ON SPECIAL CASES OF GENERAL GEOMETRY

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Abstract

We find relations between quantities defining geometry and quantities defining the length of a curve in geometries underlying Electromagnetism and unified model of Electromagnetism and Gravitation. And we show that the length of a vector changes along a curve in these geometries.

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1 Introduction

In paper [1] a new geometry called General Geometry is formulated and it is shown that its the most simplest case is geometry underlying electromagnetism. However, relation between quantities defining geometry $F^{\sigma}{}_{\lambda}$ and the length of a curve A_μ was assumed. In paper [2] it is shown that geometry underlying unified model of electromagnetism and gravitation is also a special case of General Geometry. And relations between quantities defining geometry $F^{\sigma}{}_{\lambda}$, $\Gamma^{\sigma}{}_{\mu\nu}$ and the length of a curve $g_{\mu\nu}$, A_μ were also assumed.

In the present paper we prove relations assumed in [1] and [2]. We show that the length of a vector changes along a curve in geometries underlying electromagnetism and unified model of electromagnetism and gravitation. In Riemannian geometry the length of a vector does not change and this makes it be an underlying geometry for Gravitation. If the length of a vector changes in Riemannian geometry then it fails to be an underlying geometry for Gravitation. Riemannian geometry with changing length of a vector has been used by H. Weyl [3] in order to unify electromagnetism and gravitation. Predictions of his theory is in contradiction with experimental results [4]. In our opinion this is because Weyl tried to geometrize electromagnetism in the framework of Riemannian geometry which is impossible as has been demonstrated in [1]. However, his idea that the length of a vector changes along a curve in the presence of electromagnetic field turned to be true.

In summary, Geometry of Electromagnetism [1] with changing length of a vector has physical interpretation as geometry underlying Electromagnetism. Riemannian geometry with constant length of a vector has physical interpretation as geometry underlying Gravitation. Combination of Geometry of Electromagnetism and Riemannian geometry with changing length of a vector is geometry underlying unified model of Electromagnetism and Gravitation.

In the next section we prove relations assumed in [1] and [2] and show that the length of a vector changes along a curve in geometries underlying Electromagnetism and unified model of Electromagnetism and Gravitation.

2 On Special Cases of General Geometry

In paper [1] the relation between A in (2) and F in (1) was assumed. Here we prove that this relation holds to be true, provided that the length of a vector changes along a curve. We also prove relation between Γ and g which was assumed in [2].

Geometry of Electromagnetism [1] is defined by

$$\frac{d\xi^{\sigma}}{du} = -F^{\sigma}{}_{\lambda}(x)\xi^{\lambda}. \tag{1}$$

We consider the following metric

$$ds = \sqrt{\eta_{\mu\nu}dx^{\mu}dx^{\nu}} + \frac{q}{cm}A_{\mu}(x)dx^{\mu}, \quad \eta_{\mu\nu} = diag(1 - 1 - 1 - 1). \tag{2}$$

Accordingly, the length of a vector $V=\xi^\lambda\frac{\partial}{\partial x^\lambda}$ is

$$dl=\sqrt{\eta_{\mu\nu}\xi^\mu\xi^\nu}+\frac{q}{cm}A_\mu(x)\xi^\mu. \tag{3}$$

And we assume that

$$\frac{dl}{du}=\Phi_\nu(A_\lambda,F_{\mu\sigma})\xi^\nu, \tag{4}$$

where A_μ are some functions of x , Φ_ν are functions of A_μ and $F_{\mu\nu}$, and q, c, m are some parameters. Equation (4) means that the length of a vector changes along a curve due to Φ_ν. Substitution of dl in (4) by (3) leads to equations

$$\xi^\mu\xi^\nu(F_{\mu\nu}+F_{\nu\mu})=0, \quad \frac{q}{cm}(\partial_\mu A_\sigma x^\mu_u-A_\mu F^\mu{}_\sigma)\xi^\sigma=\Phi_\nu(A_\lambda,F_{\mu\sigma})\xi^\nu. \tag{5}$$

The most general solution to the first one is any antisymmetric tensor

$$F_{\mu\nu}=-F_{\nu\mu}.$$

We choose Φ_ν such that the second equation has solution[1]

$$F_{\mu\nu}=\frac{q}{cm}(\partial_\mu A_\nu-\partial_\nu A_\mu).$$

As it is shown in [1], curvature vector R_λ is equal to $R_\lambda=\partial^\mu F_{\mu\lambda}$. Equation $R_\lambda=0$ coincides with Maxwell equation for electromagnetic field A_μ and equation for geodesics coincides with the equation for a particle interacting with electromagnetic field A_μ. This allows us to interpret A_μ as electromagnetic field and geometry defined by (1) with (2) and (4) as geometry underlying electromagnetism. q is identified with charge, m with mass of a particle interacting with electromagnetic field A_μ, c is the speed of the light.

