

UNIFICATION OF ELECTROMAGNETISM AND GRAVITATION

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Abstract

We propose a unified model of electromagnetism and gravitation in the framework of General Geometry. It reproduces Electromagnetism and Gravitation and predicts that electromagnetic field is a source for gravitational field. This theory is formulated in four dimensional spacetime and does not contain additional fields.

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1 Introduction

In this paper we consider the problem of unification of electromagnetism and gravitation. After it was realized that the underlying geometry for gravitation is already known Riemannian geometry [1],[2] at the beginning of XX century, many physicists and mathematicians tried to geometrize electromagnetism and unify it with gravitation. The first unified theory was proposed by Weyl [3] in 1918 who considered Riemannian geometry with changing length of a vector and was generalized in [4]. As it is discussed in [5] almost all prediction of this theory are in contradiction with experiments. The next attempt has been done by Kaluza [6] who included electromagnetic field in metric tensor of five dimensional space-time and accepted Riemannian geometry as underlying geometry for his theory. Unfortunately, this theory also failed to geometrize electromagnetism and did not give satisfactory unified model because besides five dimensionality it does not reproduce electromagnetism exactly, contains unobserved additional dilaton field, and has charge/mass problem. Later, Kaluza's theory was extended by Klein [7] and generalized by Einstein and coworkers [8], [9], [10]. An attempt based on Riemannian metric and distant parallelism has been done by Einstein in [11]. Consideration of non symmetric metric tensor whose antisymmetric part is to be associated with electromagnetism has been performed in [12]. Later, attempts have been done in the framework of generalized Einstein manifolds [13], quantized fractal space time [14] and etc. All these approaches considered unification in the framework of Riemannian geometry and failed to satisfy the requirement of a unified theory to reproduce Electromagnetism and Gravitation exactly, because they are experimentally well known theories. Accordingly, the problem of unification of electromagnetism and gravitation remained open.

In the present paper we propose a unified model of electromagnetism and gravitation in the framework of General Geometry [15] which is completely different from Riemannian geometry. This model reproduces electromagnetism and gravitation and predicts that electromagnetic field is a source for gravitational field. It is formulated in four dimensional spacetime and does not contain additional fields. Moreover, we show that Riemannian geometry is not appropriate for unification of electromagnetism and gravitation.

In the next section, we consider a special case of General Geometry [15]. Equation for geodesics in this geometry coincides with the equation of motion for a particle interacting with electromagnetic and gravitational fields. We find an action for the unified model from curvature function. The action is automatically gauge invariant and invariant under general transformations of coordinates. Next, we find equations from action. The equation for gravitational field predicts that electromagnetic field is a source for gravitational field. In the weak gravitational and strong electromagnetic field approximation we find that total energy of electromagnetic field produces gravitational field.

In the last section, we demonstrate that Riemannian geometry is not appropriate for unification of electromagnetism and gravitation. Here we show that geometry underlying the proposed model is created by interacting particles and sources for electromagnetic and gravitational fields unlike geometry underlying gravitation, which is created by sources for gravitational field only. We also discuss results obtained in sec.2.

2 Unification of electromagnetism and gravitation

In this section we propose a unified model of electromagnetism and gravitation using geometrization principle. For geometry underlying the proposed model we choose a particular case of General Geometry [15] defined by

$$\frac{d\xi^\sigma}{du} = -(F^\sigma{}_\lambda(x) + \Gamma^\sigma{}_{\lambda\nu}(x)x_u^\nu)\xi^\lambda, \quad (1)$$

with the length of a curve

$$ds = \sqrt{g_{\mu\nu}(x)dx^\mu dx^\nu} + \frac{q}{cm}A_\mu(x)dx^\mu,$$

where $F^\sigma{}_\lambda(x)$, $\Gamma^\sigma{}_{\lambda\nu}(x)$, A_μ and $g_{\mu\nu}$, are some functions of x . q , m and c are some parameters. ξ^λ are coordinates of a vector on a manifold with coordinates x^λ , $\lambda = 1, \dots, n$, $x_u = dx/du$. Indices are raised and lowered by $g^{\mu\nu}$ and $g_{\mu\nu}$ respectively, $g^{\mu\lambda}g_{\lambda\sigma} = \delta^\mu_\sigma$. We substitute ξ^σ in (1) by x_u^σ and obtain equation for geodesics

$$\frac{d^2x^\sigma}{du^2} = -F^\sigma{}_\lambda(x)x_u^\lambda - \Gamma^\sigma{}_{\mu\nu}(x)x_u^\mu x_u^\nu.$$

It coincides exactly with equation of motion for a particle with charge q and mass m interacting with electromagnetic and gravitational fields if we choose

$$F_{\mu\nu} = \frac{q}{cm}(\partial_\mu A_\nu - \partial_\nu A_\mu), \quad 2\Gamma_{\lambda,\mu\nu} = \frac{\partial g_{\lambda\nu}}{\partial x^\mu} + \frac{\partial g_{\lambda\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^\lambda}. \quad (2)$$

In this paper we assume these relations and declare A_μ as electromagnetic field and $g_{\mu\nu}$ as gravitational field. These relations between F and A , and Γ and $g_{\mu\nu}$ are proven in [16] and it is shown that A_μ can be identified with electromagnetic field, q with charge, m with mass of a particle interacting with A_μ , c with the velocity of the light, and $g_{\mu\nu}$ with gravitational field.

