity function is the first stage of the two-stage method proposed by Huang and Shah (1997) for the model estimation.

Using Equation (4) in Equation (1), the output from the closed loop system can be expressed as:

$$y(t) = p(z)\varepsilon(z)w(t) + \varepsilon(z)v(t) \tag{5}$$

Multiplying both sides of Equation (5) by inverse of the sensitivity function, i.e., $1/\varepsilon(z)$ Equation (5) can be re-written as,

$$\frac{1}{\varepsilon(z)}y(t) = p(z)w(t) + v(t) \tag{6}$$

Writing $\frac{1}{\varepsilon(z)}$ $y(t)$ as $y_f(t)$, Equation (6) becomes,

$$y_f(t) = p(z)w(t) + v(t) \tag{7}$$

In Equation (7), assuming that the signals $w(t)$ and $\upsilon(t)$ are uncorrelated to each other, a consistent estimate of $p(z)$ can be obtained by using $y_f(t)$ and $w(t)$. Van den Hof and Schrama (1993) have proposed a similar method wherein they propose filtering of the dither signal $w(t)$ by the sensitivity function $\varepsilon(z)$ (instead of filtering the output by inverse of the sensitivity function), for the estimation of nominal model using closed-loop data. For a relative comparison of the *y*- and *w*- filter based methods, the reader is referred to Huang and Shah (1997). Since the *y*-filtering approach has been shown to yield the same accuracy expressions with respect to bias and variance errors under closed-loop and open loop conditions, we propose the use of the *y*-filter method in our approach.

Extension to Control-relevant Model Estimation

The filtering of the output by the inverse of the sensitivity function decouples the bias on the estimation data due to the presence of the controller, and a nominal model of $p(z)$ can be estimated. However, as shown by Huang and Shah (1997), in the two-stage approach, plant $p(z)$ is estimated with an objective of achieving identical asymptotic bias and variance errors as that achieved by an open loop identification scheme. This has been achieved by an appropriate choice of the order for the model that yields satisfactory validation results. In the control relevant identification, we seek a reduced order model $\tilde{p}(z)$, with an additional requirement that the identified model yields performance as per the given specification, when deployed in a model-based scheme. The estimation of reduced order model inherently introduces large model-plant mismatch. Therefore, the next issue that needs to be addressed is the shaping of the bias error such that the model-plant mismatch is minimum in the frequencies of interest from the closed-loop performance viewpoint. Since, the identification methodology of Huang and Shah (1997) presented above converts the closed-loop identification to an equivalent open loop identification, open loop data pre-filtering schemes, that have been proposed to shape the identification data to highlight the frequency range of importance from closed-loop performance perspective, and hence, to minimize the bias error in a control-relevant manner, can be used for the estimation of control-relevant model. Here we develop the methodology for the synthesis of the control-relevant pre-filter for the above case starting from Equation (7).

Equation (7) is equivalent to an open loop plant representation with respect to the signals $y_f(t)$ and $w(t)$. For estimating the control-relevant model we use a pre-filter $L(z)$ to shape the bias error to be minimum in the closed-loop frequency range of interest, at the cost of increased bias error at other frequencies, which are not relevant from closed-loop performance perspective. Let signals $y_f(t)$ and $w(t)$ be filtered by $L(z)$ and denote the pre-filtered signals as,

$$y_{sf}(t) = L(z)y_f(t) = \frac{L(z)}{\varepsilon(z)}y(t$$

and $w_s(t) = L(z)w(t)$. Let the prediction error in the estimation of the reduced order model $\tilde{p}(z)$ using $y_f(t)$ and $w(t)$ be denoted by $e(t)$. The estimation of $\tilde{p}(z)$ is performed so as to satisfy Equation.

$$y_f(t) = \tilde{p}(z)w(t) + \tilde{p}_e(z)e(t) \tag{8}$$

where, $\tilde{p}_e(z)$ is the noise model. For further details the reader is referred to Rivera et al. (1992b).

The objective of control-relevant identification is to minimize the shaped prediction error $e_f(t)$,

where, $e_f(t) = L(z)e(t)$. Using Equations (7) and (8), the expression for $e_f(t)$ can be written as,

$$e_f(t) = L(z)p_e^{-1}(z)[(p(z) - \tilde{p}(z))w(t) + v(t)] \tag{9}$$

The objective function for identification can be expressed as,

$$J = \frac{1}{N}\sum_{t=1}^{N}(e_f(t))^2 \tag{10}$$

where, N is the number of identification data points considered. The objective function as given by Equation (10) can be written equivalently in the frequency domain by using Parseval's theorem as follows (Ljung, 1987),

$$\lim_{N\to\infty} V = \frac{1}{2\pi}\int_{-\pi}^{+\pi}\left[\left|p(e^{j\omega}) - \tilde{p}(e^{j\omega})\right|^2 \Phi_w(\omega) + \Phi_v(\omega)\right]\cdot\frac{\left|L(e^{j\omega})\right|^2}{\left|\tilde{p}_e(e^{j\omega})\right|^2}d(\tag{11}$$

In writing Equation (11), it has been assumed that the dither signal $w(t)$ and the effective disturbance $\upsilon(t)$ are uncorrelated.