If we choose $\Phi_\nu=0$ then the second equation in (5) reduces to

$$\partial_\mu A_\sigma x^\mu_u-A_\mu F^\mu{}_\sigma=0.$$

Multiplication by A^σ gives

$$A^\sigma\partial_\mu A_\sigma=0.$$

This equation is a constraint for A_μ. Therefore in order to consider general functions A_μ of x we have to allow $\Phi_\nu\neq 0$. Hence, the length of a vector must change along a curve in Geometry of Electromagnetism.

Next we consider geometry underlying unified model of electromagnetism and gravitation [2] defined by

$$\frac{d\xi^\sigma}{du}=-(F^\sigma{}_\lambda(x)+\Gamma^\sigma{}_{\lambda\mu}(x)x^\mu_u)\xi^\lambda, \tag{6}$$

and choose metric as

$$ds=\sqrt{g_{\mu\nu}dx^\mu dx^\nu}+\frac{q}{cm}A_\mu(x)dx^\mu,$$

[1] If we choose $ds=\sqrt{\eta_{\mu\nu}dx^\mu dx^\nu}$ and $\frac{dl}{du}=0$ we obtain that $F_{\mu\nu}$ is an arbitrary antisymmetric tensor and electromagnetic field A_μ has to be introduced artificially.

where $g_{\mu\nu}(x)$ is a metric tensor and the length of a vector V is

$$dl = \sqrt{g_{\mu\nu}\xi^\mu\xi^\nu} + \frac{q}{cm}A_\mu\xi^\mu, \tag{7}$$

and it changes as

$$\frac{dl}{du} = \Phi'_\nu(A_\lambda, F_{\mu\sigma})\xi^\nu. \tag{8}$$

Note that in this geometry $\xi_\rho = g_{\rho\mu}\xi^\mu$. Substitution of dl in (8) by (7) gives rise to

$$\Gamma_{\nu,\sigma\lambda} + \Gamma_{\lambda,\sigma\nu} = \partial_\sigma g_{\lambda\nu}, \quad \frac{q}{cm}(\partial_\mu A_\sigma x^\mu_u - A_\mu F^\mu{}_\sigma) = \Phi'_\sigma.$$

Solutions to the first equation are

$$2\Gamma_{\lambda,\mu\nu} = \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda}.$$

We choose Φ'_ν so that the second equation solves as

$$F_{\mu\nu} = \frac{q}{cm}(\partial_\mu A_\nu - \partial_\nu A_\mu).$$

According to the results obtained in [2] we interpret $g_{\mu\nu}$ as gravitational field and A_μ as electromagnetic field.

3 Conclusion

In this paper we considered only two special cases of General Geometry [1]. Its special cases, discussed in this paper, with appropriate metrics are underlying geometries for physical theories. The most simplest case of General Geometry

$$\frac{d\xi^\sigma}{du} = -F^\sigma{}_\lambda(x)\xi^\lambda,$$

with metric

$$ds = \sqrt{\eta_{\mu\nu}dx^\mu dx^\nu} + \frac{q}{cm}A_\mu(x)dx^\mu$$

is geometry underlying Electromagnetism.

Next order in x_u, Riemannian geometry,

$$\frac{d\xi^\lambda}{du} = -\Gamma^\sigma_{\lambda\nu}(x)x^\nu_u\xi^\lambda$$

with metric

$$ds = \sqrt{g_{\mu\nu}dx^\mu dx^\nu} \tag{9}$$

is geometry underlying Gravitation. Combination of two previous geometries

$$\frac{d\xi^\sigma}{du} = -(F^\sigma{}_\lambda(x) + \Gamma^\sigma{}_{\lambda\mu}(x)x^\mu_u)\xi^\lambda$$

with metric

$$ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu} + \frac{q}{cm} A_\mu(x) dx^\mu$$

is geometry underlying unified model of Electromagnetism and Gravitation proposed in [2].

We do not discuss the other special cases in this paper. Riemannian Geometry with metric $ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu} + A_\mu(x) dx^\mu$ instead of (9) has been considered in [5][2] and applied to Kaluza-Klein theory. As we demonstrated in [1] any attempt to geometrize electromagnetism in geometries like Riemannian independent of the chosen metric must fail. By choosing different metrics we do not change geometry.

References

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[2] I thank Prof. M. Anastasiei for informing me about [5] after [1], [2] and this paper have been posted on the Internet.