The corresponding curvature function is

$$\begin{aligned} R^\sigma{}_\lambda &= (\partial_\mu F^\sigma{}_\lambda - \Gamma^\rho{}_{\lambda\mu}F^\sigma{}_\rho + \Gamma^\sigma{}_{\rho\mu}F^\rho{}_\lambda)(x_u^\mu - x_u^\mu) + \\ &\frac{1}{2}(\partial_\nu \Gamma^\sigma{}_{\lambda\mu} - \partial_\mu \Gamma^\sigma{}_{\lambda\nu} + \Gamma^\sigma{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\sigma{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu})(x_u^\nu x_u^\mu - x_u^\nu x_u^\mu). \end{aligned} \quad (3)$$

From (3), we see that gravitational field is coupled to $F^\sigma{}_\lambda$ through covariant derivative

$$\Delta_\mu F^\sigma{}_\lambda = \partial_\mu F^\sigma{}_\lambda - \Gamma^\rho{}_{\lambda\mu}F^\sigma{}_\rho + \Gamma^\sigma{}_{\rho\mu}F^\rho{}_\lambda.$$

We have $g_{\mu\nu}$, A_μ and curvature function to use to find an action for the unified model. First, we construct a tensor from (3)¹

$$R^\sigma{}_{\lambda\mu\nu} = \frac{cm}{4q}(\Delta_\nu F^\sigma{}_\lambda A_\mu - \Delta_\mu F^\sigma{}_\lambda A_\nu) + \frac{1}{16\pi G}(\partial_\nu \Gamma^\sigma{}_{\lambda\mu} - \partial_\mu \Gamma^\sigma{}_{\lambda\nu} + \Gamma^\sigma{}_{\rho\nu}\Gamma^\rho{}_{\lambda\mu} - \Gamma^\sigma{}_{\rho\mu}\Gamma^\rho{}_{\lambda\nu}),$$

¹One may construct different tensors from (3).

where G is gravitational constant. Finally we have a scalar

$$R = g^{\lambda\nu} R^\mu{}_{\lambda\mu\nu}$$

and action

$${}^{\text{eg}}S = \int dx \sqrt{-g} R = \frac{c^2 m^2}{4q^2} \int dx \sqrt{-g} F^{\mu\nu} F_{\mu\nu} + \frac{1}{16\pi G} \int dx \sqrt{-g} {}^{\text{g}}R,$$

$${}^{\text{g}}R = g^{\mu\nu} {}^{\text{g}}R_{\mu\nu}, \quad {}^{\text{g}}R_{\lambda\nu} = \partial_\nu \Gamma^\mu{}_{\lambda\mu} - \partial_\mu \Gamma^\mu{}_{\lambda\nu} + \Gamma^\mu{}_{\rho\nu} \Gamma^\rho{}_{\lambda\mu} - \Gamma^\mu{}_{\rho\mu} \Gamma^\rho{}_{\lambda\nu}, \quad g = \det g_{\mu\nu},$$

where use has been made of

$$\Delta_\nu F^{\mu\nu} = \frac{1}{\sqrt{-g}} \partial_\nu (\sqrt{-g} F^{\mu\nu}), \quad \Delta_\nu g_{\mu\lambda} = 0.$$

Note, that the action is invariant under gauge transformations of fields and general transformations of coordinates. Covariant derivative appears naturally in this formalism. Hence, geometrization principle leads to an action which is invariant under gauge transformations of fields and general transformations of coordinates. We conclude that geometrization principle is more general than gauge principle.

Equation of motion for gravitational field is

$$\frac{c^2 m^2}{4q^2} \left(\frac{1}{2} F^{\rho\sigma} F_{\rho\sigma} g^{\mu\nu} - 2 F^{\nu\sigma} F^\mu{}_\sigma \right) + \frac{1}{16\pi G} (-{}^{\text{g}}R^{\mu\nu} + \frac{1}{2} {}^{\text{g}}R g^{\mu\nu}) = 0. \quad (4)$$

From (4) it follows that ${}^{\text{g}}R = 0$ for $n = 4$ (in the rest of the paper we restrict ourselves to four dimensional spacetime) and (4) becomes

$$\frac{c^2 m^2}{4q^2} \left(\frac{1}{2} g^{\rho n} g^{\sigma m} F_{nm} F_{\rho\sigma} g_{\mu\nu} - 2 g^{\sigma m} F_{\nu m} F_{\mu\sigma} \right) - \frac{1}{16\pi G} {}^{\text{g}}R_{\mu\nu} = 0. \quad (5)$$