To get an expression for the pre-filter $L(z)$, Equation (11) is compared with the control error minimization objective function. The expression for the control error minimization problem is derived by following an approach similar to Rivera et al. (1992b) as:

The control objective is to minimize the 2-norm of the control error $e_c(= r - y)$, where, r is the set point and y is the output of the closed loop system to be implemented using the control-relevant model based controller. That is, we seek to minimize the 2-norm of the control error e_c given by,

$$\|e_c\|_2 = \left(\sum_{k=0}^{\infty} e_c^2(k) \right)^{\frac{1}{2}} \tag{12}$$

For the single degree of freedom feedback control of SISO systems with the controller $c(z)$ designed based on the nominal model $\tilde{p}(z)$ acting on the true plant $p(z)$, the control error can be expressed as (Morari and Zafiriou, 1989),

$$e_c(z) = \frac{\tilde{\varepsilon}(z)}{1+\tilde{\eta}(z)e_m(z)}.(r-v)(z) \tag{13}$$

where,

$$\tilde{\varepsilon}(z) = \frac{1}{1+\tilde{p}c}; \; \tilde{\eta}(z) = \frac{\tilde{p}c}{1+\tilde{p}c}; \; e_m(z) = \frac{p(z)-\tilde{p}(z)}{\tilde{p}(z)} \tag{14}$$

The stability of the closed-loop system with controller $c(z)$ acting on the true plant $p(z)$ is ascertained by Nyquist stability criteria on $\tilde{\eta}(z)e_m(z)$. A computationally simpler stability criteria is defined using the small gain theorem expressed as (Rivera et al., 1992b),

$$\left|\tilde{\eta}(e^{j\omega})e_m(e^{j\omega})\right| < 1 \; \forall -\pi \le \omega \le \pi \tag{15}$$

For systems satisfying (15), the control error given in Equation (13) can be approximately written as:

$$e_c(z) \approx \tilde{\varepsilon}(z)(1-\tilde{\eta}(z)e_m(z)).(r-v)(z) \tag{16}$$

In writing (16), $(1+\tilde{\eta}(z)e_m(z))^{-1}$ is approximated as $(1-\tilde{\eta}(z)e_m(z))$ (Rivera et al.,1992b), considering only the linear term in the Taylor's series expansion of $(1+\tilde{\eta}(z)e_m(z))^{-1}$. Using Equation (16) in Equation (12), and using Parseval's theorem, the frequency domain expression for control error minimization function can be written as:

$$\left\|e_c(e^{j\omega})\right\|_2^2 \approx \frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|\tilde{\varepsilon}(e^{j\omega})\right|^2 \left|1-\tilde{\eta}(e^{j\omega})e_m\right|^2 \left|(r-v)(e^{j\omega})\right|^2 d\omega \tag{17}$$

Using the triangle inequality property of the norm, Equation (17) can be equivalently written as,

$$\left\|e_c(e^{j\omega})\right\|_2^2 \approx \frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|\tilde{\varepsilon}(e^{j\omega})\right|^2 \left|(r-v)(e^{j\omega})\right|^2 d\omega \tag{18}$$
$$+\frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|\tilde{\varepsilon}(e^{j\omega})\right|^2 \left|\tilde{\eta}(e^{j\omega})e_m\right|^2 \left|(r-\upsilon)(e^{j\omega})\right|^2 d\omega$$

The first term in Equation (18) is independent of e_m and hence it contributes to the control error due to the degradation of the nominal performance with respect to perfect control by the introduction of the IMC filter (Rivera et al., 1992b; Shreesha and Gudi, 2001a, 2001b). Only the second term that contains $e_m(z)$ in Equation (18) contributes to control error due to the model-plant mismatch. Hence, the control error minimization objective function (18) transforms into control-relevant parameter estimation objective function, which can be written as,

$$\min_{\tilde{p}} \frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|\tilde{\varepsilon}(e^{j\omega})\right|^2 \left|\tilde{\eta}(e^{j\omega})\right|^2 \left|(r-v)(e^{j\omega})\right|^2 \left|e_m\right|^2 d\omega \tag{19}$$

The pre-filter required to be used for minimizing the bias error in the control-relevant frequency band is obtained by comparing the frequency domain expression for the pre-filtered prediction error minimization function given by Equation (11) with the integrand of the control-relevant parameter estimation problem given by Equation (19). It is assumed here that the dither signal $w(t)$ is a white noise with power spectrum ($\Phi_w(\omega) = 1 \; \forall$, frequency ω) and the dithersignal to noise ratio $\frac{\Phi_w(\omega)}{\Phi_v(\omega)} >> 1$. Hence, the pre-filter $L(z)$ can be expressed as follows:

$$L(z) = \tilde{p}_e(z)\tilde{p}^{-1}(z)\tilde{\varepsilon}(z)\tilde{\eta}(z).(r-v)(z) \tag{20}$$

The presence of nominal model $\tilde{p}(z)$ in Equation (20) makes the implementation of this pre-filter iterative. However, by choosing $\tilde{p}(z)$ based on the dominant open loop time constant of the plant, one step implementation can also be performed as described in Rivera et al. (1992b). Nominal sensitivity function $\tilde{\varepsilon}(z)$ and nominal complementary sensitivity function $\tilde{\eta}(z)$ are evaluated based on the IMC design methodology in terms of the desired closed-loop specification (Rivera et al., 1992b; Shreesha and Gudi, 2001a, 2001b).

