4. Find all natural solutions to $x^2 - 17y^2 = 1$

Find the ZSCF for $\sqrt{17}$

$x_0 = \sqrt{17}$ $\qquad a_0 = 4$

$x_1 = \frac{1}{\sqrt{17}-4} = \frac{\sqrt{17}+4}{1}$ $\qquad a_1 = 8$

$x_2 = \frac{1}{\sqrt{17}+4-8} = \frac{1}{\sqrt{17}-4} = \frac{\sqrt{17}+4}{1}$ $\qquad a_2 = 8$

So $\sqrt{17} = [4; \overline{8,8}]$.

0	1		
1	0		
4	1	4	0
33	8	8	1
264	65	8	2

a fundamental solution is $(33, 8)$

So all natural solutions are given by

$$\left\{ \begin{array}{l} x_n = \frac{(33+8\sqrt{17})^n + (33-8\sqrt{17})^n}{2} \\ y_n = \frac{(33+8\sqrt{17})^n - (33-8\sqrt{17})^n}{2\sqrt{17}} \end{array} \right. \quad \Big| \quad n \in \mathbb{N} \Big\}.$$