We see that electromagnetic field is a source for gravitational field. In the weak gravitational and strong electromagnetic field approximation $g^{\mu\nu} \sim \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and

$${}^{\text{g}}R_{00} \sim -\partial_\mu \Gamma^\mu{}_{00} = -\frac{1}{2} \Delta g_{00}, \quad \Delta = \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^i}, \quad i = 1, 2, 3.$$

The 00 component of equation (5) gives

$$\Delta \Phi = 4\pi c^2 G (E_i E_i + H_i H_i) + O(hF), \quad g_{00} = 1 - 2\frac{\Phi}{c^2}, \quad (6)$$

where Φ is the Newtonian potential, $E_i = \partial_0 A_i - \partial_i A_0$ and $H_i = \frac{1}{2} \epsilon_{ijk} (\partial_j A_k - \partial_k A_j)$ are electric and magnetic fields respectively and ϵ_{ijk} is antisymmetric tensor. Accordingly, total energy of electromagnetic field produces gravitational field.

3 Discussion

For Riemannian geometry it is possible to make a change of coordinates so that $d\xi^\lambda/du = 0$. In new coordinates x' equation for geodesics in Riemannian geometry

becomes $\frac{d^2x'^\sigma}{du^2} = 0$. From physical point of view this corresponds to finding a reference frame where trajectory of particle is a strait line, because this equation must coincide with equation for geodesics. For gravitational interaction we can find a reference frame where gravitational interaction is absent. Accordingly, Riemannian geometry is suitable for gravitational interaction only. For electromagnetic interaction it is not possible to find a reference frame where it is absent. Therefore, all attempts to geometrize electromagnetism or unify it with gravitation in the framework of Riemannian geometry must fail, as it also follows from unsuccessful attempts [3]-[14]. On the other hand for geometry (1) it is not possible to eliminate its right hand side by changing coordinates because of $F^\sigma{}_\lambda$ term. And this property makes it be underlying geometry for the proposed unified model.

In general relativity, geometry underlying gravitation and metric are independent of properties of an interacting particle. This is a consequence of equivalence principle. Geometry and metric depends on the characteristics of sources for gravitational field $g_{\mu\nu}$ only. For electromagnetic interaction there is no equivalence principle, so geometry and metric underling electromagnetism and unified model of electromagnetism and gravitation must depend on characteristics of interacting particles, because particles of different charges move in electromagnetic field differently. If we replace (2) into (1) we obtain

$$\frac{d\xi^\sigma}{du} = -\left(\frac{q}{cm}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \Gamma^\sigma{}_{\lambda\nu}(x)x_u^\nu\right)\xi^\lambda.$$

Accordingly, geometry underlying unified model of electromagnetism and gravitation depends not only on the characteristics of sources for A_μ and $g_{\mu\nu}$ but also on the characteristics of interacting particles q and m . This means that geometry and the length of a curve (metric) $ds = \sqrt{g_{\mu\nu}(x)dx^\mu dx^\nu} + \frac{q}{cm}A_\mu(x)dx^\mu$ are created by interacting particles too, together with sources unlike gravitational interaction.

The proposed theory gives exactly Gravitation when electromagnetic field is equal to zero and Electromagnetism when gravitational field is equal to zero. Equation for geodesics coincides with the equation of motion for a particle interacting with electromagnetic and gravitational fields. There are no additional fields and extra dimensions. The most interesting and important thing is the prediction of the theory. It predicts that electromagnetic field is a source for gravitational field. This prediction can be confirmed by experiments in strong electromagnetic field.

Because, up to now, there is no experimental indication that there might be extra dimensions and unobserved additional fields we accept that this theory is veritable model of four dimensional spacetime with electromagnetic and gravitational fields, provided that its prediction will be confirmed by experiments.

The next step is to describe weak and strong interactions geometrically. We hope that we will be able to find their geometrical model using correspondence between physical properties of interactions and mathematical properties of geometries as it was done for gravitation and electromagnetism in [15]. Geometrization of weak and strong interaction may lead to a different way of quantization because these interactions are quantum processes. This new quantization method, description of quantum processes, can lead to quantization of gravitation. Electromagnetism is known to be a part of experimentally well known unified electroweak theory[17]-[19]. This theory is based on gauge principle. Unfortunately, geometry of weak and strong interactions is not

known yet. So, at the present time there is no known way to unify Yang- Mills theory with the model proposed in the paper using geometrization principle. Also, it is not known how geometrization principle is related to gauge principle. From the results of this paper we can conclude that geometrization principle is more general than gauge principle. Therefore, at the present time we do not know how theories based on gauge principle may appear in the context of theories based on geometrization principle.

In conclusion, we note that General Relativity gives the same relation between electromagnetic and gravitational fields as (4) if we include electromagnetic field in the energy momentum tensor $T^{\mu\nu}$. Energy momentum tensor is a source for gravitational field, $g_{\mu\nu}T^{\mu\nu}$, in General Relativity. Therefore, inclusion of electromagnetic field in $T^{\mu\nu}$ declares it as a source for gravitational field which is the assumption that electromagnetic field is a source for gravitational field. In contrast, our theory predicts that electromagnetic field is a source for gravitational field.

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