Single Stage Pre-filter Based Iterative Method

An alternative approach to the two-step identification and pre-filter design presented above is to directly use the control input u and the plant output y for identification of the plant transfer function $p(z)$, after suitable pre-filtering to reflect control-relevant requirements. For the closed-loop SISO system shown in Figure 1, (assuming $w(t) = 0$), by considering $r(t) = u_d(t)$, where $u_d(t)$ is dither signal of pulse type, applied at the set point the pre-filter $L(z)$ to be used for pre-filtering the identification data can be expressed by following the methodology proposed by Yang and Rivera (2001) as,

$$L(z) = \frac{\tilde{\varepsilon}(z)\tilde{p}_e(z)(r-v)(z)}{u_d(z)} \tag{21}$$

In this method the control-relevant model estimation needs to be performed iteratively, until satisfactory closed-loop performance is achieved. In successive iteration, the pre-filter has to be updated, based on the new control-relevant model and the nominal sensitivity function. The nominal sensitivity function is estimated based on the controller and the model estimated in the previous iteration.

Rivera and Bhatnagar (1993) have also obtained the pre-filter to be used when the perturbation signal (u_d) is applied at the manipulated variable rather than set point as,

$$L(z) = \frac{\tilde{p}\,(z)\tilde{\eta}(z)\tilde{p}\,(z)(r-v)(z)}{u\,(z)} \tag{22}$$

Issues Related to Plant Friendliness Index (PFI)

The perturbation (dither) signals used for identification should be designed to possess persistent excitation characteristics. In general, the maximum length sequences as well as multi-level signals exhibit persistent excitation characteristics. As discussed earlier, these signals should be carefully designed and frugally implemented, especially when the identification is to be carried out under closed-loop conditions. Care has to be taken that the specified operating condition of the plant is not significantly affected from safety, stability as well as other operating objectives such as those on quality. The identification signals designed with this consideration are called plant friendly signals. The plants to be identified in general possess low pass filtering characteristics and hence the identification input containing higher frequencies are filtered by the plants resulting in minimum output variability. Thus, these inputs being persistently exciting have low input friendliness while exhibiting high output friendliness. The plant friendliness index (*PFI*) is defined as (Parker et al., 2001),

$$PFI = \left(1 - \frac{n_T}{N-1}\right)100 \qquad (23)$$

where, n_T is the number of transitions in the input signal (change from one level to another) and N is the length of the signal (in number of sample units). In case of step input the *PFI* is very high almost approaching 100%. When a square pulse of two cycles is used in the perturbation period, the *PFI* will still be very high in the order of 99.95%. For white noise signals of maximum length binary sequence (MLBS) type, the *PFI* will be around 50% when sampling time and switching time of the sequence are the same. This is because of the property of the MLBS sequences that, a signal of length N contains $(N + 1)/2$ states in high level and $(N - 1)/2$ states in low level making number of transitions $n_T = (N + 1)/2$. However, by making switching time an integer multiple of sampling time, improved PFI can be achieved, as this makes the length of the sequence proportionately larger as the ratio of switching time to sampling time, keeping the number of transitions unchanged. This aspect of the *PFI* will be discussed later. Other important factors that are considered to define *PFI* are the duration of the signal and the amplitude of the signal. The duration of the signal needs to be minimum to make the experimentation time short. The amplitude has to be such that the product variability should be minimized while achieving persistent excitation.

Remark 1

In the two-step approach and the control-relevant pre-filter defined in Equation (20), the specifications are explicitly reflected in the pre-filter design, through the IMC filter. The first order IMC filter required for achieving realizable model-based controller is defined as,

$$F_{imc}(z) = \frac{(1 - e^{\frac{-T_s}{\tau_{cl}}})}{z - e^{\frac{-T_s}{\tau_{cl}}}} z \qquad (24)$$

where, τ_{cl} is the specification for the speed of closed-loop performance. Only a single stage of data collection using a persistently exciting dither signal followed by model identification is adequate to generate a control-relevant model that reflects the specifications. However, the accurate identification of the sensitivity function $\varepsilon(z)$ in the first step needs persistent excitation and careful dither signal design. Also, in the presence of the disturbances affecting the closed-loop, the identification of sensitivity function $\varepsilon(z)$ could be biased. As shown in the illustrative examples considered here, if the dominant time constant of the disturbance dynamics is known and by a careful design of dither signal, one can obtain accurate estimation of the sensitivity function $\varepsilon(z)$.

Remark 2

Further, unlike the pre-filter design of Equation (20), the iterative approach of Yang and Rivera (2001) does not have the requirement of persistently exciting dither input and is therefore a plant friendly identification approach. In the presence of a disturbance however, the identification could be biased and therefore care needs to be taken during data collection to ensure a representative data sets.

Remark 3

In the iterative approach of Yang and Rivera (2001) the specifications get indirectly reflected, in the pre-filter design as follows: The nominal sensitivity function $\tilde{\varepsilon}(z)$ (in Equation 21) and nominal complementary sensitivity function $\tilde{\eta}(z)$ (in Equation 22) of the previous iteration contains the model based controller term in which the specifications are incorporated through the IMC filter (refer to Equation 24).

Comparison of Pre-filter Based Methods

In the following, we present some of the results obtained using the control-relevant identification methodologies discussed in this paper.

Illustrative Example: A Simple Plant with Recycle

Figure 2 shows the block diagram of a plant containing recycle dynamics. Such dynamics are typically seen in chemical process plants and are discussed in Lakshminarayanan et al., 2001b; Kwok et al., 2001. The step response dynamics of such plants are governed by the relative time constants of the forward and recycle dynamics. The frequency response of these plants can typically exhibit multiple corner frequencies. They could also exhibit significantly large settling times and could complicate the design of the controller especially if there is a large difference between the time constants, gain and the delay in the forward and recycle dynamics. We consider here an example having a forward path transfer function $G_F(s) = \frac{0.9}{10s+1}$ and recycle path transfer function $G_R(s) = \frac{0.9}{100s+1}$, with positive feedback (see Figure 2). This plant has a settling time of 3500 s for a step perturbation.

Results with the y-filtering Approach

In the analysis performed here, a sampling time of 1 s is chosen. A MLBS type dither signal of variance 0.5 is synthesized with switching time of 1 s, 3 s and 5 s respectively as proposed by Rengasamy and Srinivasan (1999). The closed-loop data for estimation is collected by using a nominal proportional controller and by applying a dither signal at the controller output. The closed-loop system is also simultaneously subjected to a step disturbance of amplitude 0.2 and a white noise disturbance of variance 0.2 that is uncorrelated with the dither signal, at the output. The dither signal and the plant input/output data are recorded. A requirement that the closed-loop time constant (τ_{cl}) of 100 s is specified. The sensitivity function is estimated in a prediction error model (PEM) structure of appropriate order using the plant input and the dither signal. The sensitivity function estimated with each

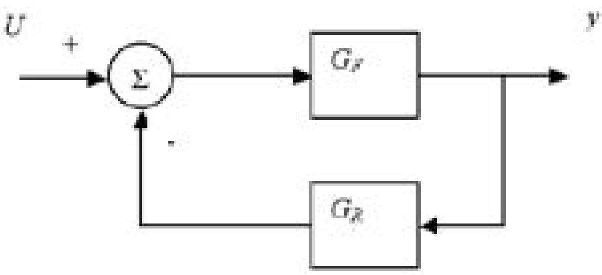

Figure 2. Schematic of a plant with recycle.

of the switching times are shown in Figure 3 along with the actual sensitivity function estimated with the nominal controller implemented on the true plant. From Figure 3, one may observe that reasonably good estimation of sensitivity function is obtained even in the presence of noise and disturbance during closed-loop data collection, with a switching time of MLBS perturbation of 5 s. When a white noise signal of variance 0.2, which is uncorrelated with the dither signal alone, is subjected at the output during the closed-loop data collection, a better estimate of sensitivity function is achieved with a switching time of 5 s. It is seen that in the presence of the disturbance, the estimation of sensitivity function deviates from the true value especially at low frequencies. However, an increase in the switching time T_{sw} brings in an increase in the low frequency content in the dither signal and enhances the estimate of the sensitivity function. In the absence of disturbance/noise at the output, the sensitivity function is estimated fairly accurately, with switching time of 1 s. The second stage of the estimation is carried out to obtain a first order model of the plant with a delay of one sample in auto regressive with exogenous input (ARX) structure using the dither signal and output filtered by inverse of the sensitivity function. To estimate a control-relevant model using closed-loop data, we use a control-relevant pre-filter given by Equation (20). The single step method for the pre-filter synthesis is adapted (Rivera et al., 1992b). The nominal model to be used in pre-filter expression is considered as a first order model with a delay of one sample, having a dominant time constant of 700 s (open loop plant dominant time constant). An ARX model structure and IMC strategy is assumed for the pre-filter design. The closed-loop performance objective is to regulate step disturbance to the specification. The pre-filter synthesized with above considerations is used to filter the output filtered by inverse of sensitivity function and the dither signal. The control- relevant model and the direct estimated nominal model (model estimated using the open loop identification data and without any pre-filtering) are compared with the actual dynamics of the plant containing recycle dynamics. Figure 4 shows the frequency domain characteristics of various models with the plant obtained with switching time of 1 s, 3 s and 5 s respectively. From Figure 4 it can be shown that with a switching time T_{sw} of 5 s, good match of the control-relevant model with the plant dynamics is exhibited in the control-relevant frequency range of interest. A specification of τ_{cl} for the closed-loop time constant implies a bandwidth (frequency at which closed-loop system magnitude response reaches -3db), of closed-loop system to be approximately $1/\tau_{cl}$ rad/s. The control-relevant frequency range is a band of frequency around the desired closed-loop bandwidth.

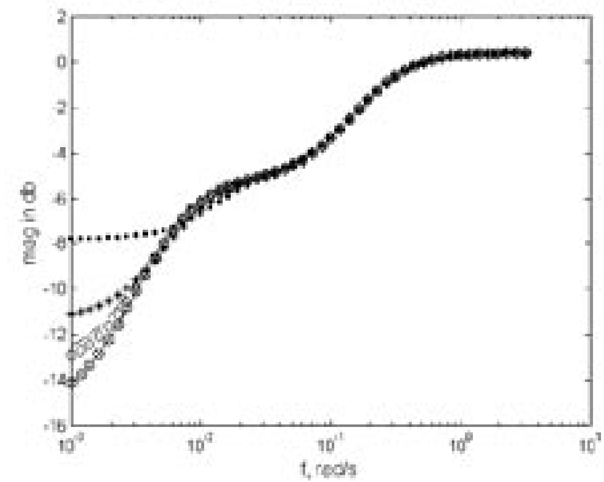

Figure 3. Comparison of frequency response of sensitivity function estimated with different switching times (T_{sw}) with sampling time of 1s: solid line- actual sensitivity function, dots- with T_{sw} = 1 s, stars- with T_{sw} = 3 s, dashed line- with T_{sw} = 5 s with MLBS dither signal in the presence of white noise and step disturbance during data collection; diamond- with T_{sw} = 5 s in the presence of only white noise disturbance, cross- and circles- with MLBS perturbation and square pulse perturbation in the absence of any disturbance during data collection.

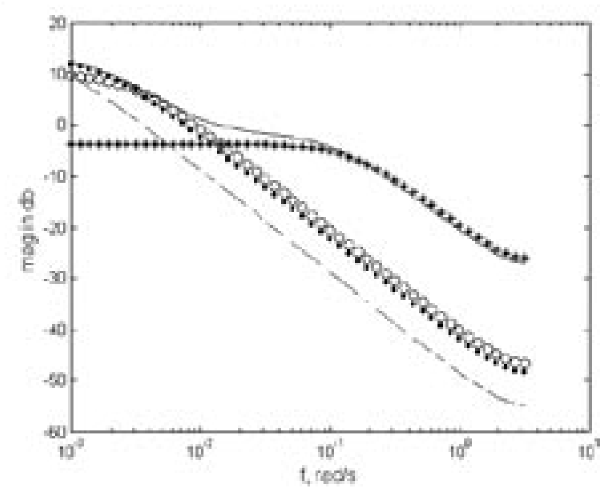

Figure 4. Frequency response comparison: solid line- plant, stars- direct estimated nominal model, dashed line-, dots-, circles- control-relevant models obtained with switching time of 1 s, 3 s, and 5 s respectively via y-filtering method with sampling time of 1 s.

Figure 4 shows a good match of the control-relevant model with the dynamics of the recycle plant at a frequency range around 0.01 rad/s, which is the desired closed-loop bandwidth in our case, at the cost of large mismatch at other frequencies. With switching times of 1 s and 3 s, even the control-relevant models exhibit a large mismatch in the control-relevant frequency range. This highlights the importance of accurate estimation of the sensitivity function.

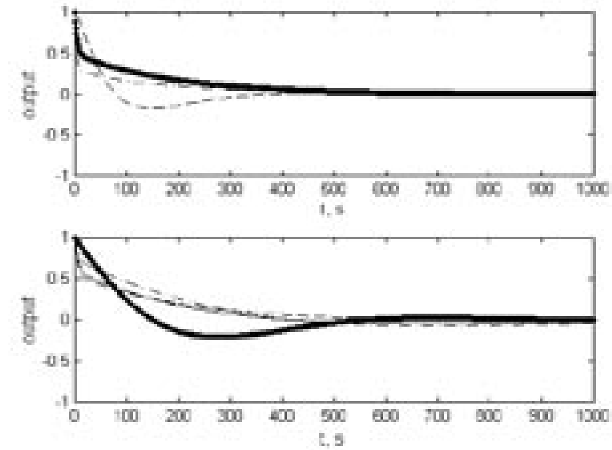

Figure 5. Closed-loop performance: a) Using models estimated with noisy data: solid line-, dotted line- and dash dotted line- using control-relevant models obtained with switching time of 5 s, 3 s and 1 s respectively and dashed line- using nominal model based system.
b) Using models estimated with noise free data and T_{sw} = 1 s: solid line- and dashed line- control-relevant model obtained with MLBS perturbation and pulse perturbation respectively, dash dotted line- and dotted line- with nominal models estimated using MLBS and Pulse perturbation respectively.

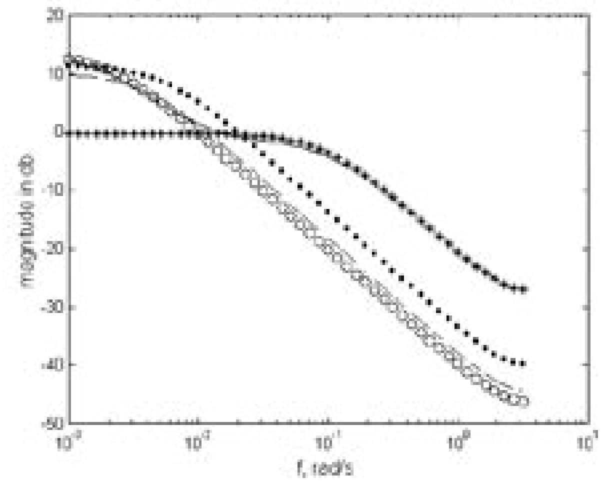

Figure 6. Frequency response comparison with T_{sw} = 1 s and estimation data collected in the absence of noise/disturbance: solid line- plant, stars- direct estimated nominal model, dashed line- control-relevant model obtained via *y*-filtering method using MLBS dither signal and dots- direct estimated nominal model, circles- control-relevant model obtained using square pulse dither signal.

The closed-loop performance obtained using the direct estimated and control-relevant model based IMC scheme is shown in Figure 5. The control-relevant model obtained with a switching time of 5 s and in the presence of noise/disturbance during closed-loop data collection, is shown to exhibit the closed-loop performance that is closest to the specification when compared with the control-relevant models estimated at other switching times. The direct estimated nominal model based system exhibits a closed- loop performance having an overshoot and does not meet the desired specification as well (refer Figure 5a). Figure 6 shows the results obtained on the model estimation when a plant friendly signal was used as dither signal. The signal was a square pulse of unit amplitude, consisting of two cycles with total experimentation time the same as that of the MLBS signal used earlier. In the absence of any disturbance or noise at the output, a consistent estimate of the sensitivity function is obtained (refer to Figure 3). Also, on comparison of the magnitude response characteristics of i) the nominal model, ii) the control-relevant model obtained using MLBS dither and iii) the control-relevant model obtained using a square pulse type of dither signal as shown in Figure 6, it is observed that good estimation of the control-relevant model is obtained only by the MLBS type dither. Similarly, from the closed-loop responses shown in Figure 5b, the control-relevant model estimated using MLBS type dither is closest to the desired closed-loop performance compared to that obtained from the control-relevant model estimated using the pulse type of dither. In the presence of noise/disturbance during the closed-loop data collection, however, the *y*-filter based approach does not yield satisfactory performance with pulse dither signal in terms of sensitivity function and model estimation. Thus, the *y*-filter based two-stage control-relevant identification method requires the persistently exciting dither signal that is uncorrelated with the disturbance to achieve good sensitivity function and model estimation.

Results with the Iterative Refinement Method

In the following, we consider the application of the method of iterative refinement (Yang and Rivera, 2001) for the control-relevant model estimation to the plant with recycle dynamics. Using the a priori knowledge of the dominant time constant of the plant, a first order model with a delay of one sample is initialized. Using this model, a nominal controller based on the IMC strategy is designed and closed-loop plant input/output data is collected by applying an appropriately designed MLBS dither acting at the set point. The pre-filter design is performed using Equation (21). The nominal sensitivity function appearing in Equation (21) is estimated based on the nominal open loop model and the nominal controller. The nominal noise model is having an ARX structure, and the closed-loop system is sought to be designed for step set point signal after the implementation of the control-relevant model based controller. The pre-filter is designed according to Equation (21) and the input/output data is filtered. An ARX model of first order with unit delay is fitted to the pre-filtered data.

The closed-loop system is adapted with the new controller estimated using the newly estimated model, in an IMC framework and the new set of input/output data is recorded with the same dither signal. The nominal sensitivity function is re-estimated and the pre-filter is redesigned. The input/output data is filtered with the re-designed pre-filter and a model of the plant is again estimated. The above iterations are continued until the closed-loop performance obtained is satisfactory and according to specification. The performance was analyzed with and without a noise/disturbance signal at the output during the closed-loop data collection in each iteration. Instead of MLBS dither, the estimation is also performed separately using square pulse type dither acting at the set point.

Results obtained with a MLBS type dither of variance 0.5 acting at the set point with a white noise disturbance of variance 0.2, which is uncorrelated with dither, acting at the output during data collection are shown in Figures 7 and 8 at the end of sixth iteration. It can be seen from Figure 7 that the control-relevant model matches the plant frequency response in the control-relevant region. As seen in Figure 8, the closed-loop response obtained using the control-relevant model based controller is as per specification and better than that obtained from the nominal controller. The results obtained with i) MLBS dither acting at set point without any disturbance at the output during the closed-loop data collection, ii) a square pulse type dither of amplitude 0.5 acting at the set point with a white noise disturbance of variance 0.2 at the output and iii) a square pulse type dither of amplitude 0.5 acting at the set point without any disturbance at the output, are also studied and seen to be almost similar. It is observed from these results that with a white noise disturbance of variance 0.2 acting at the output during data collection, a MLBS type dither at the set point yielded marginally better results than the pulse type dither.

Performance Analysis Using Plant Friendliness Index:

In the following, an analysis of the relative plant friendliness index *(PFI)* of the various perturbation signals employed and the affect of this on the results obtained are analyzed. We consider the case of two-stage control-relevant identification using γ-filtering approach first.

From Table 1 it is observed that with MLBS dither signal of variance 0.5, for unit sample time and switching time equal to five sampling times, the *PFI* is about 90%. Also it gives minimum RMS error in the sensitivity function estimation when the data is collected in the presence of disturbance. With switching time of 1 s and 3 s, the RMS error and also the *PFI* are poor compared to that for switching time of 5 s. Increase in switching time makes the signals transition minimum over a given number of samples and hence, enhances the low frequency content of the dither signal and a good estimation of sensitivity function is obtained. The results in the last two rows of Table 1 pertain to the case with no disturbance during data collection. In this case it is observed that a pulse dither of amplitude 0.5 having a plant friendliness index of 0.998 as well as a MLBS dither of variance 0.5, with unit switching time having *PFI* 0.5, gave good estimate of sensitivity function. However, in the presence of noise/disturbance during data collection, it is only MLBS type dither signal of variance 0.5, with higher switching time of five sample units gave good results.

Table 2 shows similar results for iterative refinement method of control-relevant identification. From Table 2 it can be observed that with square pulse perturbation of amplitude 0.5, and one cycle during the experimentation time, the *PFI* is in the order of 99.9%. It is also observed that with such a perturbation signal, a good estimate of control- relevant model and satisfactory performance as per specification is achieved by using data collected in the absence of any disturbance at the output. However, with square pulse perturbation the number of iterations required is more, when a white noise disturbance of variance 0.2 is present during data collection. With MLBS type perturbation, of variance 0.5, with and without the presence of white noise type disturbance, which is uncorrelated with the dither signal and of variance 0.2, the control-relevant model converged in only six iterations. The advantage of this method is that a consistent estimate is achieved with plant friendly perturbation signal as compared to γ-filtering approach. However, special care needs to be taken when a disturbance of step type is acting during closed-loop data collection, when using the iterative refinement method.

Comments on the Methodologies

In case of γ-filtering method, the estimation procedure even though has two stages, is non-iterative. Good estimation of control-relevant models and satisfactory closed-loop performance is achievable with MLBS perturbation, even in the

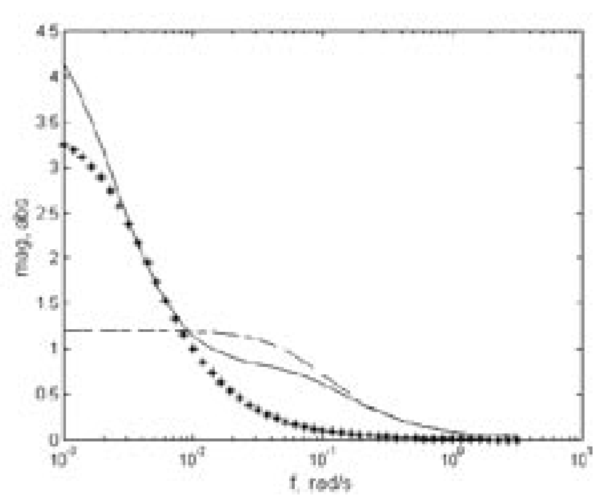

Figure 7. Frequency response characteristics of models with MLBS perturbation at set point, with white noise disturbance acting at output during data collection: solid line- plant, dashed line- direct estimated nominal model, stars- control-relevant model.

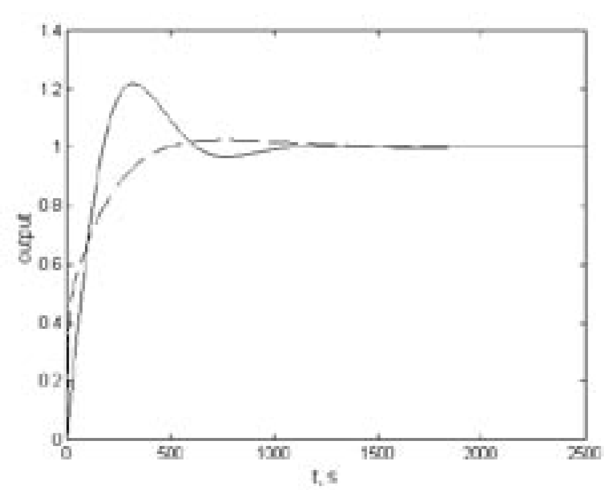

Figure 8. Closed-loop performance comparison with MLBS perturbation at set point with white noise disturbance acting during closed-loop data collection: solid line- based on nominal model, dashed line- using control-relevant model.

Table 1. *Y*-filter based approach with and without disturbance

Signal Type	N	n_r	Disturbance during data collection	RMS error in estimated $\varepsilon(z)$	*PFI*
MLBS with T_{sw} = 1 s	16383	8192	Yes	2.7675	0.5
MLBS with T_{sw} = 3 s	12288	2048	Yes	0.9884	0.8333
MLBS with T_{sw} = 5 s	10235	1024	Yes	0.2816	0.900
Square pulse with 2 cycles	16383	3	No	1.639×10^{-12}	0.9998
MLBS with T_{sw} = 1 s	16383	8192	No	7.859×10^{-13}	0.500

Table 2. Iterative refinement based approach with and without disturbance

Signal Type	N	n_r	Disturbance during data collection	RMS error in estimated $\varepsilon(z)$	Numberr of iterations	*PFI*
MLBS with T_{sw} = 1 s	8191	4096	No	0.0393	6	0.5
MLBS with T_{sw} = 1 s	8191	4096	Yes	0.0412	6	0.5
Square pulse with 1 cycle	8191	1	No	0.0393	6	0.9999
Square pulse with 1 cycle	8191	1	Yes	0.0377	8	0.9999

presence of step and/or white noise disturbance during closed-loop data collection. However, this needs an appropriate choice of switching time for white noise type of perturbation signals.

The method proposed by Yang and Rivera (2001) uses more plant friendly perturbation signals. However, the method is iterative and takes more time for estimation and special care needs to be taken to remove the effect of disturbance/noise, from the closed-loop data. In the presence of white noise disturbance during closed-loop data collection, MLBS type dither gave marginally better results in lesser numbers of iterations than square pulse type dither signal.

With plant friendly perturbation like square pulse signal, the *y*-filtering method does not estimate a good control-relevant model in the presence of either step and/or white noise disturbance or both acting together, that is, it requires persistent excitation.

Conclusions

In this paper, the design of dither signal for accurate estimation of sensitivity function from closed-loop data is analyzed. It is shown that by judiciously choosing the switching time of the dither signal, in the presence of disturbance at the output during data collection, good estimation of sensitivity function can be estimated. The two-stage plant identification method of Huang and Shah (1997), using closed-loop data which converts the closed-loop identification problem to an equivalent open loop identification in terms of asymptotic bias and variance errors, is extended to control-relevant identification. A design methodology for the control-relevant pre-filter is also presented. Analysis and comparison of this control-relevant identification method is made with that proposed by Yang and Rivera (2001), wherein they use appropriately designed pre-filter for filtering the closed-loop plant input/output data and estimate the control-relevant model in an iterative manner. It has been observed that the *y*-filtering based two-step control-relevant model estimation method results in a control-relevant model and satisfactory performance by using MLBS type dither of switching time an integral of sampling time, even when the identification data is affected by disturbance. However, the control- relevant identification method of Yang and Rivera (2001) uses pulse type dither at set point or a manipulated variable to estimate control-relevant models and results in satisfactory performance in the absence of any disturbance at the output, during closed-loop data collection. The plant friendliness of the perturbation inputs used in the two methods is also compared.

Acknowledgments

The first author would like to acknowledge the management of N. M. A. M. Institute of Technology, Nitte, Udupi, Karnataka, India, 574110, for sponsoring him to the Ph.D. programme at IIT Bombay, India and for financial assistance.

Nomenclature

ARX	auto regressive with exogenous input
$c(z)$	controller transfer function
$d'(t)$	white noise signal of zero mean and known variance
$d(t)$	disturbance at output
$e(t)$	prediction error in the estimation of $\tilde{p}(z)$ using $y_f(t)$
e_c	control error in the closed-loop system
$e_m(z)$	multiplicative uncertainty in plant estimation
$e_f(z)$	$e(t)$ filtered by the control-relevant pre-filter $L(z)$
$F_{imc}(z)$	IMC filter transfer function
G_F, G_R	forward and reverse path transfer function of recycles plant
IMC	internal model control
J	mean squared filtered prediction error
$L(z)$	control- relevant pre-filter transfer function
MLBS	maximum length binary sequence

N	number of data points
n_T	number of transitions in input signals
PEM	prediction error model
PFI	plant friendliness index
$p(z)$	discrete time rational transfer function of the plant
$p_d(z)$	stable and stably invertible noise filter
$p_e(z)$	noise model transfer function
$\tilde{p}(z)$	reduced order model of the plant
RMS	root mean square
$r(t)$	set point signal
SISO	single input single output
t	time, (s)
T_s	sampling time, (s)
T_{sw}	switching time, (s)
$u_d(t)$	dither signal used for estimation either at set point or at manipulated variable of the closed loop system
$w(t)$	persistently exciting dither signal of appropriate order
$w_s(t)$	$w(t)$ filtered by the control-relevant pre-filter $L(z)$
$y(t)$	plant output
$y_f(t)$	output $y(t)$ filtered by inverse of sensitivity function
$y_{sf}(t)$	$y_f(t)$ filtered by the control-relevant pre-filter $L(z)$
z	discrete time operator
$*(e^{f\omega})$	frequency domain representation of the function *

Greek Symbols

ε	sensitivity function
$\tilde{\varepsilon}(z)$	nominal sensitivity function
$\tilde{\eta}(z)$	nominal complementary sensitivity function
$\upsilon(t)$	effective disturbance signal at the output.
τ_{cl}	specification for speed of closed-loop response, (s)
$\Phi_*(\omega)$	power spectra of signals *
ω	angular frequency in rad/s

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Manuscript received March 19, 2002; revised manuscript received May 26, 2003; accepted for publication August 1, 2003.